A Model of Low-risk Piracy

2019

Amil Camilo
University of Central Florida

A MODEL OF LOW-RISK PIRACY

by

AMIL CAMILO
University of Central Florida, 2019

A thesis submitted in partial fulfilment of the requirements
for the Honors in the Major Program in Economics
in the College of Business Administration
at the University of Central Florida
Orlando, Florida

Spring Term, 2019

Thesis Chair: Roberto Burguet

ABSTRACT

Heterogeneous consumers make the decision to buy a durable good or to download a replica, and a monopolist chooses to price and protect their intellectual property in the form of an authentication cost. An optimal price and authentication cost is derived, and shown to be higher than the efficient outcome for a uniform distribution of consumers. The optimal selection of price and protection are shown to be commensurate with his authenticating technology, and the searching ability of consumers. As an extension, a layout for a monopolist problem where consumers have different searching abilities is shown to be indistinct from a homogeneous case when consumers are uniformly distributed.

DEDICATION

Dedicated to Amil in the future, wherever he may be.

ACKNOWLEDGMENTS

I give thanks to Roberto Burguet, who mentored me through this thesis, Eric Schmidbauer, who provided insightful suggestions, and R. Michael Sumpter, who inspired me while watching television. Additional credit is due to all who provided helpful academic or inspirational advice along the way, including Michael Caputo, Naimul Chowdhury, Colin Harris, Spencer Grice, Andrew Hutchens, Jordan Izenwasser, Iliana Moore, Andrew Nevai, Weston Richey, Michael Tseng, and the graduate students in the 2018-2019 master's program, including Kala Bryant, Michael Pabon, and Adam Walter. For emotional support, I again thank Iliana Moore.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

For most software on the Internet, lawful buyers and pirates download the same files, usually at the same speed, and largely have the same user experiences. Because file-sharing allows users to make perfect copies, pirates identically enjoy the benefits of a legal good. The difference between an original and a replica is usually not that the replica has a lower quality, but instead that the original has additional costs. A buyer, in addition to paying a pecuniary cost, is subject to additional authentication costs, including, but not limited to, product key activation, localization restrictions, and online-only use requirements. These authentication costs are intended to protect the intellectual property of a good's producer, and manifest in the form of digital rights management (DRM) technologies that impede buyers from flexibly using software. A pirate, on the other hand, is exempt from these authentication costs because replicas have these components removed to allow them to be freely distributed on the Internet. As a result, original goods, all else equal, are less valuable than their replica counterparts.

With this in mind, producers in a digital market must consider how to optimally protect their intellectual property and price their goods, knowing that consumers will prefer the replicas. Furthermore, social planners must also consider how consumers may be affected by a producer's decisions. In this thesis, I consider a two-period durable goods monopoly where a continuum of consumers decide between buying a good today, and searching for a replica tomorrow. The consumers' payoffs from buying depends on their valuation of the good, and the price and authentication cost level decided by the monopolist. Their payoffs from searching for a replica depend on (again) their valuation, the cost of searching, and the likelihood for a replica to be available in the second period. In this set-up, I assume the authentication cost determines both the level of DRM and the likelihood of replicas being available. I find that the monopolist's selection of authentication cost depends on the cost of searching, the discount factor specified, and the authenticating technology available to him. If the optimal level of authentication cost is positive, it exceeds the socially optimal level unless a social planner chooses to punish the welfare of pirates. I provide a basic framework to extend the model to evaluate consumers with different searching abilities and conclude by considering the model's limitations.

Related literature

This thesis differs from the current literature on piracy by assuming inferior original copies. Johnson (1985) first addressed the subject as a follow-up to Novos and Waldman's (1984) work on copyright protection. Earlier models of "copying" postulate replicas with little quality degradation, while contemporary models emphasize quality differences by assuming the copy is weakly inferior to the original. Bae and Choi (2006) examined the subject by modeling a software pirate facing degradation and reproduction costs, and argued that unlicensed users may be unable to access the complementary qualities of the licensed alternative, including online services, software updates, and technical support. Yoon (2002) makes a similar argument when deriving an optimal level of copyright protection[1], and finds that firms overprotect their goods by producing less than would be socially optimal. The literature on durable goods is also worth considering, especially the work of Coase (1972) on market power. This thesis avoids the Coase conjecture by assuming that the monopolist will commit to a price throughout both periods, against their

[1] In this thesis, the analogue is the authentication cost.

incentive to renege on it. The approach of this thesis abstracts away from other models of piracy by assuming pirates can costlessly avoid substantive legal penalties and malware risks; in the real world, this could be achieved by the use of a virtual private network (VPN) that ensures anonymity, or the use of a private tracker[2] that screens for malware within warez[3] communities. Therefore, this thesis' pirates do not resemble the related literature's since we assume them to be "low-risk".

[2] Private trackers, as opposed to public trackers, are password-protected websites that provide torrents or links to "cracked" (replica) software.

[3] The Warez scene refers to the extended online community of "crack teams" that specialize in ("cracking") reproducing and distributing licensed software by deactivating digital rights management protections.

CHAPTER 2: THE CONSUMER

In this section, I introduce the consumer's decision model and derive the legal demand for the monopolist's good. A continuum of consumers decides to download a durable good. At an initial period, a monopolist releases the good to the public. A crack team produces a replica of the original with some probability of success; if it succeeds, it is released in the second period. If consumers buy the original, they pay prices and authentication costs. If consumers attempt to download the replica, they must wait for the second period and search for it, since it does not release in the first period. If the replica exists, the search succeeds. The level of authentication cost is assumed to measure the level of DRM; higher authentication costs map to a lower likelihood of the replica existing, but a greater burden on the buying consumer.

Let the continuum of consumers be distributed uniformly with a mass of 1, according to their non-negative per-period valuations of the good, $V \in [0,1]$. Let $p, a \geq 0$ be the price and authentication cost decided by the monopolist. Assume payoffs are discounted in future periods by a factor $\delta \in (0,1)$. All consumers decide to buy the original, download the replica, or do neither, whichever maximizes their two-period payoff U. A consumer who buys in the initial period has a payoff of

$$U = (1 + \delta)V - (a + p) \tag{2.1}$$

Let $\alpha(a) \in [0,1)$ be the probability of a replica existing in the second period, where $\alpha'(a) < 0$, $\alpha''(a) > 0$, and $\alpha(0) = 1$. Colloquially, I will refer to $\alpha'(a)$ as the authenticating technology, since it measures the effectiveness of an authentication cost in decreasing the likelihood of the replica existing. Let $s > 0$ be the cost incurred from searching.

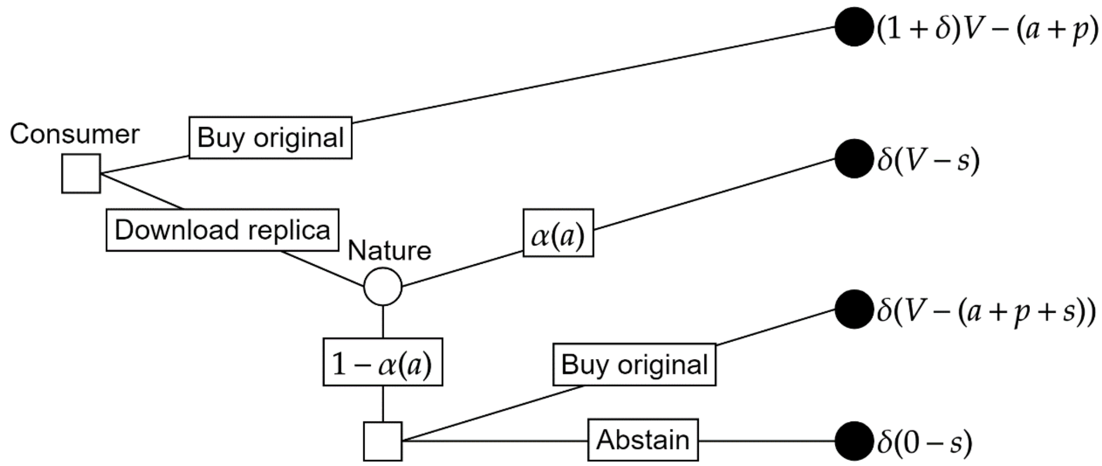

Figure 3.1: A decision tree summary of the consumer's decision problem.

A consumer who attempts to download the replica has a payoff of

$$U = \begin{cases} \delta(V-s), & \alpha(a) \text{ probability} \\ \delta(0-s), & 1-\alpha(a) \text{ probability.} \end{cases} \tag{2.2}$$

If the consumer fails to download the replica, they can still choose to buy in the second period, but with a discounted payoff. In expectation, the consumer's payoff from attempting to download the replica is

$$E(U) = \delta\big(\alpha(a)V + \big(1-\alpha(a)\big)\max(V-(a+p),0) - s\big). \tag{2.3}$$

The legal demand for the monopolist's good

A consumer with valuation $\bar{V} \in (0,1)$ satisfying

$$(1+\delta)\bar{V} - (a+p) = \delta\big(\alpha(a)\bar{V} + \big(1-\alpha(a)\big)\max(\bar{V}-(a+p),0) - s\big) \tag{2.4}$$

is indifferent between buying and attempting to download the replica. Equation (2.4) has two forms depending on whether $\bar{V} > a + p$. If $\bar{V} > a + p$, then there could be a non-monotonic group of first-period buyers and second-period buyers who will buy after failing to download a replica. If $\bar{V} \leq a + p$, on the other hand, a set of consumers buy in the first period while the rest attempt to download the replica in the second period, never buying in the second period, even if the replica does not exist.

Lemma 1. *A consumer who is indifferent between buying and attempting to download a replica has a valuation $\bar{V} \leq a + p$. Explicitly, their valuation is $\bar{V} = \frac{a+p-\delta s}{1+\delta-\delta\alpha(a)}$.*

Proof. Suppose $\bar{V} > a + p$. Then the consumer's expected payoff from downloading the replica is

$$\delta\big(V - \big(1 - \alpha(a)\big)(a + p) - s\big). \tag{2.5}$$

and their valuation satisfies that

$$(1 + \delta)\bar{V} - (a + p) = \delta\big(V - \big(1 - \alpha(a)\big)(a + p) - s\big). \tag{2.6}$$

Solve for $\bar{V}$ in 2.6 and

$$\bar{V} = (a + p)\Big(1 - \delta\big(1 - \alpha(a)\big)\Big) - \delta s \tag{2.7}$$

Since $\bar{V} > a + p$, then $\bar{V} = a + p + \epsilon$, where ϵ is some positive number Then, equation (2.6) becomes

$$a + p + \epsilon = (a + p)\Big(1 - \delta\big(1 - \alpha(a)\big)\Big) - \delta s \tag{2.8}$$

Solve for ϵ in (2.8) and

$$\epsilon = (a + p)\big(\delta(\alpha(a) - 1)\big) - \delta s \tag{2.9}$$

But since $\alpha(a)$ is at highest 1, the RHS is either zero or negative, which contradicts that $\epsilon > 0$. Therefore, $\bar{V} \leq a + p$ and the consumer's expected payoff from downloading the replica is

$$\delta(\alpha(a)\bar{V} - s), \tag{2.10}$$

and their valuation satisfies that

$$(1 + \delta)\bar{V} - (a + p) = \delta(\alpha(a)\bar{V} - s). \tag{2.11}$$

Solve for $\bar{V}$ in (2.11) and

$$\bar{V} = \frac{a + p - \delta s}{1 + \delta - \delta\alpha(a)} \tag{2.12}$$

Which is the intended result. $Q.E.D.$

Note that for any $V > \bar{V}$ the payoff-maximizing decision rule is to buy, and for any $V < \bar{V}$, the rule is to download the replica. The result is that all consumers who intend to buy do so in the first period, and (equivalently) all consumers who fail to download the replica always choose to abstain from buying. Recall that V is uniformly distributed from 0 to 1. Then, the mass of consumers with a $V > \bar{V}$, meaning the mass of buyers, is $1 - \bar{V}$.

CHAPTER 3: THE MONOPOLIST

In this section, I define the monopolist's profits and derive the optimal price as a function of his selected authentication cost. I also consider the optimal authentication cost level. The monopolist commits to a price and an authentication cost in the first period and seeks to maximize his profits. He observes a demand equal to the mass of buying consumers $1-\bar{V}$. The monopolist faces no production costs or fixed costs at the period of decision making, since development costs are already sunk.

The monopolist's profits are defined as

$$\pi = p(1-\bar{V}) \tag{3.1}$$

The firm selects (a,p) to maximize its profits; since $\bar{V}$ depends on the selection of (a,p), $\bar{V}$ is treated as a function. The firm's optimal profits are

$$\pi^* = \max_{a,p} \pi = p\big(1-\bar{V}(a,p;\delta,s)\big). \tag{3.2}$$

In order for π to have sufficient concavity for a global maximum on the domain, assume that the determinant of the Hessian matrix of π is positive, meaning,

$$\det(H) = \begin{vmatrix} -2\dfrac{\partial\bar{V}}{\partial p} & -\left(\dfrac{\partial\bar{V}}{\partial a}+\dfrac{\partial^2\bar{V}}{\partial a\partial p}\right) \\ -\left(\dfrac{\partial\bar{V}}{\partial a}+\dfrac{\partial^2\bar{V}}{\partial a\partial p}\right) & -p\dfrac{\partial^2\bar{V}}{\partial a^2} \end{vmatrix} > 0 \tag{3.3}$$

or explicitly, simply that

$$2p\delta\alpha''(a)(a+p-\delta s)\big(1+\delta-\delta\alpha(a)\big)+4p\delta\alpha'(a)\big(1+\delta-\delta\alpha(a)+\delta\alpha'(a)(a+p-\delta s)\big)$$
$$-\big(1+\delta-\delta\alpha(a)\big)^2-2\delta\alpha'(a)(a+p-\delta s)\big(1+\delta-\delta\alpha(a)\big)-$$
$$\delta^2\alpha'(a)^2(1+a+p-\delta s)^2>0 \quad (3.4)$$

for all positive (a,p). No intuitive guarantee is provided that this assumption will be satisfied.

The optimal price and authentication cost level

Proposition 1. *For a given a, the optimal price p^* exists and generates positive revenues if and only if $a+\delta\alpha(a)<1+\delta+\delta s$. In that case, $p^*=\frac{1}{2}(1+\delta-\delta\alpha(a)+\delta s-a)$.*

Proof. Differentiate equation (3.1) wrt to p and set it equal to 0, treating $\bar{V}$ as a function.

$$\frac{\partial\pi}{\partial p}=-p\frac{\partial\bar{V}}{\partial p}=1-\frac{a+2p-\delta s}{1+\delta-\delta\alpha(a)}=0 \quad (3.5)$$

Solve for p in equation (3.5) and

$$p=\frac{1}{2}(1+\delta-\delta\alpha(a)+\delta s-a). \quad (3.5)$$

Differentiate (3.1) wrt to p twice.

$$\frac{\partial^2\pi}{\partial p^2}=-\frac{2}{1+\delta-\delta\alpha(a)}<0 \quad (3.6)$$

The second derivative is negative[4] for all parameters in the domain. Therefore, since it satisfies the necessary and sufficient conditions for a maximum in p, irrespective of the assumption in (3.3), the p satisfying equation (3.5) maximizes π globally, i.e., this $p=p^*$. $Q.E.D.$

---

[4] This will also imply that, in order to satisfy the assumption in (3.3), $\frac{\partial^2\pi}{\partial a^2}=-p\frac{\partial^2\bar{V}}{\partial a^2}<0$.

Note that from Proposition 1, the monopolist can only choose a limited amount of a that generates positive revenues, regardless of whether an a is optimal. If $a = 0$, then the firm generates profits from its pricing strategy since $1 + \delta s > 0$, but if a became sufficiently high, up to where $a + \delta\alpha(a) = 1 + \delta + \delta s$, then $1 - \bar{V} = 0$. For a given a, the optimal price will also depend on the discount factor δ and the cost of searching s. By the envelope theorem, an increase in s and `δ induces a greater demand for the original, which then enables the monopolist to charge higher prices with a greater mass of buyers.

Proposition 2. *An authentication cost level* $a_0 > 0$ *that makes* $-\alpha'(a) = \frac{1}{\delta\bar{V}}$ *is the optimal authentication cost level* a^* *and it decreases in s when using the pricing strategy* p^*.

Proof. Differentiate (3.1) wrt to a and set it equal to 0.

$$\frac{\partial\pi}{\partial a} = -\frac{p\big(1 + \delta - \delta\alpha(a) + \delta\alpha'(a)(a + p - \delta s)\big)}{1 + \delta - \delta\alpha(a)} = 0 \tag{3.7}$$

Since $p > 0$ and $1 + \delta - \delta\alpha(a) > 0$, then we derive the necessary condition that

$$1 + \delta - \delta\alpha(a) + \delta\alpha'(a)(a + p - \delta s) = 0 \tag{3.8}$$

Solve for $-\alpha(a)$ in (3.8) and

$$-\alpha'(a) = \frac{1 + \delta - \delta\alpha(a)}{\delta(a + p - \delta s)} \tag{3.9}$$

Recall that $\bar{V} = \frac{(a+p-\delta s)}{1+\delta-\delta\alpha(a)}$. Then, equation (3.9) may be rewritten as

$$-\alpha'(a) = \frac{1}{\delta\bar{V}} \tag{3.10}$$

To confirm that this a is the maximum, note that from the assumptions in (3.3), π is concave over the domain and so any (a, p) that satisfies $\frac{\partial\pi}{\partial p} = 0$ and $\frac{\partial\pi}{\partial a} = 0$ will maximize π. Suppose the firm is at this maximum, so $(a, p) = (a_0, p^*(a_0))$. Then the optimal profit is:

$$\pi^* = p^*(\cdot)\left(1 - \frac{a_0 + p^*(\cdot) - \delta s}{1 + \delta - \delta\alpha(a_0)}\right). \tag{3.11}$$

With some algebra and substitution, one can obtain

$$= \frac{((1 + \delta - \delta\alpha(a_0) + \delta s - a_0)^2)}{\left(4\big(1 + \delta - \delta\alpha(a_0)\big)^2\right).} \tag{3.12}$$

Since a_0 is an interior solution, then the following identities and inequalities hold:

$$\frac{\partial\pi^*}{\partial a_0} = -\left(\frac{1 + \delta - \delta\alpha(a_0) + \delta s - a_0}{4\big(1 + \delta - \delta\alpha(a_0)\big)^2}\right)$$
$$\Big(2\big(1 + \delta - \delta\alpha(a_0)\big) + \delta\alpha'(a_0)(1 + \delta - \delta\alpha(a_0) + a_0 - \delta s)\Big) \equiv 0, \tag{3.13}$$

$$\frac{\partial^2\pi^*}{\partial a_0^2} = \frac{1 + \delta - \delta\alpha(a_0) + \delta s - a_0}{4\big(1 + \delta - \delta\alpha(a_0)\big)^2}$$
$$\big(\delta^2\alpha'(a_0)^2 + \delta\alpha'(a_0) - \delta\alpha''(a)(1 + \delta - \delta\alpha(a_0) + \delta s - a_0)\big) < 0. \tag{3.14}$$

By the implicit function theorem, note that $a_0 = a_0(\delta, s)$ around the neighborhood of a_0. From equation (3.13), we can then derive:

$$2\left(1 + \delta - \delta\alpha\big(a_0(\delta, s)\big)\right)$$
$$+\delta\alpha'\big(a_0(\delta, s)\big)\big(1 + \delta - \delta\alpha\big(a_0(\delta, s)\big) + a_0(\delta, s) - \delta s\big) \equiv 0. \tag{3.15}$$

Differentiate equation (3.15) wrt s and

$$2\delta\alpha'\big(a_0(\delta,s)\big)\left(\frac{\partial a_0}{\partial s}\right) - \delta\alpha''\big(a_0(\delta,s)\big)\left(\frac{\partial a_0}{\partial s}\right)\big(1+\delta-\delta\alpha\big(a_0(\delta,s)\big)+a_0(\delta,s)-\delta s\big)$$
$$-\delta\alpha'\big(a_0(\delta,s)\big)\left(\delta\alpha'\big(a_0(\delta,s)\big)\left(\frac{\partial a_0}{\partial s}\right)+\frac{\partial a_0}{\partial s}-\delta\right)\equiv 0 \tag{3.16}$$

Which can be arranged into:

$$\frac{\partial a_0}{\partial s}\big(\delta^2\alpha'(a_0)^2+\delta\alpha'(a_0)-\delta\alpha''(a)(1+\delta-\delta\alpha(a_0)+\delta s-a_0)\big)\equiv \delta^2\alpha'\big(a_0(\delta,s)\big) \tag{3.17}$$

so that solving for $\frac{\partial a_0}{\partial s}$ gets to

$$\frac{\partial a_0}{\partial s}\equiv\frac{-\delta^2\alpha'\big(a_0(\delta,s)\big)}{\delta^2\alpha'(a_0)^2+\delta\alpha'(a_0)-\delta\alpha''(a)(1+\delta-\delta\alpha(a_0)+\delta s-a_0)}. \tag{3.18}$$

The numerator of the RHS is positive while the denominator is negative. Therefore, the expression is negative and, around the neighborhood of a_0, a higher cost of searching s has a negative effect on the optimal a_0, that is, $\frac{\partial a_0}{\partial s}<0.\ Q.E.D.$

δ	s	a^*	p^*	π^*	$1-\bar{V}$	$\bar{V}$	$\alpha(a^*)$
0.8	0.0	0.1432	0.7234	0.3292	0.4550	0.5450	0.2627
0.8	0.1	0.1380	0.7612	0.3666	0.4816	0.5184	0.2745
0.8	0.2	0.1325	0.7982	0.4062	0.5089	0.4911	0.2881
0.8	0.7	0.0964	0.9721	0.6384	0.6567	0.3433	0.3998
0.8	1.2	0.0000	0.9800	0.9604	0.9800	0.0200	1.0000
0.7	1.0	0.0499	0.9601	0.7257	0.7559	0.2441	0.6157
0.5	1.0	0.0373	0.8077	0.5661	0.7009	0.2991	0.6954
0.3	1.0	0.0058	0.6557	0.4226	0.6445	0.3555	0.9419
0.2	1.0	0.0000	0.6000	0.3600	0.6000	0.4000	1.0000

Table 3.1: Optimal outcomes with given δ and s, assuming $\alpha(a) = (a+1)^{-10}$

In this case, a greater authentication cost a is inversely related with the cost of searching s. If s increases, less consumers download the replica, so the use of a is less necessary. With the right conditions[5], however, specifically that the cost of searching is sufficiently low so that $s < \frac{(2+\delta\alpha(a)(1+\delta-\delta\alpha(a))+\delta\alpha'(a)a)}{\delta^2\alpha'(a)}$, the monopolist could also choose, if unconstrained, the highest a available, or even $a = 0$ if s is sufficiently high so that $s > \frac{(2+\delta\alpha(a)(1+\delta-\delta\alpha(a))+\delta\alpha'(a)a)}{\delta^2\alpha'(a)}$, for all a.

Example 1. Let $\alpha(a) = (a+1)^{-10}$, starting with $\delta = 0.8$, and $s = 0.1$. Then, the optimal price is $p^* \approx 0.7612$ and the optimal $a^* \approx 0.138$. With a mass of consumers of 1, approximately

[5] As a brief proof, solve for s in equation (3.13) and then rewrite it as an inequality such that $\frac{\partial\pi}{\partial a} > 0$ in the first case, and $\frac{\partial\pi}{\partial a} < 0$ in the second.

0.4816 buy the legal good ($\bar{V} \approx 0.5184$) and the likelihood of the replica existing $\alpha(a^*) \approx 0.2745$. The firm's optimal profits $\pi^* \approx 0.3666$. At a higher cost of searching, say $s' = 0.2$, then $p^* \approx 0.7982$ and $a^* \approx 0.1325$. For a summary around different (δ, s), see Table 3.1

CHAPTER 4: THE SOCIAL PLANNER

In this section, I define the social planner's total welfare function and compare the optimal level of authentication cost with the necessary conditions for the efficient outcome. The social planner wishes to maximize the total welfare of the monopolist and his consumers.

A social planner observes total welfare, defined as

$$W = c_1 \int_0^{\bar{V}} \max\big(\delta\big(\alpha(a)\bar{V} + \big(1-\alpha(a)\big)\max(V-(a+p),0) - s\big),0\big)\,dV$$
$$+c_2 \int_{\bar{V}}^{1} \max(V-(a+p),0)\,dV + c_3\pi(\cdot), \tag{4.1}$$

where c_i, $i = 1,2,3$ are the weights the planner assigns to the surplus of consumers who download replicas, the surplus of consumers who buy originals, and the profits of the monopolist. Assume that $c_1 \leq c_3$ and $c_2, c_3 > 0$. The social planner selects a to maximize total welfare, while the monopolist is still permitted[6] to select p. The efficient authentication cost level $\hat{a}$ is defined as

$$\hat{a} = \arg\max_a W \tag{4.2}$$

Note that by Lemma 1, $\bar{V} \leq a + p$, so (4.1) can be rewritten as just

$$W = c_1 \int_0^{\bar{V}} \max(\delta(\alpha(a)V - s),0)\,dV + c_2 \int_{\bar{V}}^{1} \max(V-(a+p),0)\,dV \; + c_3\pi(\cdot). \tag{4.3}$$

[6] Since this is a monopolistic model, the efficient price $\hat{p}$ is trivially less than p^*. The results from this chapter are contingent on the necessary conditions of the solution, and so would hold even if the social planner could also select the price p.

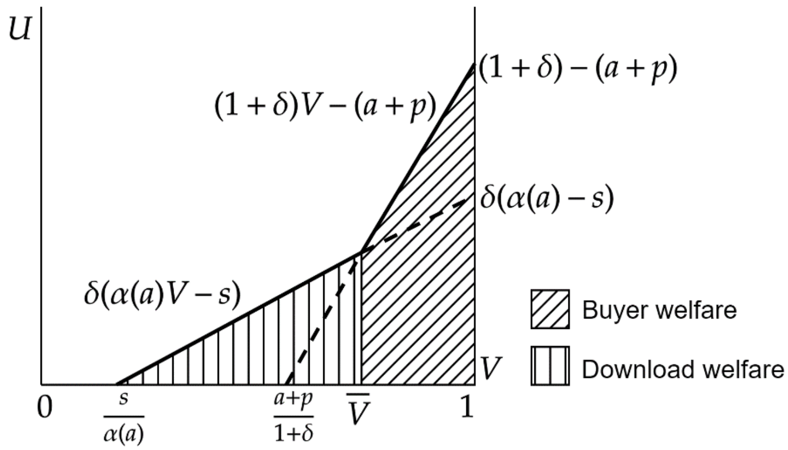

Figure 4.1: The consumer's payoffs depending on valuation, and their aggregated welfare.

Note that since each integrated function is linear in V, each welfare term can be rewritten as sums of variable right triangles and rectangles (see Figure 4.1), explicitly

$$\int_0^{\bar{V}} \max(\delta(\alpha(a)V - s), 0)\, dV = \frac{s}{\alpha(a)} \times (0) + \frac{1}{2}\left(\bar{V}(\cdot) - \frac{s}{\alpha(a)}\right)\left(\delta(\alpha(a)\bar{V}(\cdot) - s)\right)$$
$$= \frac{\delta}{2}\left(\alpha(a)\bar{V}(\cdot)^2 - 2s\bar{V}(\cdot) + \frac{s^2}{\alpha(a)}\right), \tag{4.4}$$

and

$$\int_{\bar{V}}^{1} \max(V - (a+p), 0)\, dV = \frac{a+p}{1+\delta} \times 0 + \left(1 - \bar{V}(\cdot)\right)\left((1+\delta)\bar{V}(\cdot) - (a+p)\right)$$
$$= \left(1 - \bar{V}(\cdot)\right)\left(\frac{1}{2}(1+\delta)(\bar{V}+1) - (a+p)\right). \tag{4.5}$$

Then, by combining (4.4) and (4.5), equation (4.3) can be rewritten as

$$W = c_1 \frac{\delta}{2}\left(\alpha(a)\bar{V}(\cdot)^2 - 2s\bar{V}(\cdot) + \frac{s^2}{\alpha(a)}\right)$$
$$+c_2\big(1-\bar{V}(\cdot)\big)\left(\frac{1}{2}(1+\delta)(\bar{V}+1)-(a+p)\right)+c_3\pi(\cdot). \quad (4.6)$$

The efficient authentication cost level

Proposition 3. *Let $a^* = a_0$, that is the interior solution for an optimal a and assume that $c_1 \geq 0$. Then a decrease in a^* is welfare improving, and if $\hat{a}$ is unique, $\hat{a} < a^*$.*

Remark 1. Furthermore, a necessary condition so that $a^* = \hat{a}$ is that $c_1 < 0$.

Proof. The first-order necessary conduction for $\max_a W$ is $W' = 0$. Then, it would be sufficient to show that, so long as $c_1 \geq 0$, $W'(a_0) < 0$, and so if $\hat{a}$ is within the neighborhood of a_0, $\hat{a} < a^*$. Let $a = a_0$ and differentiate equation (4.6) wrt a (note, since a_0 is an interior solution to maximizing π, $\frac{\partial \pi}{\partial a} \equiv 0$ at $a = a_0$.

$$W' = c_1\left(\frac{\delta}{2}\left(\alpha'(a_0)\bar{V}(\cdot)^2 + 2\alpha(a_0)\bar{V}(\cdot)\left(\frac{\partial \bar{V}}{\partial a}\right) - \frac{\alpha'(a_0)s^2}{\alpha(a_0)^2}\right)\right)$$
$$+c_2\big(1-\bar{V}(\cdot)\big)\left(\frac{1}{2}(1+\delta)\left(\frac{\partial \bar{V}}{\partial a}\right)-1\right)-c_2\left(\frac{\partial \bar{V}}{\partial a}\right)\left(\frac{1}{2}(1+\delta)(\bar{V}(\cdot)+1)-(a_0+p)\right) \quad (4.7)$$

Note that $\frac{\partial \pi}{\partial a} = -p\left(\frac{\partial \bar{V}}{\partial a}\right) = 0$, then $\frac{\partial \bar{V}}{\partial a} = 0$ given $p > 0$. As a result, equation (4.7) simplifies to:

$$= c_1\left(\frac{\delta}{2}\left(\alpha'(a_0)\bar{V}(\cdot)^2 - \frac{\alpha'(a_0)s^2}{\alpha(a_0)^2}\right)\right) - c_2\big(1-\bar{V}(\cdot)\big)$$

$$= c_1\left(\frac{\delta}{2}\alpha'(a_0)\left(\bar{V} - \frac{s}{\alpha(a_0)}\right)\left(\bar{V}(\cdot) + \frac{s}{\alpha(a_0)}\right)\right) - c_2\big(1-\bar{V}(\cdot)\big) < 0$$

Since $\bar{V} \geq \frac{s}{\alpha(a_0)}$ if $\delta(\alpha(a)\bar{V}(\cdot) - s) \geq 0$. $Q.E.D.$

For the social planner, the monopolist is overprotecting their good from downloading. Note that this is true even if the social planner is a "lawful" regulator that does not value the welfare of pirates, that is, if $c_1 = 0$. On the other hand, a hawkish regulator that wishes to actively punish pirates, that is a regulator with $c_1 < 0$, may actually see the optimal outcome as efficient. This is because even if this authentication cost level harms the buyers, the total welfare function benefits from decreasing the payoff of pirates.

CHAPTER 5: DISCUSSION AND CONCLUSION

In this section, I discuss the model's economic content and elaborate on its implications. I conclude by pointing out the model's limitations.

Criminality, perverse incentives, and risk

While the model does provide reasons to *not* download replicas, namely, to avoid the wait and uncertainty of costly searching, there are also other reasons that are not encapsulated by the model. In the United States, copyright infringement is punishable up to ten years of imprisonment in addition to a $250,000 fine (18 U.S.C. § 2319). There is also statistical evidence that suggests that online piracy increases exposure to malware and spyware[7]; after all, there is no reason why the incentives of the crack team producing the replica should be benign, especially when they purport to offer replicas at no cost to the consumer. It may be that many consumers avoid piracy with these risks in mind: to avoid legal consequences, damaging their computers, or even moral discomfort. Nonetheless, it may be the case that the magnitude of some of these risks, while nonzero, is rather small. Aggregating websites that provide streaming content would have trouble bringing unregistered users to a legal authority, especially when it is in their best interest to continue hosting infringing content[8]. As I mentioned before, the use of VPNs can also be used to minimize the likelihood of a legal encounter. As for the issue of malware, an extension to the model may be necessary --- while I do not provide this extension, I think this could be incorporated, as a starting point, by accounting for differences in searching ability. Since the basic model does not account

[7] See Telang (2018).
[8] See Institute for Information Law (2018) for more on how streaming affects rates of piracy.

for these considerations, its specifications exaggerate the consumer's propensity to download replicas, and bias the welfare of consumers toward the positive direction.

Cost of searching and homogeneous searching ability

In the consumer's utility, the cost of searching is exogenous, which implies that all consumers have the same searching ability. There is *some* aspect of piracy that is exogenous to the consumer; for example, popular search engines, such as Google, regularly remove content results that infringe copyright[9]. In the social planner's model, Proposition 4 implied that the monopolist overprotects their intellectual property in comparison to the efficient protection level, much in the same way that, by virtue of its market power, it prices too highly. A social planner, though, could also curb the negative effects of a by deciding (or distorting)[10] s. The Digital Millennium Copyright Act (DMCA) criminalizes the distribution of such content on the Internet, in addition to outlawing "cracking" DRM itself. These policies certainly distort the parameter s for all consumers, regardless of their endowments. Even so, consumers in the real world certainly differ in education and computer savvy, which may make some consumers better pirates than others. As I hinted in the previous section, some crack teams may want to defraud consumers by injecting viruses and computer worms into replica files, and so some pirates may want to identify the difference between a "genuine" replica and an "infected" replica. In the appendix, I provide a basic model for a continuum of consumers with both heterogeneous valuations and searching abilities, as an extension. As a conjecture, it's possible that with some monotonic threshold $\tilde{s}^*$, a consumer with

[9] See Google (2019).
[10] In the heterogeneous case, this could be achieved with some parameter θ that affects the distribution of $\tilde{s}$ and thus affects $E(\tilde{s})$.

an $\tilde{s} < \tilde{s}^*$ could update a prior belief about the authenticity of a replica based on either an involuntary signal from the crack team, or a signal provided by some curator. The extended model I provide does not change the monopolist's solution, although it may alter the form of the social planner's welfare function.

Additional limitations

To conclude, there are other limitations to the model in addition to the homogeneity in searching and the welfare bias. As I pointed out in the introduction, this thesis does not address the market power of the monopolist, and his incentives to renege on his price and authentication cost commitments. The model also does not portray the competition the monopolist may face from other firms selling differentiated goods. Even though, legally, the monopolist has an exclusive right to sell the good, there may be other players in the market selling similar substitutes that affect the demand. The model also does not consider the case where the replica may release before an original, or general situations with "leaks". Finally, this model does not look at an infinite time horizon, wherein consumers may repeat their decision and learn from the decisions of other consumers (or the past choices of the monopolist), nor does it consider the motivations of the crack team, whatever they may be.

APPENDIX A: EXTENSIONS

Extension: heterogeneous searching abilities

Let the continuum of consumers (of density 1) have uniformly distributed valuations $V \in [0,1]$ and, instead of cost of searching, have searching abilities $\tilde{s} > 0$ independently distributed according to F, where F is a density function with derivative f.

Then, the demand for the original replica is:

$$\int_0^\infty \int_{\bar{V}(\cdot)}^1 f(\tilde{s})\, dV d\tilde{s} = \int_0^\infty \left(1 - \bar{V}(\cdot)\right) f(\tilde{s}) d\tilde{s} = E\left(1 - \bar{V}(\cdot)\right)$$
$$= E\left(1 - \frac{a + p - \delta\tilde{s}}{1 + \delta - \delta\alpha(a)}\right) = 1 - \frac{a + p - \delta E(\tilde{s})}{1 + \delta - \delta\alpha(a)} \qquad (A.1)$$

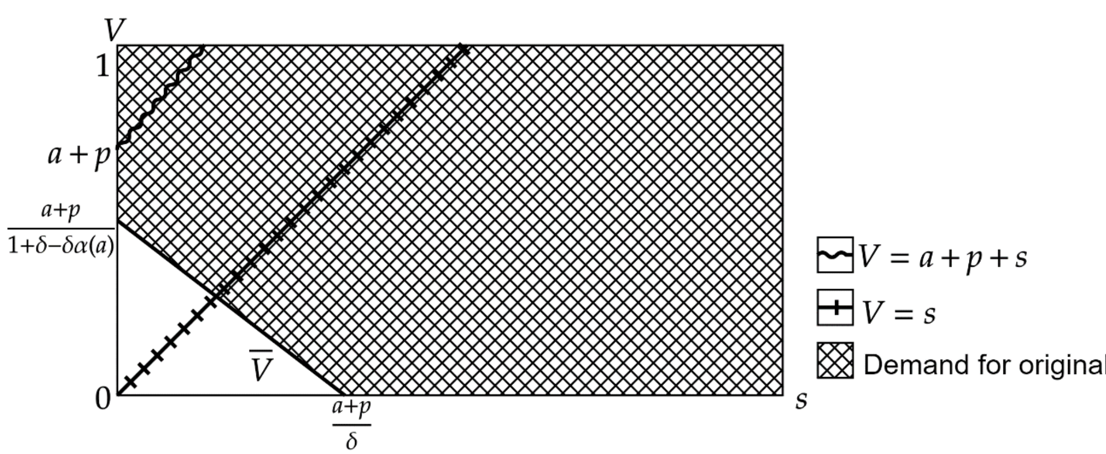

Figure A.1: The projection in $(\tilde{s}, V)$ of demand with heterogeneous searching abilities.

Therefore, the monopolist's profits would be

$$\pi^{\tilde{s}} = p\left(1 - \frac{a + p - \delta E(\tilde{s})}{1 + \delta - \delta\alpha(a)}\right) \qquad (A.2)$$

And so, all the solutions to the monopolist's problems described in Chapter 3 are also solutions to the problem with heterogenous searching ability, that is,

$$\arg\max_{a,p} \pi^{\tilde{s}}(a, p; \delta, \tilde{s}) \equiv \arg\max_{a,p} \pi\big(a, p; \delta, E(\tilde{s})\big). \qquad (A.4)$$

LIST OF REFERENCES

[1] 18 U.S.C. § 2319.

[2] Sang Hoo Bae and Jay Choi. A model of piracy. Information Economics and Policy, 2006, vol. 18, issue 3, 303-320

[3] Ronald Coase. Durability and Monopoly. University of Chicago Law School, 1972.

[4] Google. Content removals due to copyright. Google Transparency Report, 2019.

[5] Institute for Information Law. Global Online Piracy Study. University of Amsterdam, July 2018.

[6] William R. Johnson. The Economics of Copying. Journal of Political Economy, Vol. 93, No. 1 (Feb. 1985), pp. 158-174

[7] Ian E. Novos and Michael Waldman. The Effects of Increased Copyright Protection: An Analytic Approach. Journal of Political Economy, Vol. 92, No. 2 (Apr. 1984), pp. 236-246

[8] "Public Law 105-304: Digital Millennium Copyright Act." Oct. 28, 1998.

[9] Rahul Telang. Does Online Piracy make Computers Insecure? Evidence from Panel Data. Carnegie Mellon University, 2018.

[10] Kiho Yoon. The optimal level of copyright protection. Information Economics and Policy, 2002, vol. 14, 327-348