EFFECTS OF A MATHEMATICS CURRICULUM RICH IN SPATIAL-REASONING ACTIVITIES ON FIFTH-GRADE STUDENTS'ABILITIES TO SPATIALLY REASON: AN ACTION RESEARCH PROJECT

by

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ABSTRACT

The purpose of this study is to describe the effect of a curriculum rich in spatial reasoning activities and experiences on the ability of my fifth grade students to spatially reason. The study was conducted to examine 1) the effects of my practice of incorporating spatial reasoning lessons and activities in my fifth-grade mathematics classroom on the students' ability to spatially reason and 2) the effects of my practice of incorporating spatial reasoning lessons and activities on my students' ability to problem solve. Data were collected over a ten-week period through the use of student interviews, anecdotal records, photos of student work, student journals, pre- and posttests and a post-study survey.

In this study, students demonstrated a statistically significant increase on all pre- and posttests. The student interviews, anecdotal records, photos of student work, and student journals all revealed spatial reasoning was used in mathematics problem solving. The study suggests that spatial reasoning can be taught and spatial reasoning skills can be used in problem solving.

This research is dedicated to my loving husband, Mitch, for his unending support.

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CHAPTER 1

RATIONALE FOR THE STUDY

This study investigated my practice, in my mathematics classroom, of exposing students to lessons and activities that involved spatial reasoning on their ability to spatially reason. Spatial ability involves the ability to see in one's mind the manipulation and rotation of objects. I specifically looked at my practice by targeting tasks that depend on proficiency in rotating, i.e.: visualizing and imagining objects in altered positions.

In 1990, while teaching a grade 3 and 4 multiage class, I began to expose students to spatial relation activities. Dr. Grayson Wheatley, a professor at FSU, had presented a series of workshops for teachers in Orange County. He demonstrated, through his lessons, the importance for students to work on spatial reasoning. In a focus issue of NCTM's *Arithmetic Teacher* he stated, "Spatial sense is indispensable in giving meaning to our mathematical experience" (1990, p. 11). Dr. Wheatley introduced me to Quick Draws (Wheatley, 2004), a series of activities in which students draw figures after seeing them for three seconds and then describing what they saw. Students tend to visualize the figures in different ways. As the students describe in words how they visualized the figures, the activity becomes a rich learning experience as each student discovers new ways to see the figure. While some see familiar objects, others see geometric shapes and patterns. In another activity, the students are challenged by predicting what a folded piece of paper will look like when opened with cuts made in the body. Incorporating Wheatley's methods changed the way I thought of mathematics. I realized mathematics that visualization as well as an intellectual understanding of figures and is as important as visualization. A student of mathematics must see on paper as well as visualize in the mind's eye. I noticed that my own ability to spatially reason was impaired by my lack of experience in my mathematics education background. As I worked more and more, my skills improved. I have wondered whether spatial skill can be improved with experience or is it an innate ability. This question is the basis of my study.

The National Council for Teachers of Mathematics in the *Principles and Standards for School Mathematics* (NCTM, 2000) stated "Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science" (p. 23). Specifically, the standards for grades 3-5 (NCTM, 2000) include

Use visualization, spatial reasoning, and geometric modeling to solve problems:

- Create and describe mental images of objects, patterns and paths
- Build and draw geometric objects
- Identify and build a three dimensional object from two dimensional representations of that object
- Identify and draw a two-dimensional representation of a three-dimensional object
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life
- Use geometric models to solve problems in other areas of mathematics such as number and measurement (NCTM, 2000, p.164)

The development of my research in spatial reasoning is driven by my belief that this skill has a direct impact on overall mathematics achievement. Students able to visualize geometric relationships and use their spatial sense to problem solve, do better on other mathematical tasks requiring visualization such as regrouping in addition and subtraction. Research has shown that visual spatial ability has been found to be a sound predictor in the success a child will have in mathematics (Capraro, 2001; Clements, 1998; Tartre, 1990). In his book, *Spatial Ability: Its Educational and Social Significance* (1964), Smith described studies that correlate spatial ability with mathematics ability. Connor and Serbin (1985), working with seventh and tenth graders, concluded that visual-spatial skills can be taught and there is a positive correlation between spatial ability and mathematics achievement.

The NCTM standards emphasize the importance of spatial sense throughout the mathematics curriculum. Yet instruction for the sole purpose of strengthening spatial ability is at a minimum in the United States' mathematics curriculum in the elementary grades. The Third International Mathematics and Science Study (TIMSS) (Hiebert et al., 2003), conducted by Stigler and Hiebert, compared mathematics achievement and teaching methods of 38 countries. "Eighty-six percent of U. S. eighth-grade students reported that they worked from worksheets or textbooks on their own almost always or pretty often during mathematics lessons in 1999, which was higher than the international average of 59 percent" (Stigler & Hiebert, 2004, p. 14). Although worksheets can provide spatial skill practice, the ones referred to contain computation and procedural practice. The TIMMS study indicates a lack of opportunity for students to explore spatial relationships in U.S. classrooms. Spatial sense, which is critical to making mathematical connections, is ignored when students are taught procedures. The use of imagery and using one's own ability to visualize as a problem solving strategy is ignored. Stigler and Hiebert further stated,

> In the United States, teachers implemented none of the making connections problems in the way in which they were intended. Instead, the U.S. teachers turned most of the problems into procedural exercises or just supplied students with the answers to the problems. (p.15)

There is no procedure to follow to visualize spatial relations; one must rely on connections made through experiences and the ability to make mental images.

There are teaching methods and experiences that can provide opportunities for students to improve their spatial sense. These experiences include manipulating, rotating, visualizing, and transforming objects in space.

> When children examine the result of combining two shapes to form a new shape, predict the effect of changing the number of sides of a shape, draw a shape after it has been rotated a quarter or half turn, or explore what happens when the dimensions of a shape are changed, they acquire a deeper understanding of shapes and their properties. Such activities promote spatial sense. (NCTM, 1989, p. 45)

Purpose of the Study

The purpose of this study is to describe the effect of a mathematics curriculum rich in spatial reasoning activities and experiences on the ability of my fifth-grade students to spatially reason. The specific questions I attempted to answer were

Question #1: Does my practice of incorporating spatial ability lessons and activities in my classroom affect the students' ability to spatially reason?

Question #2: Does my practice of incorporating spatial reasoning lessons and activities in my classroom affect my students' ability to use this ability in their mathematics problem solving?

Definitions for the Study

Abstraction- the process by which the mind selects, coordinates, unifies, and registers in memory a collection of mental items or acts that appear in the attention field (von Glasserfeld, 1995).

Aviation and Aerospace Magnet School- a school in which the students are exposed to science lessons and activities that integrate all aspects of science to aerospace.

Imagery- constructions derived from viewing objects, reading a passage, or just reflecting and are influenced by what we know (Wheatley, 1990).

Re-presentation- the calling up of an image, which may not remain in consciousness but must be called up or represented again (Wheatley, 1990).

Spatial reasoning skills- those mental skills concerned with understanding, manipulating, reorganizing, or interpreting relationships visually (Tartre, 1990).

Spatial ability- the ability to judge the relations of objects in space, to judge shapes and sizes, to mentally manipulate objects, to visualize the effects of putting objects together, and to mentally turn objects over or around (Haydel, 1998).

Visual-spatial ability- a cognitive skill involving the ability to perceive spatial relationships and to manipulate visual material mentally (Connor & Serbin, 1985).

Visualization- the collective abilities of reasoning, figural classifications, and figural relations (Tartre, 1990)

Visualization factor- “the ability to imagine how pictorially presented objects will appear when they are rotated” (Halpern, 1986, p. 49).

Significance of the Study

> Spatial competence is a central aspect of human adaptation. Spatial knowledge is essential to life in the world, since anything concretely existing in the world must have spatial location—perhaps not a known one, but at least a potentially knowable one.” (Newcombe & Huttenlocher, 2000, p. 67)

Students use the ability to reason spatially in every subject, every day. It is essential to navigating one's location and being successful in assigned tasks. Rhoades (1981) stated it clearly:

> The ability to create a mental image of an object and then to manipulate it mentally has significant practical application in fields such as mathematics, physics, architecture, engineering and design. It is documented that success in these fields is highly predicted by one's ability to visualize and manipulate objects. (p. 247)

Studies have shown a positive relationship between mathematics achievement and the ability to spatially reason (Brown & Wheatley, 1989; Connor & Serbin, 1985; Fennema, 1974). Improving fifth grade students' ability to spatially reason might increase their mathematics achievement in general. Educators can directly instruct students through games and activities to improve their spatial ability thereby increasing their performance in mathematics.

The purpose of this study is to describe the effect of a mathematics curriculum rich in spatial reasoning activities and experiences on the ability of my fifth-grade students to spatially reason. I believe this study is important in that it will provide me with more knowledge of the better methods of teaching my students. This study will add to the research knowledge for whether or not spatial reasoning can be taught or is an innate ability one naturally possesses. A review of the literature is discussed in Chapter 2.

CHAPTER 2

REVIEW OF THE LITERATURE

Overview

A review of the relevant literature is presented in this chapter. The areas to be discussed include the theoretical framework for this study, studies involving spatial reasoning, studies involving mathematics achievement and spatial reasoning, and studies involving spatial reasoning and environmental factors.

Theoretical Background

This study was designed to explore the effect that exposing students to spatial reasoning lessons and activities would have on their ability to spatially reason. Studies done by Jean Piaget have served as a basis for other researchers. I will examine Piaget's foundational work and current researchers' criticisms of it.

Jean Piaget, a Swiss psychologist, believed that spatial awareness evolves in developmental stages. As a child progresses through one stage successfully, he goes on to the next; from topological to projective to Euclidean. Piaget's writings in *The Child's Conception of Space* (Piaget & Inhelder, 1948/1967) are quite complex on this subject. In his description of the topological stage, the child passively explores his environment. The child may touch an object and explore it to receive a tactile perception and yet touch another part of the object without making a connection that the two parts are the same object. The comparison can be made to the book *Seven Blind Mice* by Ed Young (1992), in which the blind mice touch the elephant at the tail, the tusks, the trunk, etc., and draw different conclusions about what it might be. The conclusions drawn lack connectedness and result in false perceptions. Only through the child's ability to interact with their environment can the child progress spatially. Piaget and Inhelder stated, "Hence the abstraction of shape is achieved on the basis of co-ordination of the child's actions and not, or at least entirely, from the object direct" (p.43).

In transitioning from perceptual knowledge to representational, Piaget used children's drawing to judge their level of development. The drawings would mirror what the child saw. For example, the child's representation of a face would indicate the position of the nose, mouth and eyes in relation to each other. Piaget and Inhelder (1948/1967) wrote, "…everyone has seen the kind of drawings which children produce between the age of 4 and 8, showing chimneys perpendicular to sloping roofs and men at right-angles to hills they are supposed to be climbing" (p. 379). They asserted that this drawing indicates the child is a long way from reaching the perceptual stage. Many researchers (Clements, 1998; Clements & Battista, 1992; Newcombe, 1981) have criticized Piaget's use of children's drawing to explain spatial ability. Clements and Battista countered by stating, "An obvious objection to such arguments is that inaccuracies might be attributable to motor difficulties" (p.423).

During the projective space stage, which begins at about age 7, the child begins to connect ideas about space. Children begin to draw from other points of view. Piaget explored this through the Three Mountains experiment. The child draws the mountains from a doll's different points of view. This experiment is similar to the child drawing configurations of snap cubes from the top, side, and front on triangular dot paper, a spatial ability lesson commonly used today.

In this stage, Piaget worked with geometric nets, in which the child would predict the shape of a three dimensional object from a two dimensional or flat object. Rotating and manipulating a figure in one's mind were demonstrated through this activity. The ability to mentally visualize the rotation is a key to one's success in spatial ability.

The third or Euclidean stage marks the point when the child can perceive similar relationships. The Euclidean stage is the culmination of the topological and projective stages. Piaget and Inhelder (1948/1967) presented tilted bottles in the Water Level Test, with a designated amount of liquid in them. The child must draw a line at which the liquid will level when the bottle is laid flat. Only older children were able to predict the water level. Their "frame of reference" for horizontal and vertical conception was developed.

Piaget and Inhelder (1948/1967) set out to prove the stages one must pass through to develop spatial reasoning. As Clements (1998) stated when analyzing the work of Piaget and Inhelder, "Children's ideas develop from intuitions grounded in *action*-building, drawing, moving and perceiving" (p.4). The inclusion of visual representations throughout the mathematics curriculum could result in increased understanding of mathematics concepts.

At the core of this study is the question, "Can spatial reasoning skills be taught?" The life work of Lev Vygotsky, born the same year as Piaget, 1896, is an integral part of the premise for my research. Vygotsky's Cultural-Historical Theory (Leong & Bodrova, 2001) states that child development is based upon the interactions of children and their social environment. The social environment includes the objects around them including the toys they play with. Research studies (Brosnan, 1998; Hill & Redden, 1984; Robert & Heroux, 2004) have shown that the toys and games children play with have a correlation to their ability to spatially reason. Children that play outdoor games such as baseball, play with Legos or with computer games such as Tetris, score higher on spatial reasoning tests. Vygotsky's belief that development is impacted by activities and toys supports the idea that spatial ability can be taught.

The difference between Piaget and Vygotsky is the way they view learning. While Piaget (1948/1967) believed that learning occurs in stages, Vygotsky saw learning as the child's ability to make sense of his environment and culture. According to Vygotsky, the Zone of Proximal Development (ZPD) is the zone where children can make instructional strides with the appropriate support (Leong & Bodrova, 2001). My study contains varied activities in which the student collaborates with a partner within their ZPD to make sense of spatial challenges. As one child "sees it," the other does too. While my study will use Piaget's work as a guide to the way students progress in their understanding of spatial reasoning, Vygotsky, a constructivist, serves as a model for effective teaching of spatial concepts.

Studies on Spatial Reasoning

Spatial reasoning studies tend to focus on mathematics achievement/ performance and spatial reasoning ability (Capraro, 2001; Clements & Del Campo, 1989; Fennema & Sherman, 1978; Hegarty, 1999; Reuhkala, 2001; Seng & Chan, 2000; Seng & Yeo; 2000; Tartre, 1990; Wheatley, Brown, & Solano, 1994), spatial reasoning and trainability in spatial skills (Baenninger & Newcombe, 1989; Blatter, 1983; Connor & Serbin, 1985; De Lisi & Wolford, 2002; Rhoades, 1981; Robichaux & Guarino, 2000; Sanz de Acedo Lizarraga, 2003; Vasta, Knott, & Gaze, 1996), or spatial ability and environmental factors (Brosnan, 1998; De Lisi &

Wolford, 2002; Dixon, 1997; Hill & Redden, 1984; McClurg & Chaille, 1987). My research includes these areas; therefore I reviewed the literature within each area.

Spatial Ability and Mathematic Achievement

The relationship between spatial reasoning ability and mathematics achievement was researched by Seng and Chan (2000). In their study, 127 randomly selected fifth graders' scores on the Comprehensive Test of Basic Skills (CTBS) and the Spatial Relations-Orientation Test (SR-O) and the Spatial Visualization Test (Vz) were compared. A significant positive relationship was found between mathematic achievement and spatial ability. Seng and Chen concluded that "Visual skills appear to have an important role in mathematical performance" (p.6).

Connor and Serbin (1985) used five visual-spatial tests from the Educational Testing Service Kit of Factor Referenced Tests (Ekstrom, French, Harman, & Derman, 1976) to study the relationship between different types of visual-spatial skills and mathematics achievement, including computation, algebra, and geometry. The Factor Referenced Tests included the Paper Folding Test, in which participants predict how the paper will look when it is unfolded with holes punched in it, the Card Rotations and Cube Comparisons Tests, which both measure spatial orientation; and the Vocabulary Test. A section of the Differential Aptitude Test (Bennet, Seashore, & Wesman, 1974) also measured spatial ability. Connor and Serbin administered a test developed by the researchers and used those results plus standardized test scores and school grades to measure mathematic achievement.

Although the findings show a stronger relationship between visual-spatial ability and mathematics achievement for boys than for girls, the ability to reason spatially is strongly tied to mathematic ability. Connor and Serbin (1985) also concluded that tests that directly measure visual-spatial ability were good predictors of mathematic achievement.

Tartre (1990) researched how spatial orientation skills are used to solve mathematical problems. The purpose of the study was to investigate the nature of the relationship between spatial skills and mathematics; to understand if spatial skills enable learners to organize their thoughts differently when problem solving.

A sample of 97 tenth graders were administered the Gestalt Completion Test (Ekstrom et al., 1976) and a mathematics achievement test (Houghton Mifflin, 1971). Fifty-seven students who had scored in the top or bottom third of the Gestalt Test were participants in the study. The students were presented with various visual problems, for example to calculate the area of an irregular figure on grid paper without counting the grid spaces. The students were interviewed and asked to explain the way they solved the problem. Interviews of one hour revealed the students' thinking during and after the process.

The results (Tartre, 1990) indicate that 10% of the low spatially oriented group were able to get the correct answer before a hint was given and 41% of the high spatially oriented group were able to find the correct answer. The low spatially oriented group needed the hint more often to think through to the solution. The high spatial group was able to come up with strategies more often, e.g., marking off sections of the figure to calculate the area. Tartre concluded, "The two significant spatial orientation group differences found for nongeometric problems . . . support the thesis that spatial orientation skill is manifested more broadly than just in geometric or visually presented contexts" (p.227). The study asserts that spatial skill may be an indicator of how one organizes mathematical thoughts, and therefore the skill's value may be generalized to mathematic performance in all areas.

In contrast to Tartre, Lean and Clements (1981) found that the wide use of visual images used in problem solving can lead to erroneous solutions. A study of 116 students entering the engineering program at the University of Technology in New Guinea found that the verbal logical students outperformed the spatial students on both mathematical and spatial tests. Lean and Clements explained their findings by stating the following:

> The results of the present study appear to be in conflict with other studies which suggest that it is desirable to use visual processes when attempting mathematical problems. However, this apparent conflict could be due to the use, in the present study, of straightforward, routine tasks on the “Pure” and “Applied” Mathematics tests, whereas in most other relevant studies difficult, non-routine mathematical word problems have been used. (p. 296)

The qualifying statement by the researchers (Lean & Clements, 1981) suggests that spatial reasoning skills become more important as the mathematics becomes less procedural. Although the study appears to be in conflict, it supports the theory that more complex problem solving requires spatial reasoning skills.

Hegarty’s (1999) study sought to explain the results of Lean and Clements (1981) by classifying two types of visual spatial representations in mathematical problem solving. Schematic representations involve rotating and manipulating objects and pictorial imagery, the construction of visual images to solve problems. The research was done in Ireland with 33 sixth-grade boys in an all boys’ school. The results show students who scored high in visual spatial schematic tests had a high association with mathematics achievement levels. Those students who scored well in pictorial imagery showed a negative relationship to mathematic achievement. Hegarty asserted that this finding may be the reason for the results of Lean and Clements’ study. Their use of pictorial imagery with the participants limited the findings and ignored the schematic representation. Hegarty concluded, “Instructing students to try to ‘visualize’ mathematical problems will probably not be successful. Instead, instruction should encourage students to construct spatial representations of the relations between objects in a problem and discourage them from representing irrelevant pictorial details" (p. 8).

Seng and Yeo (2000) studied 173 students in Singapore and found spatial ability performance to be independent of learning modes (concrete, abstract, reflective or active), learning dimensions (perception or processing), or learning styles. This study negates the need to categorize students when assessing spatial reasoning skills.

Wheatley, Brown, and Solano (1994) conducted a long term study of fifth graders with a follow-up study in tenth grade. The purpose of the study was to explore whether or not students who used imagery and were successful in mathematics in fifth grade continued to use imagery to solve problems in tenth grade. Thirty-two of the original 54 students participated in the follow up. The Wheatley Spatial Ability Test (WSAT) was administered at fifth grade and again in tenth grade. Tasks included the use of tangrams and visual problems. The results show that students who relied upon spatial reasoning to problem solve in fifth grade continued to use the same strategies to problem solve in tenth grade. Wheatley et al. (1994) found that although the use of imagery was not encouraged in the years since the initial fifth-grade study, the students' reliance on imagery to problem solve remained constant.

In a study done with third graders by Clements, Battista, Sarama, Swaminathan, and McMillen (1997) using a computer program called Geo-Logo (Clements & Meredith, 1994) it was found that "students with connected schemas had more powerful and flexible solution strategies at their disposal for solving spatial problems" (p.92). The schemas refer to the student's ability to connect spatial and numerical thinking to solve problems. Clements, Battista, Sarama, Swaminathan, and McMillen suggested that students who use spatial sense in harmony with numerical reasoning will perform better on mathematical tasks. The connection of mathematical ideas is a valuable mathematical activity.

Many of the reviewed studies conclude that there is a strong relationship between spatial ability and achievement in mathematics. The results of the reviewed research have inspired the work of this study. The possibility of improving spatial reasoning skills and thereby improving mathematical reasoning is a significant reason for the study.

Training and Spatial Ability

Numerous studies have linked the effect of training through spatial activities and lessons on one's ability to spatially reason.

Ben-Chaim, Lappan, and Houang (1988) studied a large sample of 1,000 fifth through eighth graders in a Midwestern town to explore the effect of instruction on spatial reasoning skills. The three-week unit of instruction was developed by the Middle Grades Mathematics Project. Instruction revolved around the manipulation of cubes and drawing the views of the cube configuration from the top, left, and right sides of the cube collections. Spatial ability was evaluated with the Spatial Visualization Test (SVT). The study was done at three sites: the first included all fifth- and sixth-grade students; the second, all students except exceptional education and half of the sixth-grade students; and third, all sixth grade students and 6 of 11 seventh-grade students.

The study (Ben-Chaim et al., 1988) found that with instruction intervention, the middle school students gained significantly in their scores on the spatial reasoning test after the training. According to the study, the optimal time for teaching spatial reasoning is in seventh grade. Although no items from the test were taught directly during the study, the participants had improved scores. The findings show that spatial reasoning can be learned and is not necessarily an innate ability. Given the findings of studies linking mathematics achievement to spatial ability, the implications are crucial for educators to understand. The inclusion of spatial activities and lessons could improve overall mathematics achievement.

While the Ben-Chaim et al. (1988) study investigated if there were differences between the trainability between males and females, there were not. Both sexes improved through training in a proportional way. Training consisted of a three-week unit developed by the Middle Grades Mathematics Project. Students were to work with small cubes to build, draw the base, front view, and side view of “buildings” made with the cubes. Additionally, the students used isometric paper to draw the different views.

Rhoades (1981) conducted a study to correlate whether or not spatial ability was trainable. The hypothesis had not been previously proven to his satisfaction. In fact, even after his initial study, he conducted another study, fearing the first one was not sufficient to correlate his theory. The initial study included 142 freshmen at Rice University. The Guilford-Zimmerman Spatial Visualization Test (Guilford & Zimmerman,1953) and the Identical Blocks Test (Stafford, 1962) were given to test spatial reasoning skills. Training consisted of mental rotation exercises. The correlation between pre- and posttest was .81, indicating a significant increase. Further, since students with varied majors were used in the study, it should be noted that students with a greater number of mathematics and physical science courses made greater gains in their pre- and posttest scores. Rhoades (1981) replicated the study and produced similar results.

The question can training affect one’s spatial ability was studied by Baenninger and Newcombe (1989). Their work, similar to many others, began with the question of sex differences in spatial ability. Finding no difference in trainability between the sexes, their work in this study included the question of trainability. The study revealed that test-specific training yielded improved results over indirect training. However, overall all participants who received training had improved test scores. They concluded that there was a reliable relationship between spatial activity participation and spatial ability.

Findings from the study of Ferrini-Mundy (1987) were not consistent with the findings of the previously discussed studies. The study investigated the affects of spatial training on achievement in calculus. A random sample of 334 students was selected from 1054 students pre-registered for a calculus course at a university. The seven classes made up two treatment classes and one control group. The treatment groups were presented six lessons by audiotape and worksheets. One treatment group used manipulatives, the other did not. The control group had no spatial training. Training consisted of mental rotations from two to three dimensional objects. The results did not support an increase in spatial ability through training.

> The case for enhancing college students' spatial scores through the type of training employed in this study was not supported by the results, but the feasibility of fostering improvement in college students' spatial scores through other means is clearly indicated. (p. 136)

Blatter (1983) found inconclusive results when exposing students to spatial training. The study included 48 ninth-grade students who were identified as gifted. Spatial ability was measured by the Differential Aptitude Test (1974) and the training program was adapted by Daily and Neyman (1967). The control group continued with their regular course of study in literature. The trained group received ten one-hour sessions of instruction in spatial relations over a period of four weeks. Significant improvement in the scores of the experimental group after training suggests students can be trained to improve their spatial ability.

Connor and Serbin (1985) found results that support training as a means of increasing spatial ability. The participants were 231 boys and 203 girls from eighth-grade mathematics classes in junior high schools in upstate New York. The training materials progressed from simple, concrete tasks to complex demanding tasks that took place in half-hour sessions. The sessions included students matching two-dimensional drawings to three-dimensional figures, predicting what a folded paper would look like when opened and manipulating tangrams. Pre- and posttests include the ETS (1976) Cube Comparisons Test, the Form Board Test, and the Paper Folding Test. These tests were administered after each training session. The study concluded that junior high students can improve their visual–spatial skills with brief training sessions. It should be noted that the study also found that it is easier to train the skill of spatial orientation than the skill of spatial visualization.

Professional articles have been written to show a relationship between spatial ability and training incorporating drawing (Olkun, 2003; Orde, 1997). Orde asserted that the skills needed to draw go hand in hand with those needed for spatial success. The ability to see proportion and perspective relate to spatial/visual criteria; noting shape, detail, and action relate to orientation; and the receding space, grouping, and imagination relate to visualization. Orde stated

> Drawing and spatial abilities share common conceptual ground. Both have been associated with similar cognitive functions. They are related in mental function through a relationship in which drawing is dependent on spatial ability and spatial ability may be enhanced through drawing. (p.276)

The work of Piaget (Piaget & Inhelder, 1948/1967) was the beginning of the connection between art and spatial ability. His work with preschoolers' drawing what they saw spatially is duplicated in Orde's work.

Clements, Battista, Sarama, and Swaminathan (1997) investigated the development of students' spatial thinking in a unit on geometric motions and area. The idea was to chart growth of spatial skills as the unit progressed. The curriculum used for the study was created as part of a curriculum development project funded by the National Science Foundation. The curriculum is based on the constructivist belief that presenting problem solving situations to students will result in strategies conceived by the students themselves and not prescribed by the teacher. The curriculum included spatial activities on the computer, including the ability to manipulate tetrominoes. Spatial ability was measured through the administration of the Wheatley Spatial Aptitude Test (WSAT, 1978). Twenty-three students from one suburban school and two urban schools participated in the field test. During the study, Clements, Battista, Sarama and Swaminathan reported, “They improved in their ability to use spatial imagery and physical motions to provide a convincing argument for congruency” (p. 176). Since the WSAT test deals with the transformation of internalized images, it was shown that the curriculum had a positive effect on test results. The researchers believe that the constructivist philosophy, like Vygotsky’s, will foster the development of spatial reasoning.

The training used for these studies was the impetus for my study. Mental rotation, drawing, and the use of tangrams, Legos®, and cubes are all part of the training I provided my participants. My goal was to include as many different sources as possible, to have the best of the studies I reviewed.

Spatial Training and Environment

A study that investigated the reason why Chinese students perform better on spatial tasks than American students was done by Li (2001). The study included interviews of 29 Chinese teachers with varying years of experience. The questions and responses included:

1. In China, what life experience might facilitate the development of spatial ability to solve the Water Level Task?

The question refers to the work of Piaget and Inhelder (1948/1967) in which students had to describe the water level of tilted bottles. The response was that containers used in China are transparent. Children in rural areas carry containers over their shoulders. Therefore children have real world experience with tilted water bottles.

2. What aspects of education in China might facilitate students' spatial ability?

Subtle changes in the characters used for the alphabet must be carefully formed by children at a young age. These subtle differences foster right brain development.

3. Specifically, what content from what curricula might facilitate the development of spatial ability?

The small living space in the home enables children to pay more attention to their surroundings than American children.

4. What teaching process, methods, equipment or environment might facilitate the development of spatial ability?

Traditional Chinese culture focuses on philosophy, relational and imagery thinking.

Li believed that the exceptional spatial abilities of the Chinese people support Piaget and Inhelder's assertion (1948/1967) that a "frame of reference" is needed to develop spatial sense. The Chinese culture provides an anchor for spatial sense to develop. In a sense, the culture serves as a source of training for the society. Despite the fact that a formal method of teaching spatial sense is not established, the culture itself supports spatial training methods.

In addition to cultural differences, several studies (Brosnan, 1998; De Lisi & Wolford, 2002; Dixon, 1997; Haydel, 2000; Hill & Redden, 1984; McClurg & Chaille, 1987) have attempted to make a connection between spatial ability and activities with which children participate. Legos®, computer games, and athletics have all been researched to test their role in spatial training.

De Lisi and Wolford's (2002) study focused on the mental rotation ability of third-grade students. During one month of instruction, students participated in eleven class sessions of spatial activity computer games such as Tetris, while the control group played Carmen San Diego, a non-spatial computer game. The French Card Rotations Test (1976) was used to evaluate increases in spatial ability. Students who were considered low ability in the Tetris group outscored the high ability participants in the control group on the posttest. This study provides evidence that a spatial training method using the computer can improve the mental rotation ability in third-grade children.

The research of Caldera, Culp, O'Brien, Truglio, Alvarez, and Huston (1999) does not support the theory that children who play with blocks are more adept at spatial skills than those children who do not play with blocks. The study was conducted with sixty children who attended nine university- affiliated preschool classrooms. They ranged in age from 47 to 69 months of age. The play ranged from unstructured block play to structured block play. The Wechsler Preschool and primary Scale of Intelligence for Children (1967) was used to evaluate the child's ability to analyze and reproduce abstract forms. The children who scored high in spatial performance did show a preference for art activities.

Summary

Several studies have provided evidence that spatial ability is linked to mathematics achievement. The ability to train students and thereby improve their spatial ability has been researched and documented. Exposing students to tasks and games that require spatial skill can improve spatial reasoning. From the research studies reviewed and the theoretical framework discussed, the basis for my study has been established.

This study goes beyond the research done by Ben-Chaim et al. (1988) by including spatial tasks and lessons taught directly and indirectly. Computer experiences, games, puzzles, lessons, and activities used in my study expanded the scope of the training of the studies I've reviewed.

The testing instruments used in this study were taken from recommendations used in several of the reviewed studies (Clements, Battista, Sarama & Swaminathan, 1997; Connor & Serbin, 1985; De Lisi & Wolford, 2002). The specific instruments will be discussed later in this study.

The environment for this study reflected the constructivist teachings of Vygotsky. The tasks were open ended and students created their own meanings and solutions.

The selection of the tasks and lessons were a result of the reviewed literature and NCTM publications and on-line resources. The methods and procedures used in this study will be discussed in Chapter 3.

CHAPTER 3

METHODOLOGY

This study: (1) investigated the effects of my practice of incorporating spatial relations activities and lessons into my mathematics teaching on my students' ability to spatially reason and (2) explored the effect of my practice of incorporation of spatial relations activities and lessons into my mathematics teaching on my students' problem solving strategies. This was an action research study which drew on the strengths of quantitative and qualitative research methods. Mills (2003) defined action research as "any systematic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teaching/learning environment to gather information about how their particular schools operate, how they teach, and how well their students learn" (p.5). The study employed triangulated methods of data collection with student journals, anecdotal notes and student interviews, a post-study survey, and pre- and posttests.

Design of the Study

The design of this study utilized both qualitative and quantitative methods for collecting data to acquire a detailed description of students' ability to increase their spatial reasoning abilities. The quantitative data included pre- and posttests using the Wheatley Spatial Ability Test (WSAT, 1976) and the Paper Folding Test, the Cube Comparisons Test and the Card Rotation Test (Ekstrom et al., 1976). The qualitative data included student journals, student interviews, anecdotal records, and post-study surveys. The triangulation of data increased the credibility and trustworthiness of the results. The student journals included responses and reactions to the spatial lessons and activities given to the participants. The journals served as a springboard for discussion between me and the participants. In addition, they helped to explain the students' thinking. The student interviews gave a one-on-one view into the reasoning behind the product. Students had to develop a language to represent their thoughts. Anecdotal records were kept by me during and after the lessons. These records provided me with information I could reflect on as well as a record of the day's tasks. The post-study surveys provided insights into how the participants viewed the study. I consider their self concept in the mastery of spatial reasoning tasks as important as the test results.

Limitations

Several weeks into the study, a fourth-grade teacher at the school resigned. The result was that a fifth-grade teacher was reassigned to fourth grade. Five of the displaced fifth-grade students were placed in my classroom. These students are not a part of the findings of the study, since they did not have permission to participate. Their presence, at times, limited the attention that I gave to my participants. They did not have the "buy in" to the study and often served as a distraction. One additional student was moved from another fifth-grade classroom to my room during the later part of the study.

During the study, our community was hit by three major hurricanes. The hurricanes were the cause of a loss of school time and disruption of the school year. As a result the study lacked the continuity for which I had planned.

Setting

This study was conducted in a suburban central Florida public elementary school. The school was built in 1960 on 17 acres of land. The school population was approximately 479 students in grades K-5. There were ninety staff members including instructional, clerical, educational paraprofessional, extended day, custodial, and cafeteria personnel. In addition to the regular K-5 curriculum, the site serves as an Exceptional Education Center School providing services to students with special needs, including physical handicaps. Art, music, physical education and science lab are offered as special area classes. The students are distributed ethnically as follows: 36.7% White, 14.4% Black, 43.8% Hispanic and 5% other. The mobility rate is fairly high at 50%.

During the 2001–2002 school year, the school became a magnet school for Aviation and Aerospace. Students outside the school's designated zones can apply for enrollment. Bus service is provided. The process to enroll includes the completion of an application, current teacher recommendation, letters of interest from the student and parent, and approval from the selection committee. Students in the magnet program receive 85 minutes of instructional time in the science lab per week as part of their special area time. Several of the field trips foster knowledge of aviation and aerospace. Classroom instruction includes the study of topics in aviation and aerospace.

The participants consisted of 16 students from my fifth-grade class. They were randomly selected from 34 magnet students. During the course of the study, one student returned to his home school, exiting the magnet program. Shortly after that, six students from two other non-magnet classes were added to my class. These six students were not included in the study, although they did participate in many of the lessons and activities on spatial reasoning.

The classroom is totally self contained. The students receive 90 minutes of mathematics instruction time per day. The spatial reasoning lessons and activities were in addition to the regular mathematics instruction time. While mathematics instruction is the first instruction in the day, the spatial lessons took place for the most part in the afternoon. For six weeks, spatial lessons and activities were presented two to three times a week. These lessons and activities would last about 20–30 minutes. Students worked independently with privacy screens or with a partner, depending on the goal of the lesson.

The desks were arranged in groups of four, two by two facing each other. Each student was given a folder that served as a journal for keeping materials and observations.

Procedure

The research design process began in July 2004 with the submission and approval of a proposal to the University of Central Florida Institutional Review Board (IRB). The principal of the school approved the study before it was sent to the IRB for approval. On the night of Meet the Teacher, prior to school's official opening, the parents signed permission forms for their children to participate. A child assent script form was read to the participants to affirm consent from the participants to be a part of the study. All forms are included in the appendixes.

Measures

At the beginning of my ten-week study, I received permission and administered several pretests. These same tests were administered at the end of the study. The Wheatley Spatial Ability Test (WSAT, 1996) is a 100-item test designed to measure the ability to mentally rotate and compare figures. A figure is pictured to the left of the page and participants must decide whether the subsequent five congruent figures match it. Figures match if they can be rotated; they do not match if they were flipped. The test is normed for grades three, four, and five. It is a valid and reliable test with a high internal consistency (K-R= .92). The test has a time limit of eight minutes.

In addition, the participants were pre- and posttested on three tests from the Kit of Factor-Referenced Cognitive Tests (Ekstrom et al., 1976). The Card Rotation Test, the paper folding test, and the Cube Comparisons Test were used to measure students' visualization. The Card Rotation test, similar to the WSAT, requires the participant to determine whether the eight figures are the same or different from the original figure on the left. If the figures have been rotated, an "S" is shaded in; if it has been flipped a "D" is shaded in. The Paper Folding Test requires the student to visualize in three dimensions. The student imagines a piece of paper being folded and holes are punched through all the thicknesses according to drawings. The student selects the appropriate result from five choices when the paper is opened. The Cube Comparisons Test pictures two blocks, the kind children play with. The six-sided block has a different design or number on each face of the block. Three of the faces—the front, a side, and the top—can be seen on the first block; the second block is flipped or turned. The student must decide whether the flipped or turned block is the same or different from the first block. All three of these test consists of two sections, three minutes long.

Lessons and Activities

Quick Draw

Quick Draw is an activity developed by Wheatley (2004). A figure is presented on the overhead projector for three seconds. The student draws the figure using visualization skills. The same figure is shown for three more seconds. After a minute, the figure is shown again. The richness of this activity arises from students' sharing how they remembered how to draw the figure. They reveal what they saw in their "mind's eye." As students explain what they saw, they affirm each other's point of view and see the figure in a new way. The descriptions of the figure often include rotation of the figure. Quick Draws were completed 3 to 4 times a week prior to the start of our daily mathematics lesson. Students verbally shared what they saw with the class and wrote notes describing what they saw.

Pentominoes

Pentominoes are pentagonal polygons. Before I distributed the pentominoes, the students worked with a partner to arrange color tiles in as many different ways as they could to create five-sided figures. Each figure had to be completely different with no rotations or reflections. The five-sided figures were drawn and cut out on centimeter paper. When they were satisfied that all possibilities were represented, they were given a set of pentominoes to compare their results with.

After the introduction of pentominoes, the participants were given a perimeter of a shape drawn with solid lines. The objective was to fill in the space with various pentominoes. Visualization skills were developed by attempting ways to fill in the space completely without going outside the lines. This activity was done as a group for four days. The students recorded which ones were more challenging and why. They shared strategies for solving the puzzle. Pentominoes were done as a class and left on the table to be worked with individually at students' leisure.

Tangrams

Tangrams are an ancient Chinese set of triangles, a parallelogram, and a square, seven pieces in all. I introduced tangrams by reading the book *Grandfather Tang's Story* by Ann Tompert (1990) aloud. In this book, a grandfather is telling a story to children, and he describes how various animals are changing into one another. As each transition occurs, the tangrams are changed to depict each animal. I modeled these animals with magnetic tangrams as the participants imitated the shapes with their own tangrams. This activity provided immediate success and experience with the tangrams. Tangrams were an open-ended activity that lasted for the entire time of the study. In the lessons to follow, students were given a series of outlines to create with their own tangrams. A book was provided with 500 shapes to make with tangrams. The tangrams and the book were left on the table to be interacted with as desired.

Thinkcards

The Aims Education Foundation (Wiebe, 1997) published a book titled *Spatial Visualization*. One of the activities in the book uses rhombuses of three colors to create the illusion of three-dimensional cubes. The rhombuses are places in a predetermined outlined shape to show a continual pattern of top, side, and front views of the cube. To begin, the students were given the tri-colored rhombuses and a shape outline. On the overhead projector, I modeled, without verbally explaining, how the top of the cubes should be all the same color, the sides should be the same of a second color and the fronts should be the same of a third color. I repeated this modeling with the participants copying and observing several times. Following the modeling lessons, the students were given the rhombuses with an outline to fill in with cubes independently. The students checked their results with the model on the overhead. The students were given outlines to fill in on hard copies. When the solution was found, they colored in isometric paper to match their solution. This activity lasted four days.

Spatial Problem Solving

Paper-folding activities provided practice in visualization. An 8 1/2 x 11-inch paper was folded in half and in quarters. Cuts were made on the folds and the edges. Students drew their predictions of how the paper would look when it was opened. Additional practice was provided by the *Spatial Problem Solving With Paper Folding and Cutting* book (1984) by Davidson and Willcutt. Students matched a cutout to the paper it originated from. Another activity featured a design on the left side of the paper. Five pictures depicted what the paper would look like folded. The student was to select one. These activities correlated with the paper folding on the ETS Test.

Flip Flop Triangles

A series of three linear squares with diagonals dividing each into four triangles was provided for each participant. The participants' task was to manipulate the triangles, without cutting, into various configurations. Students answered questions about how they created the configurations.

Grid Game

The participants were given a game board with a grid picturing a series of squares divided diagonally into four triangles. The players took turns filling in one of the triangles. Each player used a different color. Players were given one point for creating a square, two points for a triangle, and three points for a rectangle. Each shape had to be constructed by shading in four triangles with one color. If the opponent created a square, triangle, or rectangle and didn't see it, his opponent could claim it. This game developed visual rotation. Anecdotal notes were used for data collection.

Building A Box

One of the lessons that provided applying spatial skills to problem solving was Building a Box. The student created as many different nets as they could to create a three-dimensional box with centimeter grid paper. Student interviews, written questions, and anecdotal records were used for data collection.

Math Stars

Math Stars are a collection of problem solving and spatial reasoning mathematics extensions created by the North Carolina Department of Education. Students were given these once each week to provide problem solving that involved spatial reasoning skills.

Computer Activities

The students were taken to the computer lab to practice with online tangrams. This enabled them to manipulate and rotate each piece to create the desired shape.

Online geoboards were also accessed to create various polygons. Students were able to use the pegs of the geoboard to demonstrate rotations, reflections, and translations.

The classroom had Legos®, jigsaw puzzles, and other types of hands-on puzzles. Nets were available for students to fold to create three-dimensional figures. All materials were available for students to use whenever they had time or at dismissal time. Seven of the classes' magnet students leave thirty minutes early to take the bus, which allowed time for others to check out materials or work on the computer.

Summary

The pre- and posttests administered as well as the lessons and activities were analyzed and recorded to show students' understanding of spatial reasoning. All data were analyzed to look at trends and themes at the end of the study. The findings of the research were analyzed and are discussed in Chapter 4.

CHAPTER 4

DATA ANALYSIS

Introduction

Throughout this research study, my goal was to collect data that would inform me of how my practice of exposing students to lessons and activities in spatial reasoning would affect students' ability to spatially reason. A personal goal was to improve students' spatial ability capabilities and thereby observe them using their spatial ability in mathematics problem solving.

The purpose of the study was to describe the effect on students' spatial reasoning ability through the use of activities and lessons. The activities and lessons selected provided a variety of experiences to the participant. Anecdotal records, student journals, interviews with students, photos of student work and pre- and posttests were analyzed throughout the study but reexamined at the conclusion of the study to look with more of a researcher's view. I was able to analyze the data as a whole when the study was concluded. The students were given a survey at the conclusion of the study to reflect on their experiences.

The 16 students who participated in the study were given spatial reasoning pre- and posttests: the Wheatley Spatial Ability Test (1996) with norms for grade 5, and the ETS (1976) Paper Folding, Cube Comparisons, and Card Rotation Tests. These pre- and posttests provided me with quantitative data to compare each student's spatial ability before and after the study. The qualitative and quantitative data were analyzed to answer the questions posed in the study.

Analysis

This section includes an analysis of the data for each research question. The analysis includes efforts to triangulate the findings in order to build confidence in the validity and reliability of the assessment method.

Question 1

Question 1: Does my practice of incorporating spatial reasoning lessons and activities in my classroom affect my students' ability to spatially reason?

The first theme related to question 1, found from analyzing the data collected from the pre- and posttests, was that on each of the four posttests students showed a statistically significant difference when compared to the pre-test. The increased scores would indicate an improvement in spatial reasoning ability (see Figures 1, 2, 3, and 4). The Wheatley Spatial Ability Test, using norms for fifth-grade students, revealed that the mean on the pretest for the participants was 11.75. On the posttest the mean score was 50.75. The participants' score increased 39%. One-hundred percent of the participants showed an increase on the posttest. On the Paper Folding Test, the participants had a mean score of .41 on the pretest and .51 on the posttest. Sixty-nine percent of the participants showed an increase on the posttest. The Cube Comparisons Test is the test students indicated was the most difficult. The participants had a mean score of .31 on the pretest and .47 on the posttest. Seventy-five percent of the participants showed an increase on the posttest. The Card Rotation Test pretest the participants had a mean score of .44 and on the posttest .53. Sixty-eight percent of the participants showed an increase on the posttest.

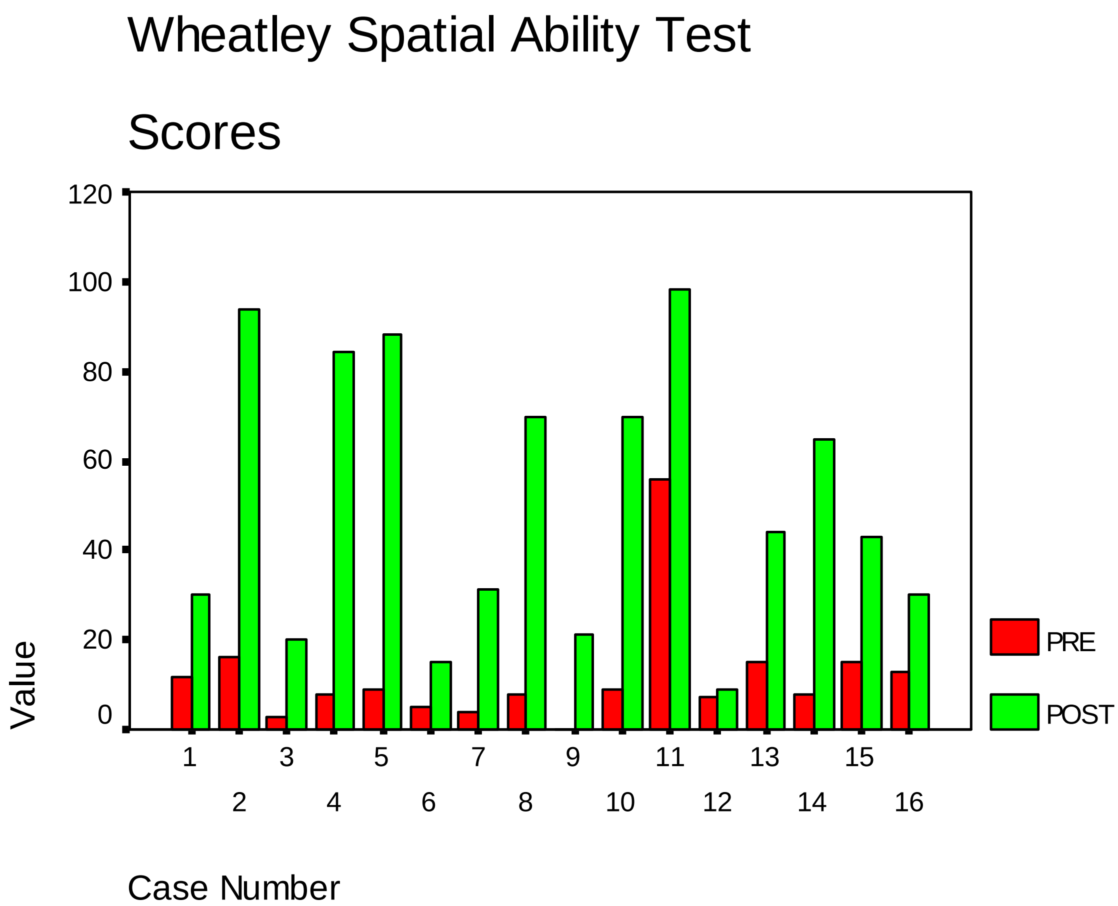

Figure 1: Wheatley Spatial Ability Test: Listed by Participant's Number

A paired t-test was run to analyze the data. The mean score from the pretest is the 11.75th percentile; the posttest mean is the 50.75th percentile. The significance (2-tailed) for the pre- and post Wheatley test was 0.00. Therefore $p <. 05$ and indicates a statistically significant difference. The degrees of freedom (df) is 15 and the t-value is -5.999.

I would hypothesize that this test revealed the greatest increase because its bold illustrations and page setup were student friendly.

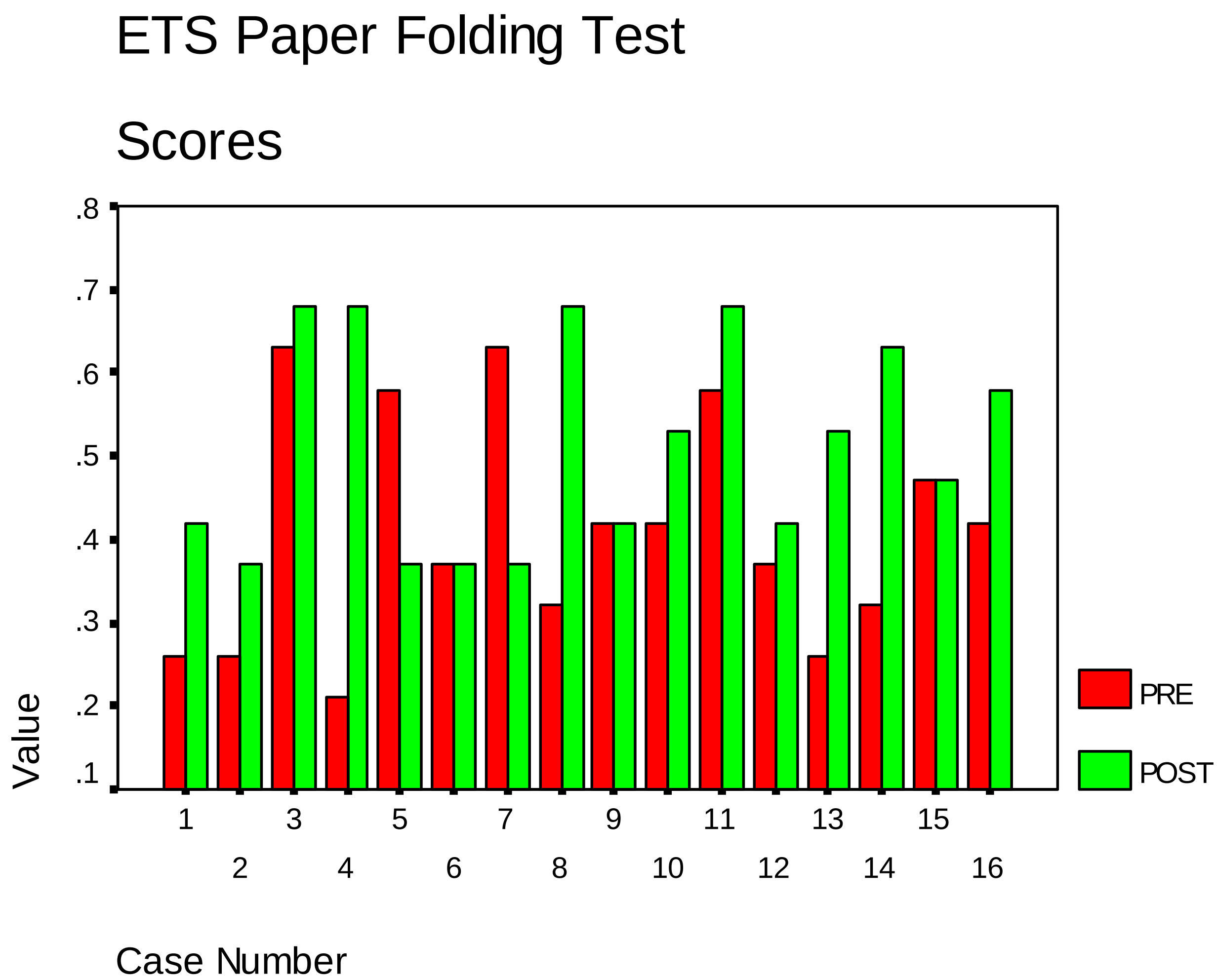

Figure 2: ETS Paper Folding Tests Listed by Participant's Number

A paired t-test was run to analyze the data. The mean score from the pretest is 0.4075; the posttest mean is 0.5125. The significance (2 tailed) for the pre- and post–ETS Paper Folding Test was 0.044. Therefore $p <. 05$ and indicates a statistically significant difference. The degrees of freedom (df) is 15 and the t-value is -2.203.

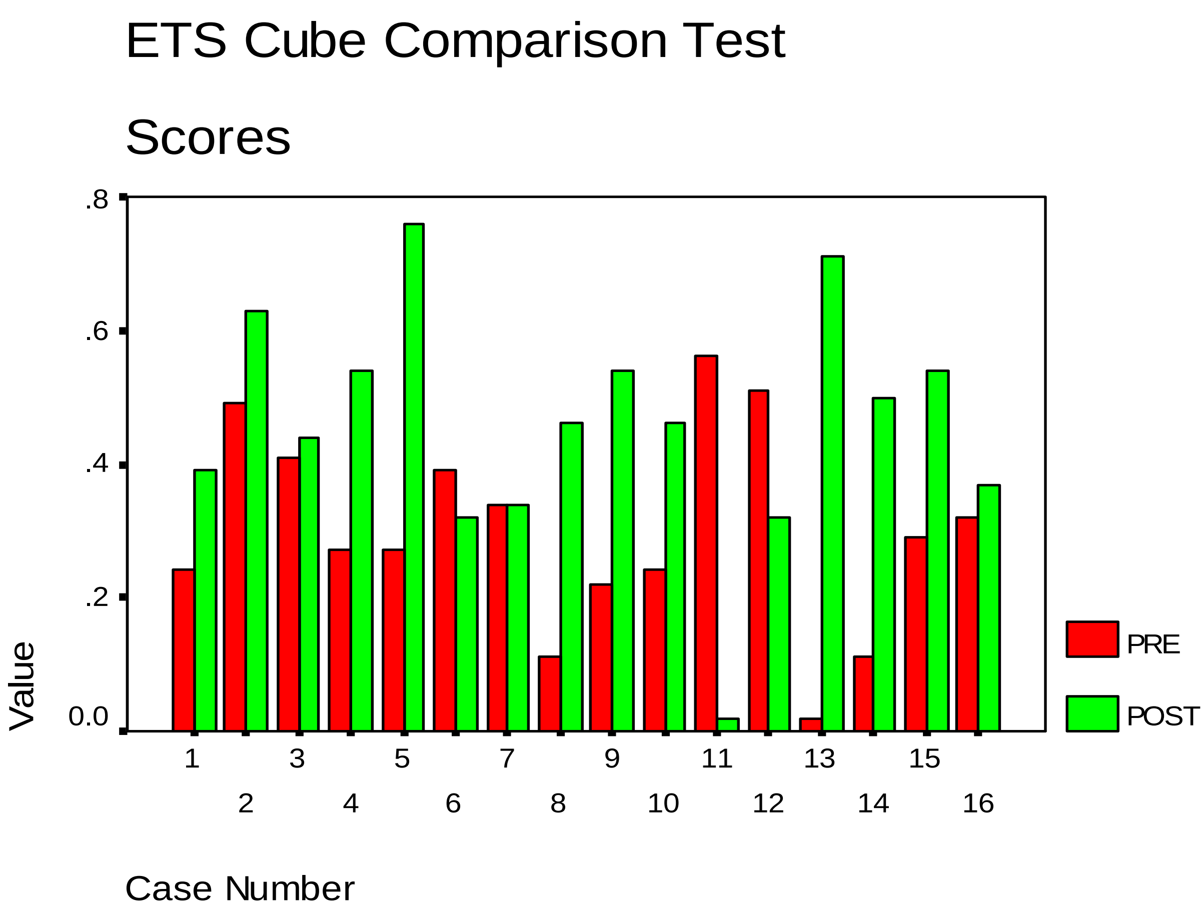

Figure 3: ETS Cube Comparisons Test Listed by Participant's Number

A paired t-test was run to analyze the data. The mean score from the pretest is 0.3131; the posttest mean is 0.4725. The significance (2-tailed) was 0.015 on the pre- and posttest of the Cube Comparisons Test. Therefore $p < .05$ and indicates a statistically significant difference. The degrees of freedom (df) is 15 and the t-value is -2.753.

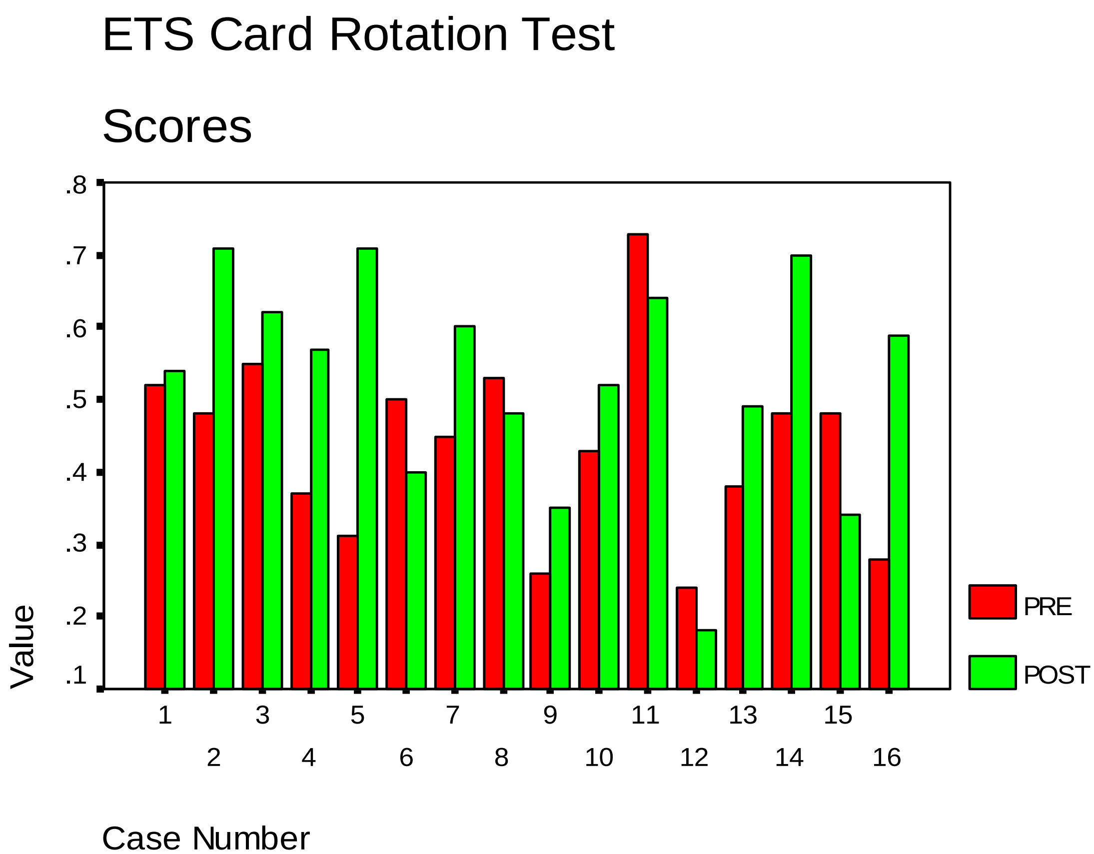

Figure 4: ETS Card Rotation Test Listed by Participant's Number

A paired t-test was run to analyze the data. The mean score from the pretest is 0.44; the posttest mean is 0.53. The significance (2 tailed) was 0.036. Therefore $p < .05$, which indicates a statistically significant difference. The degrees of freedom (df) is 15 and the t-value is -2.205.

Following the administration of the posttests, students remarked how much easier the posttest was compared to the pretest. "I was able to finish the whole thing." "I was able to do it in my head more easily." The students shared their reactions following the tests during student interviews.

While the pre- and posttests clearly indicate a significantly improved ability to spatially reason at the end of the study, the researcher's anecdotal records were also reviewed to add confidence to the conclusions drawn. As the study progressed students were completing spatial tasks more quickly. My notes reveal a gradual improvement in their ability to complete the tasks accurately with less time required.

In addition to the pre- and posttests and the anecdotal records, I triangulated my results with the post-study surveys. One-hundred percent of the participants responded affirmatively to the question, "Do you think you have improved in spatial reasoning skills?"

Student #6: "I think I'm getting better and better every time we do spatial reasoning. I get the answer to Quick Draws more. I think I've improved."

Student #11: "I learned how to keep pictures or words in my mind longer from Quick Draws."

Student #12: "I think I been doing great. Some of my grades improved."

Student # 16: "My level would probably be an eight (out of ten). I know because I've gotten faster at things we've done."

The results from the pre- and posttests, anecdotal records, and the post-study survey all reveal an increase in my fifth-grade students' ability to spatially reason as a result of the lessons and activities presented during the study.

Question 2

Question #2: Does my practice of incorporating spatial reasoning activities and lessons in my fifth-grade classroom affect my students' ability to use spatial reasoning in their mathematics problem solving?

During the course of the study several problem solving lessons were presented to the students. One lesson posed the question of how many different nets they could make using centimeter grid paper that created a box (see Figure 5). Students were given a choice of working independently or with a partner. Most students choose to work with a partner.

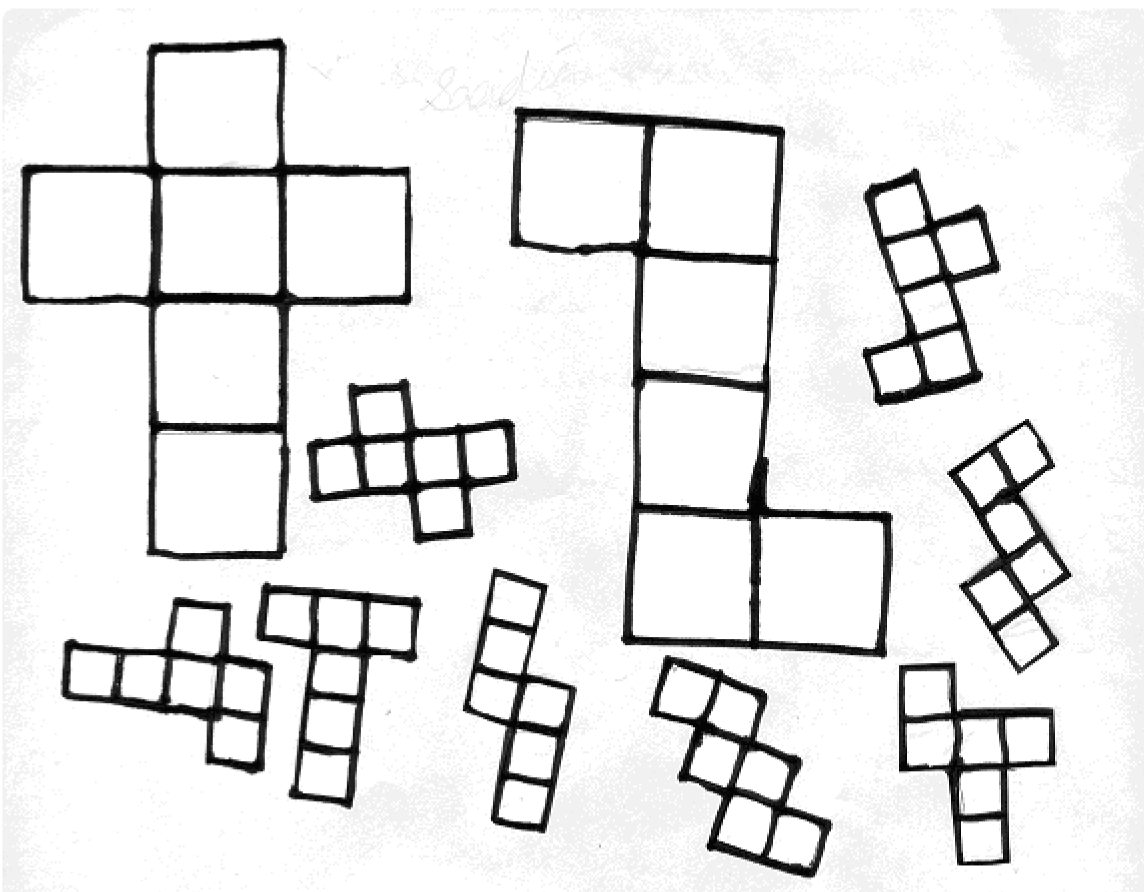

Figure 5: Participants # 8 and # 13's Work

In the anecdotal notes, I documented observing several different techniques to finding the solution. Some groups cut out a set of three-centimeter grids in a row and used another three individually to use as floaters to test.

I asked, “What is your thinking for this strategy?”

Student # 6, “It makes it easier to visualize because you can see what you’re doing.”

Teacher: “Explain.”

Student #6, “You can move some of these around and turn it different ways to see it better.”

I took note that students were using their ability to view objects from different perspectives to solve the problems.

Other students used a variety of ways to solve the problem. One pair of students quickly noticed it took six centimeter squares to become a box. However, they could not figure out why some configurations worked and why some did not. Through observation and further experimentation, they realized the position of the outside tabs was essential. At the end of the lesson, students were asked to write their strategy for solving. Here is a sampling of the responses.

Students #2 and 4: “We drew them. We experimented. If one doesn’t work we cut the block that is causing it not to work and place where it should be on the grid paper, then test it.”

Student #13: “The method I used was to first imagine it, then cut it out, but to tell you the truth sometimes the way I thought didn’t come out, but that’s my method to imagine it.”

Student# 17: “I draw the net and do it in my mind.”

The written responses clearly indicate the use of spatial reasoning and mental rotation to solve the box problem. My observations and anecdotal records triangulate evidence of the use of spatial reasoning skills.

Another lesson taken from the AIMS Foundation spatial reasoning book gave students the task of arranging rhombuses of three colors on a template so that cubes were stacked. The students were to position the cubes to look three dimensional and all the sides were the same color, the tops were the same color and the tops were the same color.

Figure 6 is representative of a common error frequently noted in my observations. The positioning of the top two rhombuses, turned vertically instead of horizontally, placed the whole figure incorrectly. The blue rhombus should have been rotated 90 degrees clockwise to serve as the top of the cube.

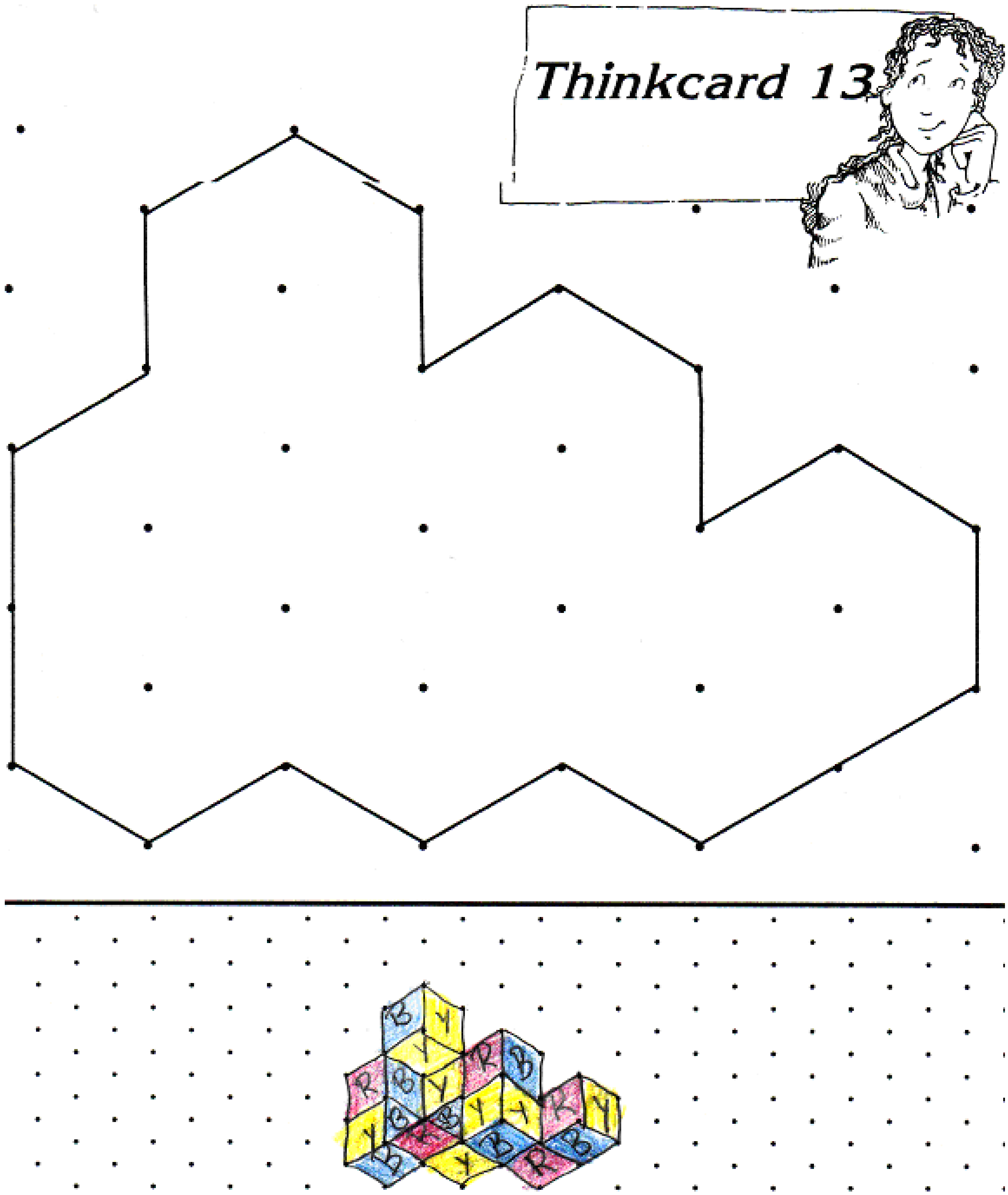

Figure 6: Student #15 Aims Education Foundation
Used by Permission

Interviews with students who positioned the top rhombus incorrectly revealed that they could always see that something was wrong but were not sure what. Once the first one was incorrectly placed, the rest would follow.

Student # 14: “I know it doesn’t look right, but I don’t know why.”

Teacher: “Why doesn’t it look right?”

Student #14: “The top doesn’t look like a top.”

Teacher: “How could you change it to look like a top?’

Student #14: “I don’t know.”

Each time this theme was repeated, I observed the student carefully move the rhombuses to find the solution. Sometimes after several days, the student self corrected. I also observed that at times the student had to draw the configuration to “see” they had incorrectly solved the problem. Piaget’s initial theory that drawings reflected the child’s spatial interpretations was made clear to me. Student # 14 worked while I observed for two days. On the second day, she made the connection and expressed pure glee. She was able to complete all the other Thinkcards with ease. Her experiences gave her the insight to know “it didn’t look right” but up to that point she did not have enough experiences with rotation. The theme of rotation will be discussed later in this chapter.

A second theme I observed in this lesson was that a small minority of students, who were successful in the orientation of the rhombuses, did not incorporate the color designation for the top, side, and front of each cube (see Figure 7). The colors did not follow a pattern indicating a visual misunderstanding. My observations revealed that about 5 of the students made this error and about 3 self corrected. Students who did not self correct did not see a problem with the colors they selected. While most students could visually discriminate the color differences a small minority could not.

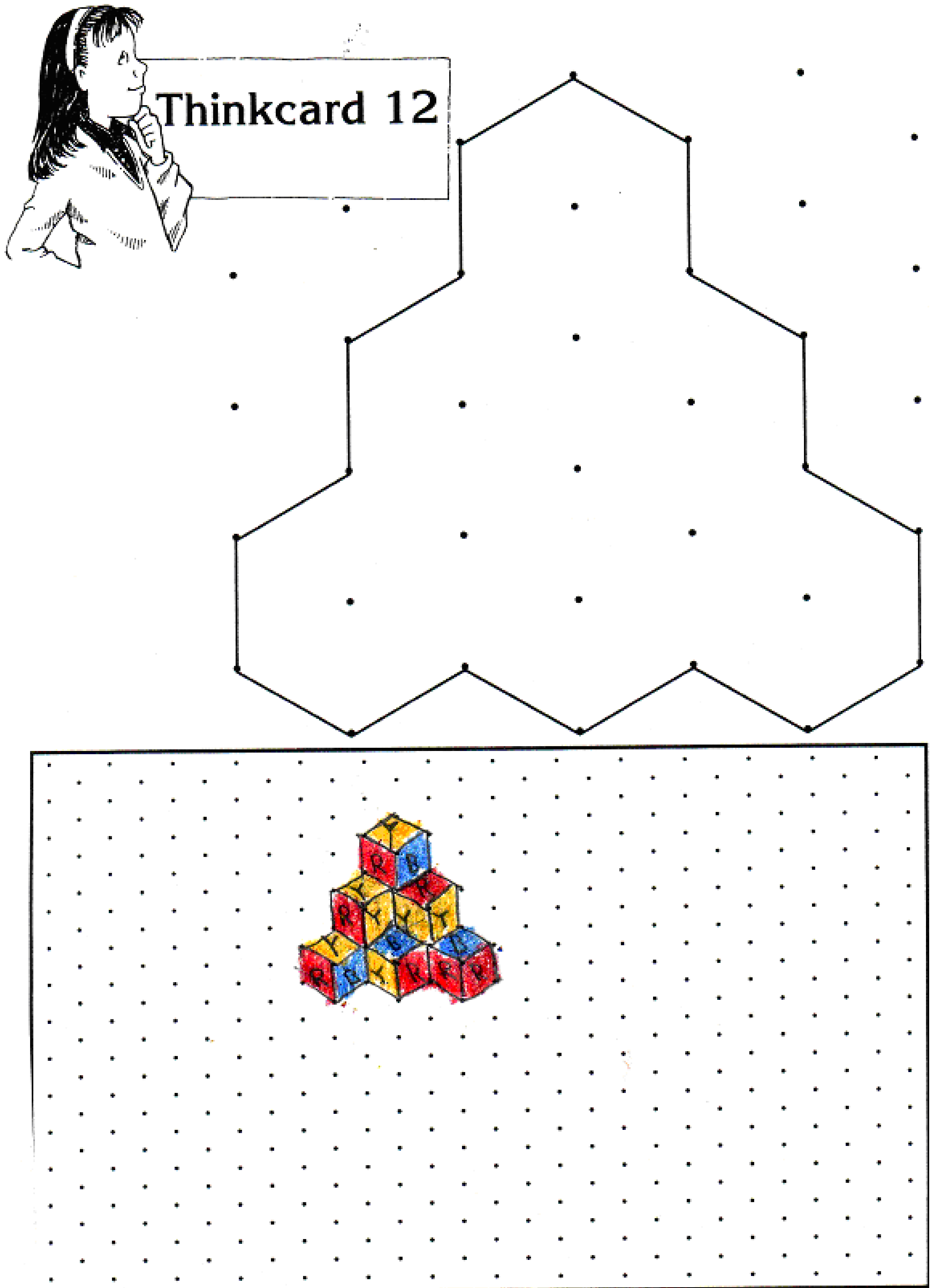

Figure 7: Student #11 Aims Education Foundation Used by Permission

Additional Themes

Throughout the study several themes emerged that were not connected to the research questions posed. These themes will be discussed in this section.

The students' mathematics vocabulary changed during the course of the study. The change was evident as we used Quick Draws. Quick Draws are figures shown for three seconds and the students draw what they see. The figure is shown another three seconds and the students describe what they see in their mind to remember the image. Student #1 is representative of the majority of students who began to use more geometric terms to describe what they saw. Figures 8 and 9 are examples of descriptions given at the beginning of the study. Student #1 describes objects such as a fish. The visual description reflects the image she remembered to transform to paper. Figure 10 is a description given by the same student in the middle of the study. Her descriptions become more mathematical with images of geometric shapes and angles. Without prompting, the students are incorporating their geometric knowledge with their spatial imagery.

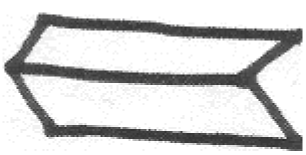

Figure 8: Student #1 Quick Draw Drawing at the Beginning of the Study

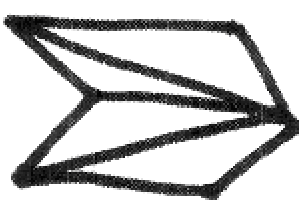

Figure 9: Student #1 Quick Draw Drawing During the Study

Student #1 written description of Figure 8: “What I saw is a somewhat like a fish with one stripe or a tent with an opening.”

Student #1 written description of Figure 9: “What I saw is a house with a big triangle door and an origami mouth.”

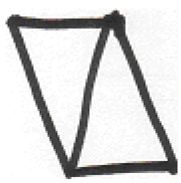

Figure 10: Student #1 Quick Draw Drawing During the Study

Student #1 written description of Figure 10: “What I did is think of two big triangles put together the opposite ways.”

Student #11 in one description of a Quick Draw wrote: “I saw a parallelogram with a diagonal line.”

In student interviews, students began to apply terms we studied in our study of geometry as a natural application without being prompted. Terms such as rotate instead of turn, reflection instead of the opposite of and quadrilateral instead of four sides.

Newcombe in her book, *The Making of Space* (2002), discussed how the child’s use of language in describing spatial relationships evolves. At the beginning of the study, the students were speechless when asked to describe the appearance of figures or their strategy for solutions. Through the use of pictures at first, the language of describing spatial reasoning began to emerge. During the study, I observed the development of language to describe spatial images.

Another theme observed was the students' ability to mentally rotate figures to see them in different orientations. At the beginning of the study, I asked the students to draw various polygons such as a right triangle, a pentagon, and a rectangle. We discussed how every student had drawn them with the same orientation. I posed the question of whether or not they could identify them with a different orientation. Alternate orientations were shown to the students. From that point on Quick Draws descriptions were overflowing with "If you look at it by rotating it you will see….." The class response was always, "Oh, yeah…" The Grid Game reinforced the practice of changing the orientation.

Student #8: "When I turned the paper I could see other shapes that were upside down or side to side."

Student #14: What helped me to visualize-turning paper. Occasionally my partner would point them out."

Rotating or changing the shape's orientation was a common strategy I observed in all activities particularly tangrams, pentominoes, Quick Draw and the Grid Game.

Student Attitudes

Student attitudes toward mathematics were improved in the study. On the post-study survey, 100% of the participants wrote that they enjoyed the study. Sample comments include:

Student #13: I learned that people can see stuff different ways than other people. I loved!!!! the study.

Student #9: "In this study I learned how to use my imagination and how to picture things in my mind easily. I enjoyed this study a lot! It was fun!

Student #2: "I did enjoy this study and I learned that what you do in life is spatial reasoning."

I observed that whenever we had a spatial reasoning task there was a collective, "Yes!" throughout the classroom. Despite the occasional comment "This is hard," the tasks compelled them to persist. This contributed to a positive attitude toward mathematics and the ability to see the task to completion. Schlechty (2001) in his book *Shaking up the School House: How to Support and Sustain Educational Innovation* states,

> The proper focus of the schools, then, is on the quality of the work provided student and the capacity of that work to engage students, to cause students to persist when they have difficulty, and to produce in students a sense of satisfaction and accomplishment. (p. 53)

The lesson of finding engaging work and the students' persistence was exemplified in the course of this study.

Summary

Based on the triangulation of the sources of data, I found that my practice of incorporating spatial reasoning lessons and activities in my fifth-grade classroom positively affected my students' ability to spatially reason. This finding was triangulated with a statistically significant increase shown in all of the pre- and posttests, student interviews, anecdotal records and post-study surveys. Further, I found that my practice of incorporating spatial reasoning lessons and activities in my fifth-grade classroom positively affected my students' ability to use spatial reasoning in their problem solving. This finding was triangulated through analyzing student work, student interviews, student journals, and anecdotal records.

Several themes emerged during the course of the study. Among them were the increased use of geometric terms to describe drawings, the development of a language to describe what students saw spatially, the ability to mentally rotate and change the orientation of figures, and a more positive attitude toward mathematics. All the themes were derived from data collected from analyzing student work, student journals, anecdotal records, and post-study surveys.

Chapter 5 will complete this study on the effect of spatial reasoning lessons and activities presented in my fifth-grade classroom on students' ability to spatially reason. Chapter 5 will review current research sources as they relate to my findings and provide recommendations for future research in this area.

CHAPTER 5

CONCLUSION

Summary

The purpose of this study was to investigate the effectiveness of my practice of exposing students to lessons and activities that involved spatial reasoning on their ability to spatially reason. The specific questions I attempted to answer were

Question #1. Does my practice of incorporating spatial ability lessons and activities in my classroom affect the students' ability to spatially reason?

Question #2. Does my practice of incorporating spatial reasoning lessons and activities in my classroom affect my students' ability to use this ability in their problem solving?

The results of this study suggest that incorporating lessons and activities in my classroom had a significant effect on my students' ability to spatially reason. These findings are consistent with the findings of Ben-Chaim et al. (1988), which concluded that with instruction intervention students could improve their ability to spatially reason. In their study, as in this one, little or no direct instruction on test questions was taught, yet the students improved their test scores. Their findings as well as my own conclude that spatial ability is not solely innate but can be taught.

The study by Baenninger and Newcombe (1989) found that direct training of test items yielded improved results over non-specific training and overall all participants improved spatial ability with training. These findings are also consistent with my study.

Although the study of Ferrini-Mundy (1987) disagreed with findings that support the trainability of spatial reasoning, there may be several reasons for this. The researcher's use of the Differential Aptitude Test to measure spatial ability could account for the results. Ferrini-Mundy herself stated "The Differential Aptitude Test measures only the ability to visualize a constructed object from a picture or a pattern, and the ability to imagine how an object would appear if rotated in various ways" (p. 137). The participants for her study were university calculus students, which may also account for the opposing findings.

The findings of this study are consistent with the findings of Connor and Serbin (1985). Their use of the ETS tests to pre- and posttest and the inclusion of tangrams closely resembled this study. Their tasks ranged from simple, concrete tasks to demanding tasks as did this study.

Clements, Battista, Sarama, and Swaminathan (1997) used the WSAT as did this study to measure spatial reasoning trainability. The training included the computer as well as manipulating tetrominoes, which I incorporated in my study.

The studies of Tartre (1990) and concluded that students with high spatial ability were able to construct more schemas to solve mathematical problems. This study extends those findings by observing students using these skills to solve problems. The students in my study used various strategies to problem solve and as time evolved more and more were engaged in spatial reasoning.

The positive finding is also consistent with the theoretical aspect of the study. Numerous findings of Piaget (Piaget & Inhelder, 1948/1967) were themes throughout my study. The use of drawings in the early stages for students to explain spatial relationships could be seen in the students' work. As their spatial ability improved, they were able to express verbally the strategies they used for solutions. Students were able to connect the concepts they learned such as looking at images from different orientations and incorporated the learning to other tasks. Piaget wrote about the child connecting ideas about space in the projective stage (Piaget & Inhelder, p. 9).

Vygotsky's (Leong & Bodrova, 2001) belief that development is impacted by activities and toys was demonstrated in the study by the use of puzzles, Legos®, tangrams, and pentominoes that were left out for students to interact with. The puzzles became part of their environment and therefore a part of their daily life. Students were able to construct their own understanding using the spatial activities.

Recommendations

Recommendations for further research include a larger population because findings in one fifth-grade classroom cannot be generalized to other classroom settings. Further research is needed in the ability to train students in spatial reasoning.

The findings of this and numerous other studies (Baenninger & Newcombe, 1989; Blatter, 1983; Connor & Serbin, 1985; De Lisi & Wolford, 2002; Rhoades, 1981; Robichaux & Guarino, 2000; Sanz de Acedo Lizarraga, 2003; Vasta et al., 1996) suggest that spatial reasoning is a trainable skill. The lack of success in spatial tasks has been linked to the number of experiences one has. I recommend that teachers at all educational levels incorporate spatial lessons and activities into their daily mathematics lessons. Research shows our students can only benefit from being consistently engaged in these tasks.

Several studies have shown spatial reasoning skills are linked to achievement in mathematics (Capraro, 2001; Clements, & Del Campo, 1989; Fennema & Sherman, 1978; Hegarty, 1999; Reuhkala,2001; Seng & Chan, 2000; Seng & Yeo; 2000; Tartre, 1990; Wheatley et al., 1994). Educators need to pursue this connection and conduct action research to test these findings. All of us are stakeholders in our children's mathematic achievement.

Further research must be done to discover the ways that spatial skills are tied to mathematics performance. With this knowledge educators could pinpoint how the mind processes spatial skills and could incorporate these skills into the curriculum more effectively as part of their daily teaching.

Spatial reasoning skills are a necessary component of many occupations (Smith, 1964). Careers in architecture, mathematics, mathematics education, and mechanical engineering must attract those adept at spatial skills. Perhaps we can encourage interest and an aptitude for these careers by including spatial skills in our K-12 mathematics curriculum.

As an extension to this study, I would recommend that a more intense analysis be done of the development of language used during spatial reasoning tasks. If we can understand the development of words and phrases used to describe the mental imagery used to work through spatial tasks, perhaps we can understand how spatial ability develops and is perceived in the mind.

Conclusion

This study emphasized the use of spatial reasoning lessons and activities to improve spatial reasoning skills. Throughout the study, spatial skills increased to a measurable statistically significant degree on all four of the pre- and posttests administered. Students became more adept at solving puzzles and tasks which involved spatial reasoning. Student attitudes improved toward mathematics and problem solving. The students were persistent and positive in their quest to solve problems. Mathematics education in our classrooms must strive to present mathematics using less procedural and more open-ended tasks. Spatial reasoning tasks are a start, and it is my hope that we will begin to change the way we deliver mathematics in our classrooms.

The information provided to me in this study led me to further questions. Are there more accurate means to assess individuals' proficiency in spatial reasoning? Since spatial sense involves nonverbal skills are there ways to assess nonverbally? How important is the ability to use the language of spatial descriptions to one's understanding of a spatial task? The study of spatial reasoning intrigues me even more than prior to the start of the study.

As a result of this study, I will continue to expose my students to spatial reasoning tasks and encourage my colleagues to do the same. It is my hope that spatial tasks will increase their mathematics ability and their ability in the mathematics curriculum.

APPENDIX A

INSTITUTIONAL REVIEW BOARD FORMS

UCFIRB Form

1. Title of Project: Effect of Teaching Fifth Grade Students Spatial Reasoning Skills On Their Ability To Perform Spatial Reasoning Tasks.

2. Principal Investigator(s):

   Signature:

   Name: Mrs.Theresa Varn
   Degree: B.S. Elementary Education
   Title: Fifth Grade Teacher
   E-Mail tvarn@cfl.rr.com
   Telephone:
   Home Telephone:
3. Supervisor
   Signature:

   Name: Dr. J.K. Dixon
   Department: Teaching and Learning Principles
   College: College of Teaching and Learning
   Degree: Ph. D. Education -Mathematics Education
   Title: Associate Director, Lockheed Martin UCF Academy
   E-Mail: jkdixon@mail.ucf.edu
   Phone: (407) 823-4140
   Facsimile: (407) 823-2815

4. Dates of Proposed Project (cannot be retroactive): From: 8/04 To: 8/05

5. Source of funding for the Project: None

University of Central Florida

Office of Research

July 12, 2004

Mrs. Theresa Varn
3716 Dinghy Court
Orlando, FL 32812

Dear Mrs. Varn:

With reference to your protocol entitled, "Effect of Teaching Fifth Grade Students Spatial Reasoning Skills on their Ability to Perform Spatial Reasoning Tasks," I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur. Further, should there be a need to extend this protocol, a renewal form must be submitted for approval at least one month prior to the anniversary date of the most recent approval and is the responsibility of the investigator (UCF).

Should you have any questions, please do not hesitate to call me at 823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Barbara Ward

Barbara Ward, CIM
Institutional Review Board (IRB)

Copies: Dr. Juli Dixon, Department Education
IRB office

12443 Research Parkway • Suite 207 • Orlando, FL 32826-3252 • 407-823-3778 • Fax 407-823-3299

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IRB # 04-1972

THE UNIVERSITY OF CENTRAL FLORIDA
INSTITUTIONAL REVIEW BOARD (IRB)

IRB Committee Approval Form

PRINCIPAL INVESTIGATOR(S): Theresa Varn

PROJECT TITLE: Effect of Teaching Fifth Grade Students Spatial Reasoning Skills on their Ability to Perform Spatial Reasoning Tasks

Committee Members:

Dr. Theodore Angelopoulos: ____________
Ms. Sandra Browdy: ____________
Dr. Jacqui Byers: ____________
Dr. Ratna Chakrabarti: ____________
Dr. Karen Dennis: ____________
Dr. Barbara Fritzsche: ____________
Dr. Robert Kennedy: ____________
Dr. Gene Lee: ____________
Ms. Gail McKinney: ____________
Dr. Debra Reinhart: ____________
Dr. Valerie Sims: ____________

[ ] Contingent Approval
Dated: ____________

[ ] Final Approval
Dated: ____________

[X] Expedited
Dated: July 7, 2004

[ ] Exempt
Dated: ____________

[x] Expiration
Date: 6 July, 2005

Chair, IRB

Signed: ____________
Dr. Sophia Dziegielewski

NOTES FROM IRB CHAIR (IF APPLICABLE): ____________

APPENDIX B

WHEATLEY E-MAIL

terri Varn

From: GHWHEATLEY@aol.com
Sent: Wednesday, May 19, 2004 12:12 PM
To: tvarn@cfl.rr.com
Subject: Re: spatial sense

Terri,

The Wheatley Spatial Ability Test has been used for many research projects. The test assesses mental rotations, one component of spatial ability. It is appropriate for grades 3-8. I have written many articles on spatial ability and can send you the references if you wish. I can send you a copy of the test but will not be back in town until June 4 to post it to you.

Please provide an address.

Stay away from the Differential Appitude Test (DAT) since students can score high using non spatial reasoning. There is a French (name, not country) written at ETS which is good. Cubes comparison is not a good measure.

Grayson Wheatley

APPENDIX C

AIMS EDUCATION FOUNDATION PERMISSION

AIMS Education Foundation

March 23, 2005

Theresa Varn

Dear Theresa,

Thank you for your support of the AIMS Education Foundation programs and publications. The AIMS Education Foundation is a non-profit organization that seeks to enrich the education of students in grades K-9 through hands-on activities that integrate mathematics, science, technology, and other disciplines.

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If you have any further questions or concerns, please do not hesitate to contact me at the Foundation office.

Sincerely,

Michelle Pauls
Assistant Editor
Copyright Requests Coordinator

E-mail: mpauls@aimsedu.org

AIMS Education Foundation P. O. Box 8120
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APPENDIX D

ETS LICENSE

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T. Varn

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LICENSEE	EDUCATIONAL TESTING SERVICE
Theresa Varn	Lorraine Carmosino
Name: Theresa Varn	Name: Lorraine Carmosino
Title: Teach	Title: Copyright Permissions Administrator
Date: 7/22/04	Date: 7/15/04

ETS

APPENDIX E

CONSENT FORMS

Parental Consent

August 6, 2004

Dear Parent/Guardian:

I am your child's fifth grade teacher and a graduate student at the University of Central Florida under the supervision of faculty member, Dr. Juli K. Dixon. I am conducting research in my classroom from August 2004-August 2005. The purpose of the research study is to examine my practice of providing spatial reasoning tasks to my students and help them make meaningful connections to understanding mathematics. The results of this study will help me to better understand the ways I can help students to improve their ability to reason spatially and allow me to design my instructional practices accordingly.

Students in the study will receive the same instruction as students who do not participate. Students participating in the study will be interviewed by me, and I will collect work samples from them to use in my research. With your permission, your child will be audio taped during instruction and interviews. The audio tapes will be destroyed after the study is completed. Although the children will be asked to write their names on the work samples for matching purposes, their identity will be kept confidential and will not be used in the report of the research. I will replace their names with fictitious names. Results will be reported in the form of individual data and group data. Participation or non participation in this study will not affect the student's grades or placement in any programs.

You and your child have the right to withdraw consent for your child's participation at any time without consequence. There are no known risks to the participants. No compensation is offered for participation. Group results of the study will be available in June upon request. If you have any questions about this research project, please contact me at (407) 858-3110 or my faculty supervisor, Dr. Juli K. Dixon, at (407) 823-4140. Questions or concerns about research participants' rights may be directed to the UCFIRB office, University of Central Florida Office of Research, Orlando Tech Center, 12443 Research Parkway, Suite 207, Orlando, Fl 32826. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The phone number is (407) 823-2901.
Sincerely,

Theresa Varn

I have read the procedure described above and I voluntarily give my consent for my child, ________________________, to participate in Mrs. Theresa Varn's action research study.

____

Parent/Guardian Date

____

2nd Parent/Guardian Date

Child Assent Script

My name is Mrs. Varn and I am your teacher and a graduate student at the University of Central Florida. I am studying the ways that students use their spatial reasoning skills and how they apply them to spatial tasks to learn mathematics.

During my study, I will ask you to take part in audio taped recordings of you and other classmates working on spatial reasoning tasks. I may ask you to take part in audio taped interview sessions with me. Only my research team and I will have access to the audio tapes that are used. During the interviews, I will ask you questions about the strategies you used to get your answer. I will ask you to keep a journal in which you will explain the strategies you used. Your name will not be revealed. If you choose to take part, you may stop at any time. You will not have to answer any questions you do not want to answer. Taking part in this study will not affect your grades. Would you like to participate?

APPENDIX F

POST-STUDY SURVEY

Spatial Reasoning Survey #

1. What is spatial reasoning?

2. What activity or lesson helped you to develop spatial reasoning the most?

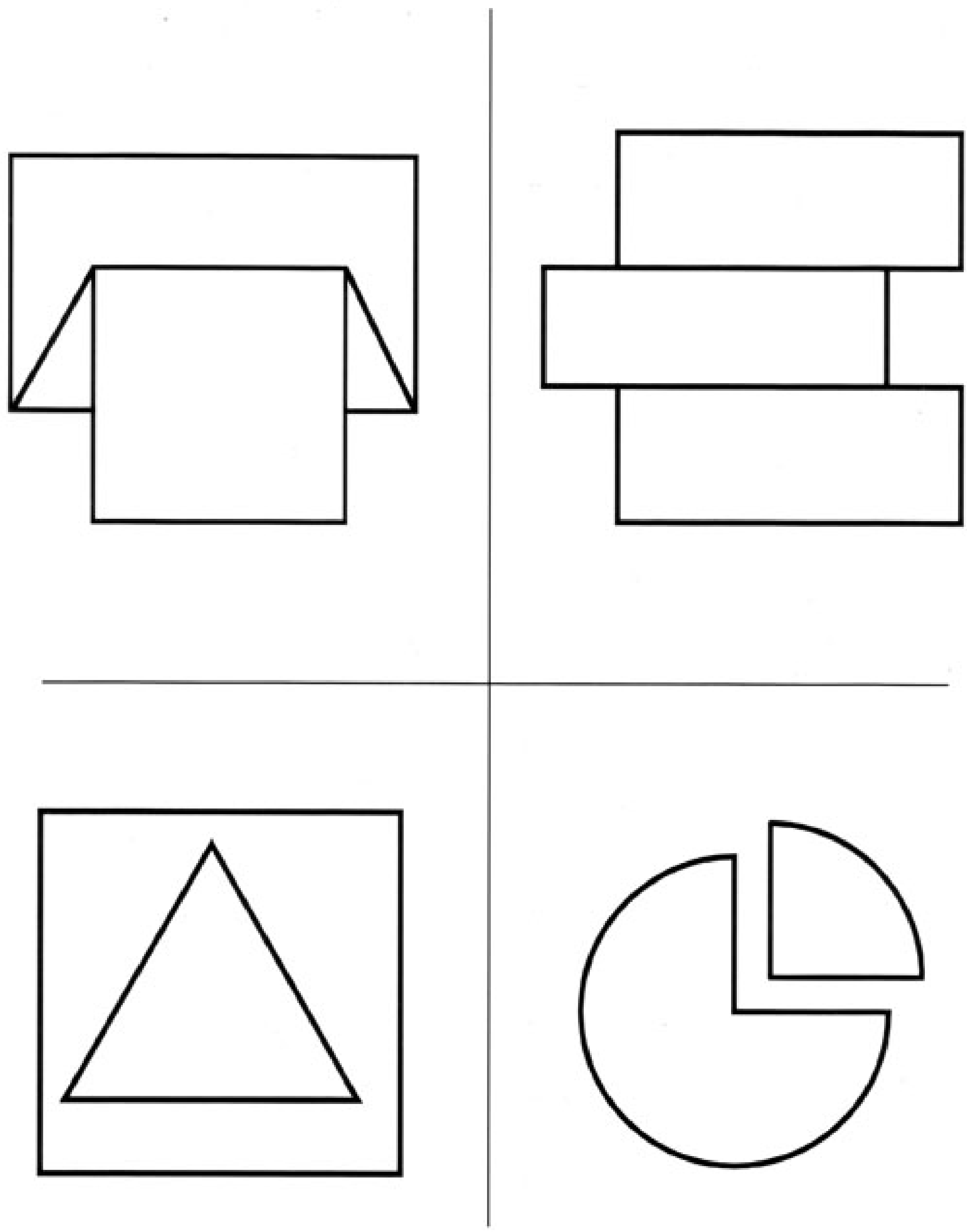
3. Describe your skill level in spatial reasoning? How do you know? Do you think you have improved?

4. What activities do you participate in that involve spatial reasoning?

5. What did you learn in this study? Did you enjoy the study?

APPENDIX G

QUICK DRAW



3

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