THE EFFECT OF COGNITIVELY GUIDED INSTRUCTION ON STUDENTS' PROBLEM SOLVING STRATEGIES AND THE EFFECT OF STUDENTS' USE OF STRATEGIES ON THEIR MATHEMATICS ACHIEVEMENT

by

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ABSTRACT

The purpose of this study was to investigate the effect of teachers attending Cognitively Guided Instruction (CGI) professional development on students' problem solving strategies and the effect of students' use of strategies on their mathematics achievement as measured by a standardized test. First, the study analyzed the differences in students' use of strategies between treatment and control groups. The treatment was CGI professional development, and the teachers in the treatment group attended CGI workshops whereas the teachers in the control group did not. The students, both in the classes of treatment teachers (treatment students) and in the classes of control teachers (control students) were classified into the strategy groups according to their use of strategies. Student interviews were used to identify the strategies used by the students and to classify them into the strategy groups. The strategies that were analyzed in this study are; (a) concrete modeling, (b) counting, and (c) derived facts / recall for single-digit numbers; and (a) unitary, (b) lower standard algorithm, (c) concrete modeling with tens, (d) higher standard algorithm, and (e) invented algorithms for multi-digit numbers. The analyses were performed separately for first and second grade students.

Next, the study analyzed the differences in the mathematics achievement of students between different strategy groups. A student posttest, which was ITBS (*Math Problems* and *Math Computation*), was used to compare students' mathematics achievement. A student pretest was used as a covariate.

The literature indicates that instruction has an effect on students' use of strategies. However, two studies reported conflicting results related to the students' use of strategies between students of CGI and students of non-CGI teachers. While one study reported no significant differences in students' use of strategies between the two groups, the other study reported that students of CGI teachers used advanced strategies significantly more often than students of non-CGI teachers. In addition, the literature about student-invented strategies indicates that students who are able to use their own invented strategies have a better understanding of place value and number sense. To add to the literature about students' strategies, this study investigated the effect of students' use of strategies on their mathematics achievement as measured by a standardized test.

The results of this study showed that there were statistically significant differences in students' use of strategies between the treatment and control groups at the second grade level. A greater percentage of treatment students used *derived facts / recall* strategies (the most advanced strategy for single-digit addition and subtraction) than control students did, and a greater percentage of control students used *counting* strategies than treatment students did. This study concluded that the treatment students showed more progression towards the use of the most advanced strategy for single-digit addition and subtraction. The results of this study suggest that all first and second grade teachers should have the knowledge of students' thinking and the progression that they show in dealing with numbers. One way to accomplish this is to provide teachers with CGI professional development.

The results related to the effect of students' use of single-digit strategies on their mathematics achievement showed that second grade students who were in the *derived facts / recall* strategy group had significantly higher mathematics achievement than the students in the *counting* and *concrete modeling* strategy groups. For multi-digit strategies, the students in the invented algorithms group had significantly higher mathematics achievement than the students in the standard algorithm groups (*lower standard algorithm* and *higher standard algorithm* groups). The results of this study suggest that all students should be provided with sufficient opportunities and time to develop their own strategies, and teachers should facilitate their progression towards the use of more advanced student-invented strategies before teaching them the procedures of standard algorithms so that students have better mathematics achievement.

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CHAPTER ONE: INTRODUCTION

The Problem and Its Underlying Framework

"In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures" (National Council of Teachers of Mathematics, 2000, p. 50). Therefore, mathematical achievement is an important goal for all students. A broad base of literature indicates that one of the most important factors of student achievement is the knowledge and skill of classroom teachers (Carey, 2004; Darling-Hammond, 2002; Marzano, 2003; Nye, Konstantopoulos, & Hedges, 2004).

Cognitively Guided Instruction (CGI) is a professional development program for teachers based on a research focused on students' mathematical thinking and teachers using students' thinking as a guide to design their instruction (Carpenter, Fennema, Franke, Levi, & Empson, 2000). Cognitively Guided Instruction has been found to have a positive effect on student achievement by enhancing teachers' knowledge of students through a series of professional development experiences (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). The current study will explore the effect of teachers' attending the CGI professional development on their students' problem solving strategies, and the effect of students' use of different problem solving strategies on their mathematics achievement. It is important to note that this study was conducted at the end of the first year of a two-year planned CGI professional development. Therefore the results of this study should be interpreted cautiously.

Background of the Problem

The mathematics achievement of students in the United States (U.S.), when compared with the performance of students in other high achieving countries, leads one to deduce that there is a need for improvement in mathematics education (Ball, 2003). The Trends in International Mathematics and Science Study (TIMSS, 2007) reported that US fourth-grade students' average mathematics score was lower than eight Asian and European countries that are considered high achieving countries. Additionally, TIMSS has shown that in the U.S. students spend a large amount of time during mathematics instruction by reviewing the materials they already learned, and the focus of most lessons was to practice the mathematical procedures rather than developing a conceptual understanding (Stigler & Hiebert, 2009). When videos of teachers' instruction from TIMSS were analyzed, the U.S.'s motto for mathematics instruction was classified as "learning terms and practicing procedures", whereas Germany's motto was classified as "developing advanced procedures", and Japan's motto was classified as "structured problem solving" (Stigler & Hiebert, 1999, p. 27). It was common for students to share multiple solution strategies in a typical Japanese classroom (Stigler & Hiebert, 1999). It has been reported that high achieving countries frequently used a problem solving approach with an emphasis on conceptual understanding (Hiebert et al., 2003). Therefore, the results of TIMSS have revealed the need to improve school mathematics in the U.S.

With the aim of improving mathematics education in the U.S., the National Council of Teachers of Mathematics (NCTM) standards based reform movement began in 1989 with the release of *Curriculum and Evaluation Standards for School Mathematics* and have continued. These standards recommended that the focus of school mathematics should be on problem solving, reasoning, communications, and connections (NCTM, 1989). Another milestone was the release of the *Principles and Standards for School Mathematics* by NCTM in 2000 in which they refined and clarified the *Standards* document (Herrera & Owens, 2001). More recently, The Council of Chief State School Officers (CCSSO, 2010) released a set of mathematics standards, called the Common Core State Standards for Mathematics (CCSSM). The CCSSM provides a foundation to develop more focused, coherent, and rigorous mathematics curricula and instruction that promote conceptual understanding and skill fluency (NCTM, 2013). Following the release of CCSSM, NCTM (2014) released *Principles to Actions: Ensuring Mathematical Success for All*. The primary purpose of *Principles to Actions* is to provide a direction to fill the gap between the adoption of rigorous standards and the enactment of practices, programs, and actions that are required for the successful implementation of those standards (NCTM, 2014).

The CCSSM consist of two types of standards: (a) standards for mathematical content (SMC) and (b) standards for mathematical practices (SMP). Standards for mathematical content include a set of grade-specific standards for grades K-8 in which the goal is more focus and coherence with the content. Standards for mathematical practices describe a set of mathematical habits that teachers should develop in their students. The goal of the SMP is to guide teachers to improve their instructional methods so that students can learn mathematics with understanding (CCSSO, 2010). The eight SMP's are the following:

1. Make sense of problems and persevere in solving them,
2. Reason abstractly and quantitatively,
3. Construct viable arguments and critique the reasoning of others,
4. Model with mathematics,

5. Use appropriate tools strategically,

6. Attend to precision,

7. Look for and make use of structure, and

8. Look for and express regularity in repeated reasoning (CCSSO, 2010, pp. 6-8).

Parallel to the goal of the SMP, CGI seeks to address the need to improve students' mathematical proficiency through professional development of teachers (Carpenter, Fennema, Franke, Levi, & Empson, 1999). Carpenter, Fennema, and Franke (1996) stated that several other projects have also focused on teachers' understanding of mathematical learning and used it as a base to help teachers to make notable changes in their instructions. (e.g., the Summer Math for Teachers Project - Schifter & Fosnot, 1993; Simon & Schifter, 1991; the Purdue Problem Centered Mathematics Project - Cobb et al., 1991).

Cognitively Guided Instruction differs from other projects in that students' thinking is used as a context for teachers to enhance their own understanding (Carpenter et al., 1996). Therefore, the goal of CGI is not to show teachers the representations that they can directly teach to their students, rather the goal is to help teachers understand the ways students intuitively solve problems, even if those are not the most efficient ways (Carpenter et al., 1999). Franke and Kazemi (2001) stated that knowing the sequence of how children develop problem solving strategies enables teachers to pose problems that challenge their students' thinking.

Existing research shows that CGI is effective in raising student achievement under specific professional development models and teachers reaching higher levels of implementation of CGI within their practice (Carpenter et al., 1989). CGI helps teachers to understand how students think about word problems involving the four basic operations and what strategies they use to solve different types of problems by watching videos of children who use variety of strategies to solve those problems (Wilson & Berne, 1999). Children's strategies progress from *direct modeling* to *counting* and then to *derived facts or recall* for single digit problems (Carpenter et al., 1999). For problems involving multi-digit numbers, children's progression progress from *counting single units (unitary)* to *direct modeling with tens* and then to *invented algorithms* (Carpenter et al., 1999). The *derived facts/recall* and *invented algorithms* strategies are based on some fundamental properties of arithmetic operations. The progression of students through these strategies represents increased levels of sophistication and efficiency in dealing with numbers (Medrano, 2012).

According to CCSSM, students are expected to have a learning progression in which they develop efficient and generalizable methods based on the properties of operations and place value understanding (Fuson & Beckman, 2012). Therefore, it suggests that conceptual understanding should precede procedural understanding. Conceptual understanding also plays an important role in selecting a procedure, monitoring the selected procedure, and transferring of the procedural knowledge to new situations (Hiebert, 1986). Rittle-Johnson and Alibali (1999) found that children who received conceptual instruction were able to generate multiple procedures and adapt their existing procedures to novel problems. Geary (1995) concluded that conceptual understanding and flexible use of solution strategies are closely related.

On the other hand, when students learn about the procedures of the standard algorithms in early grades, some may perform algorithmic computation as a series of *concatenated single-digit* operations (Blote, Van der Burg, & Klein, 2001), which are responsible for children's misconceptions (Fuson, 1992). Unlike *invented algorithms*, students are not likely to invent the procedures of standard algorithms. Therefore, they need to be explicitly instructed how to use those procedures. Learning about the procedures of standard algorithms prior to make sense of invented algorithms deemphasizes the learning of properties of numbers and place value, since the number properties that those procedures are based on are not apparent to the students (Kilpatrick, Swafford, & Findell, 2001).

Students who use invented algorithms to solve problems think about and apply knowledge of fundamental properties of number operations (Carpenter, Franke & Levi, 2003), since invention and application of invented algorithms involves facets of number sense like decomposition, re-composition, and understanding of number properties (McIntosh, Reys, & Reys, 1992). It can be proposed that possession of number sense in a technological age is one major attribute that distinguishes human beings from computers (McIntosh, Reys, & Reys, 1992).

McIntosh, Reys, and Reys (1992) define number sense as “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations” (p. 2). In their framework, they suggested that number sense involves: (a) knowledge of and facility with numbers such as place value understanding and decomposition / re-composition, (b) knowledge of and facility with operations such as understanding mathematical properties and relations between operations, and (c) applying knowledge of and facility with numbers to operational settings. How students use number sense while they invent their own algorithms can be illustrated with an example of a student who adds

36 + 58 by using combining tens and ones strategy. The steps and corresponding number properties a student might use are listed in Figure 1.

# of steps		Operation in each step	Corresponding number property
1.	36 + 58	= (3 x 10 + 6) + (5 x 10 + 8)	Representation of base ten numbers
2.		= (3 x 10 + 5 x 10) + (6 + 8)	**Involves associative and commutative property**
3.		= (3 + 5) x 10 + (6 + 8)	**Involves distributive property**
4.		= 8 x 10 + (6 + 8)	Execution of addition
5.		= 80 + (6 + 8)	Execution of multiplication
6.		= (80 + 6) + 8	**Involves associative property**
7.		= 86 + 8	Execution of addition
8.		= 86 + (4 + 4)	Renaming a number
9.		= (86 + 4) + 4	**Involves associative property**
10.		= 90 + 4	Execution of addition
11.		= 94	Execution of addition

Figure 1: Steps in calculation and corresponding number properties.
Adapted from Thinking Mathematically: Integrating arithmetic and algebra in elementary school (p.113) by T. P. Carpenter, M. Loef Franke, and L.Levi, 2003, Portsmouth, NH: Heineman. Copyright 2003 by T. P. Carpenter, M. Loef Franke, and L. Levi.

When students use such an invented algorithm, they do not necessarily posses a complete understanding of the number properties or their definitions. However, it does imply some level of understanding of those properties (Carpenter, Levi, Franke, & Zeringue, 2005), which might serve as a bridge to generalize these basic principles when they deal with algebraic expressions and equations in later grades (Carpenter et al., 2003).

Existing research on students' use of different strategies has concluded that instruction has an effect on students' actual use of strategies (Carpenter, Hiebert, & Moser, 1983; Villasenor & Kepner, 1993; Fuson, Smith, & Lo Cicero, 1997), as well as on students' ability to use them flexibly (Blote et al., 2001; De Smedt et al., 2010;). Blote et al. (2001) conclude that students who initially learn to use one standard procedure continue to use the same procedure even after they are taught other procedures and become inflexible problem solvers with limited understanding. Peters, Smedt, Torbeyns, Ghesquière, & Verschaffel, (2012) suggested that mathematics textbooks and lessons should include more word problems and external representations to stimulate children to make flexible strategy choices, rather than using a single strategy for all problems.

Statement of the Problem

Problem solving ability and thinking critically are highly regarded as essential skills in the 21st century (Hargreaves, 2003). Mathematics problem solving has been a long concern with the mathematics achievement of U.S. students. In 2006 U.S. was ranked 21st of 30 countries in the Organization for Economic Cooperation and Development (OECD) in the international assessment conducted by the Program in International Student Assessment (PISA) (Darling-Hammond, 2010). American students fell even further behind on PISA tasks that required problem solving. Nations who significantly outperform the U.S. on mathematics achievement have classrooms where focus is on mathematical reasoning and problem solving with students (Darling-Hammond, 2010).

Studies examined the relationship between numbers of mathematics courses taken by the teachers, which refer to teachers' content knowledge (TCK), and student achievement failed to show significant correlations (Begle, 1979; Monk, 1994). On the other hand, Hill, Rowan, and Ball (2005) found that teachers' pedagogical content knowledge, specifically knowledge of content and teaching (KCT), which refers to a teacher's ability to deliver clear mathematical explanations, listen to students' reasoning to guide their next instructional steps, and build mathematical representations of problems, had a positive effect on student achievement.

The need for improvement in mathematics instruction is well documented in the literature. High achieving countries in international studies are determined to have curriculum with focus on problem solving. Similarly, CGI emphasizes the importance of basing mathematics curriculum on problem solving and giving students the opportunity to be actively involved in deciding how to solve a mathematics scenario (Carpenter et al., 1999).

At least two experimental studies have examined the impact of CGI on students' mathematics achievement. For both studies the teachers in the treatment group attended the CGI professional developments whereas control teachers did not. The studies found significant differences in students' mathematics achievement between the students of treatment and control teachers (Carpenter et al., 1989; Villasenor & Kepner, 1993). The original CGI study, which was an experimental study, did not report any differences in students' solution strategies between the two groups (treatment and control) (Carpenter et al., 1989). However, the study conducted in 1993 reported significant differences between the treatment and control groups, and the authors stated that treatment students used more advanced strategies significantly more often. (Villasenor & Kepner, 1993). Recently a replication study of CGI has been started to re-examine the impact of this intervention on student achievement and teachers' beliefs when implemented with a larger and more diverse sample of students (Schoen, LaVenia, Tazaz, et al., 2014).

Purpose of the Study

Peters et al. (2012) suggested that more research is needed to evaluate the success of powerful instructional settings on students' use of strategies. The current study seeks to address this gap in the literature and will explore the impact of teachers' attending the CGI professional developments, which can be considered as one type of powerful instructional setting, on students' problem solving strategies and the impact of students' use of different problem solving strategies on their mathematics achievement as measured by a standardized test. In the current study, the teachers in the treatment group attended the CGI professional developments whereas the teachers in the control group did not. The results of this study will provide empirical evidence regarding the effect of teachers' attending CGI workshops on students' use of strategies, and the effect of students' use of different strategies on their mathematics achievement. The results may be helpful for mathematics educators, stake holders, and policy makers to highlight the necessity of using a problem solving approach in mathematics education and for students' being encouraged to use their invented algorithms in early elementary grades.

Research Questions

The following questions will guide the direction of this study:

1. Are there statistically significant differences in the number of first grade students in different strategy groups between treatment and control groups?
2. Are there statistically significant mean differences in first grade students' mathematics achievement (as measured by Iowa Test of Basic Skills) between different strategy groups controlling for students' prior mathematics achievement (as measured by student pretest)?
3. Are there statistically significant differences in the number of second grade students in different strategy groups between treatment and control groups?
4. Are there significant mean differences in second grade students' mathematics achievement (as measured by Iowa Test of Basic Skills) between different strategy groups controlling for students' prior mathematics achievement (as measured by student pretest)?

Organization of Dissertation

This dissertation is organized into five chapters. Chapter one includes the introduction which reviews the problem and its underlying framework, background of the study, the statement of the problem, and the purpose of the study. Chapter two contains a review of relevant literature. Chapter three details research questions, methodology, and statistical procedures for data analysis. Chapter four includes the data analysis and shows the results of the data analysis. The last chapter, chapter five, discusses the results of the data analysis, limitations for the current study, and recommendations for future research.

CHAPTER TWO: LITERATURE REVIEW

This chapter begins with a review of literature about Cognitively Guided Instruction and continues with a review of literature on; (a) children's strategies for single-digit addition and subtraction, (b) children's conceptual structures of multi-digit numbers, (c) children's strategies for multi-digit addition and subtraction, (d) school-taught algorithms, and (e) research studies focusing on children's use of invented algorithms and standard algorithms.

Cognitively Guided Instruction

Cognitively Guided Instruction is a professional development program based on research focused on students' mathematical thinking and teachers using students' thinking as a guide when they design their instruction (Carpenter et al., 2000). CGI does not provide a prescription or specific ways of teaching; rather teachers make decisions for their instruction based on the knowledge of their students' thinking (Wilson & Berne, 1999). A typical CGI classroom follows the sequence where the teacher poses a problem to students and allows them to solve the problem using a strategy of their preference. Next, several students with different types of solution strategies present their strategies to their classmates. Then, the teacher asks questions to elaborate the strategies to ensure that each strategy is clear to everyone in the class. Students may then be asked to compare their strategies with one another (Carpenter et al., 1999).

According to Steve (1998) CGI is an alternative to teacher professional development that focuses on creating new activities for students' learning. Rather than providing new activities, CGI focuses on changing teachers' beliefs and practices. Several other projects have also provided professional developments for teachers (e.g., the Summer Math for Teachers Project -

Schifter & Fosnot, 1993; Simon & Schifter, 1991; the Purdue Problem Centered Mathematics Project - Cobb et al., 1991). In the first phase of the Summer Math for Teachers Project, teachers were taught mathematics in a classroom where construction of meaning was valued and encouraged (Simon & Schifter, 1991). In the next phase of the program, teachers focused on students' learning. They studied students' understanding and misconceptions through the videotaped interviews conducted with individual students (Simon & Schifter, 1991). In the Purdue Problem-Centered Mathematics Project, teachers are provided with problem-centered mathematical activities and teaching strategies to use in their classes. These activities provided teachers with opportunities to attend to and reflect on students' thinking (Carpenter, Fennema, & Franke, 1996).

In the Summer Math Project, the mathematics served as a context for teachers to learn about student thinking, and in Purdue Problem-Centered Mathematics Project the activities served as a context to understand student thinking. On the other hand, in CGI students' thinking provides a context for teachers to improve their own understanding of mathematics (Carpenter, Fennema, & Franke, 1996).

In CGI the focus is more on helping teachers understand students' thinking by assisting them to construct the models of the development of students' thinking (Carpenter, Fennema, & Franke, 1996), because researchers have found that teachers have informal knowledge about students' thinking which is not coherently organized (Carpenter, Fennema, Peterson, & Carey, 1988). The CGI project deals with this lack of focus by using research findings to identify students' thinking in a model. Furthermore, Steve (1998) argues that CGI assists teachers' paradigm shift away from a teaching perspective towards understanding of students' learning.

Teachers' understanding of students' mathematical learning is very important as research in mathematics education has consistently reported an evidence of the benefits of attending to students' thinking (Franke et al., 2009).

The researchers of CGI have built their thesis on the belief that children bring a great deal of informal knowledge of mathematics to school that can be used as a basis for developing much of the formal mathematics of the elementary school curriculum (Carpenter, Fennema & Franke, 1996). Therefore, CGI encourages students to find their own ways to solve problems rather than having teachers teaching the procedures to solve them. Carpenter and Moser (1984) found that all addition and subtraction problems are not alike for children and identified different problem types based on children's understanding. Students' solutions showed that they see important distinctions among different types of addition and subtraction problems (Carpenter et al., 1996). The researchers of CGI proposed a framework in which addition problems are categorized as "Join" problems which are further categorized as "Join Result Unknown," "Join Change Unknown," and "Join Start Unknown" problem types (Carpenter et al., 1999). Similarly, subtraction problems are categorized as "Separate" problems which are further categorized as "Separate Result Unknown," "Separate Change Unknown," and "Separate Start Unknown." Addition and subtraction problems are further categorized as "Part-Part-Whole" and "Comparison" problems along with their specific subcategories. Table 1 illustrates the various problem types with a sample problem for each.

Table 1: CGI Problem Types

Categories	Subcategories		
Join	**Result Unknown** Jamie had 7 pencils. Tom gave her 8 more pencils. How many pencils did she have altogether?	**Change Unknown** Jamie has 7 pencils How many more pencils does she need to have 15 pencils altogether?	**Start Unknown** Jamie had some pencils. Tom gave her 8 more pencils. Now she has 15 pencils. How many pencils did Jamie have to start with?
Separate	**Result Unknown** Jamie had 15 pencils. She gave 7 to Tom. How many pencils did Jamie have left?	**Change Unknown** Jamie had 15 pencils. She gave some to Tom. Now she has 7 pencils left. How many pencils did Jamie give to Tom?	**Start Unknown** Jamie had some pencils. She gave 7 to Tom. Now she has 8 pencils left. How many pencils did Jamie have to start with?
Part-Part Whole	**Whole Unknown** Jamie has 7 red pencils and 8 blue pencils. How many pencils does she have?	**Part Unknown** Jamie has 15 pencils. Seven are red and the rest are blue. How many blue pencils does Jamie have?	
Compare	**Difference Unknown** Jamie has 15 pencils. Tom has 7 pencils. How many more pencils does Jamie have than Tom?	**Compare Quantity Unknown** Tom has 7 pencils. Jamie has 8 more than Juan. How many pencils does Jamie have?	**Referent Unknown** Jamie has 15 pencils. She has 8 more pencils than Tom. How many pencils does Tom have?

Note: CGI Problem Types. Adapted from Children's Mathematics: Cognitively Guided Instruction (p.12), by T. P. Carpenter, E. Fennema, M. Loef Franke, and S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi and Suzan B. Empson.

CGI provides a guiding framework that is based on different problem types varying their level of complexity and cognitive demand on children. In CGI workshops teachers learn about the classification of addition, subtraction, multiplication, and division problems and watch videos of children who use a variety of strategies to solve those problems (Wilson & Berne, 1999). The strategies for single-digit problems progress from *direct modeling*, to *counting strategies*, and then to *derived facts or recall* as the basis for students' problem solving strategies (Carpenter et al., 1999).

The original CGI study was an experimental study comparing mathematics achievements of the students of CGI teachers (n=20) and non-CGI teachers (n=20). Results of the study demonstrated higher mathematics achievement on solving word problems for students of CGI teachers when compared with the students of non-CGI teachers (Carpenter et al., 1989). This study, however, did not report significant differences in students’ use of strategies between the two groups. Following the original study, a quasi-experimental study was conducted in 1993 with 24 first grade teachers (n=12 for treatment, n=12 for control) and their students (n=144 for treatment, n=144 for control) (Villaseñor & Kepner, 1993). This study reported that the students of CGI teachers used more advanced problem solving strategies than the students of non-CGI teachers. Both studies reported results regarding first grade students’ strategies involving single digit numbers. The current study included students from both first and second grade and investigated students’ strategies from a broader perspective including single digit and multi-digit numbers. Since the original CGI study, several qualitative and quantitative studies investigated the effect of CGI on teachers and/or their students. Next, I discuss the findings of CGI related studies published in peer review journals.

Knapp and Peterson (1995) found that CGI developed an intervention that could change teachers’ beliefs and practices remarkably. They interviewed the 20 teachers four years after they attended the original CGI study. Half of the teachers reported noteworthy changes in their instructions.

Fennema et al. (1996) conducted a longitudinal study with 21 teachers and their students. They reported fundamental changes in teachers’ beliefs and instruction where their role evolved from demonstrating procedures to helping children build on their existing knowledge by attending to their mathematical thinking and encouraging them to solve a variety of problems.

The study also reported that changes in the instruction of individual teachers were directly related to the changes in their students' achievement.

The result of a case study of a teacher revealed dramatic changes in the teacher’s engagement with children’s thinking in a period of only a few months (Steinberg, Empson, & Carpenter, 2004). Yet another study reported that the changes in teachers’ practices were related to the increased years of experience with CGI (Jacobs, Lamb, & Philipp, 2010). Therefore, teachers’ implementation of CGI principles into their instruction increases as their experience with CGI increases.

The results of a kindergarten study on students’ problem solving processes provided an existent proof that many kindergarten students can learn how to solve a variety of word problems including multiplication and division problems by directly modeling the action or relationship described in the problem (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Learning how to model at the beginning might be crucial for students because some of the most obvious signs of problem solving deficiencies in older students appear to have occurred because they did not attend to the obvious features of problem situations (Carpenter et al., 1993).

The longitudinal study about invention and understanding in Children’s multi-digit addition and subtraction strategies showed that students who were given time to master invented strategies before being introduced to standard algorithms demonstrated better knowledge of base ten number concepts than students who were first introduced to the standard algorithms (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998). Students who used invented strategies were able to transfer their knowledge to new situations and were more successful on solving extension problems (Carpenter et al., 1998).

Most CGI studies have been conducted in schools that serve predominantly white middle class students (Turner & Celedon-Pattichis, 2011) and the critical point in the literature is that CGI needs to be implemented in more diverse environments including those with bilingual, Hispanic, and African American students. Identifying this gap, Turner and Celedon-Pattichis (2011) conducted a CGI study focusing on Latino students where the students were provided with a problem solving focused curriculum (Turner & Celedon-Pattichis, 2011). The results of this study showed that when given repeated opportunities to solve a variety of word problems, the achievement of young Latino students on post tests was comparable to that of their white middle class counterparts (Turner & Celedon-Pattichis, 2011).

Recently a replication study of CGI has started to re-examine the impact of this intervention on student achievement and teachers' beliefs when implemented with a larger and more diverse sample of students (Schoen, LaVenia, Tazaz, et al., 2014). The current study is a part of this CGI study and explores the effect of teachers attending the CGI professional development on their students' problem solving strategies at the first and second grade levels and the effect of students' use of different problem solving strategies on their mathematics achievement. In the next section, I discuss the CGI framework of children's use of different strategies for single-digit addition and subtraction problems.

Children's Use of Strategies for Single-Digit Addition and Subtraction Problems

Most children are able to learn at a young age how to count and understand many of the principles of numbers on which counting is based. Children's ability to count provides a basis for them to solve simple addition, subtraction, multiplication, and division problems (Kilpatrick et al., 2001). Learning and understanding whole number concepts is the main piece of the curriculum in the first years of elementary education, and appropriate learning experiences in these grades improve children’s chances for later success. Word problems are one of the most meaningful and appropriate contexts in which young children begin to develop proficiency with whole numbers (Kilpatrick et al., 2001).

Researchers generally agree that young children have a rich repertoire of informal problem solving strategies based on their preexisting knowledge of numbers when they first enter school (Carpenter 1985; Fuson, 1992). There is evidence that many kindergarten students are able to solve a variety of word problems by directly modeling the action or relationships described in the problem (Carpenter et al., 1993). As children’s number sense develops, they begin to use counting and invented strategies, which are more abstract and more efficient (Carpenter et al., 1999).

Research on children’s strategies to solve addition and subtraction problems involving single-digit numbers has provided a highly structured analysis of the development of addition and subtraction concepts and skills. For single-digit addition and subtraction, many children in different countries show the same learning progression (Fuson, 1992). In spite of the differences in details, researchers have drawn similar conclusions about children’s solution strategies for adding and subtracting single-digit numbers (Carpenter, 1985; Carpenter & Moser, 1984; Fuson, 1992).

Carpenter et al. (1999) describe three levels of progression that most children pass through in acquiring problem solving skills for addition and subtraction problems involving single-digit numbers. Initially, children solve problems using *direct modeling* strategies. Over time, these strategies are replaced by *counting* strategies, which are more efficient and require more sophisticated counting skills. Finally, children use *derived facts/recall,* which are based on number properties, to solve problems involving single-digit numbers.

Direct Modeling Strategies

Direct modeling involves use of physical objects of some kind or drawings to represent the action or relationship described in the problem. Children who are direct modelers are not able to successfully solve all problem types that can be modeled, since some problem types are more difficult to model than others. For example, most direct modelers find it difficult to solve join-start unknown or separate-start unknown problems because they cannot start to represent the initial number since the initial quantity is unknown (Carpenter et al., 1999). Direct modelers may also use counting strategies in situations for which a counting strategy is easier to apply (e.g. when the second addend is a small number). Table 2 summarizes different direct modeling strategies associated with different addition and subtraction problems that mostly include an action since there must be an action in order to use direct modeling strategy.

Table 2: Direct Modeling Strategies

Problem	Strategy Description
Join Result Unknown Jamie had 4 pencils. Tom gave her 9 more pencils. How many pencils did she have altogether?	**Joining All** Construct a set of 4 objects and 9 objects. Then join the two sets and count them all starting from 1.
Join Change Unknown Jamie has 4 pencils How many more pencils does she need to have 13 pencils altogether?	**Joining To** Construct a set of 4 objects. Add objects on to this set until there is a total of 13 objects. Then count the number of objects being added.
Join Start Unknown Jamie had some pencils. Tom gave her 9 more pencils. Now she has 13 pencils. How many pencils did Jaime have to start with?	**Trial and Error** Construct a set of some number of objects. Add 9 more to the set. Count all the objects in the set. If the final count is 13, then the number of objects in the initial set is the answer. If it is not 13, try a different initial set and repeat the process.
Separate Result Unknown Jamie had 13 pencils. She gave 4 to Tom. How many pencils did Jaime have left?	**Separating From** Construct a set of 13 objects. Remove 4 of them and count the number of remaining objects.
Separate Change Unknown Jamie had 13 pencils. She gave some to Tom. Now she has 4 pencils left. How many pencils did Jaime give to Tom?	**Separating To** Construct a set of 13 objects. Remove objects from the set until there are 4 objects left. Then count the number of objects removed from the set.
Compare Difference Unknown Jamie has 4 pencils. Toms has 9 pencils. How many more pencils does Tom have than Jamie?	**Matching** Construct a set of 4 objects and a set of 9 objects. Match the sets 1-to-1 until one set is used up. The answer is the unmatched objects remaining in the larger set.

Note: Direct Modeling Strategies. Adapted from Children's Mathematics: Cognitively Guided Instruction (p.19), by T. P. Carpenter, E. Fennema, M. Loef Franke, and S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi and Suzan B. Empson.

Counting Strategies

Counting strategies are generally represented by students using their fingers to count on or down from an initial number (Carpenter et al., 1999). Children using counting strategies recognize that it is not necessary to construct and count the sets. They can figure out the answer by focusing on the counting sequence itself. Sometimes they might use their fingers or any other object to keep track of their counting. Table 3 summarizes different counting strategies associated with different problem types.

Table 3: Counting Strategies

Problem	**Strategy Description**	
Join Result Unknown Jamie had 4 pencils. Tom gave her 9 more pencils. How many pencils did she have altogether?	**Counting On From First** Start from 4 and count on 9 more. The answer is the last number in the counting sequence.	**Counting On From Larger** Start with 9 and count on 4 more. The answer is the last number in the counting sequence
Join Change Unknown Jamie has 4 pencils How many more pencils does she need to have 13 pencils altogether?	**Counting On To** Start counting from 4 and continue until 13 is reached. The answer is the number of counting words in the sequence.	
Separate Result Unknown Jamie had 13 pencils. She gave 4 to Tom. How many pencils did Jaime have left?	**Counting Down** Start counting backward from 13. Continue for 4 more counts. The last number in the counting sequence is the answer.	
Separate Change Unknown Jamie had 13 pencils. She gave some to Tom. Now she has 4 pencils left. How many pencils did Jaime give to Tom?	**Counting Down To** Start counting backward from 13 and continue until 4 is reached. The answer is the number of words in the counting sequence.	

Note: Counting Strategies. Adapted from Children's Mathematics: Cognitively Guided Instruction (p.23), by T. P. Carpenter, E. Fennema, M. Loef Franke, and S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi and Suzan B. Empson.

Recall or Derived Number Facts

Recall or derived facts involve students using their number sense without using any physical objects or fingers to arrive at a solution (Carpenter et al., 1999). *Recall facts* are the number facts that students retrieve from memory without doing any computation in their head. Children usually learn some number combinations such as doubles and sums of tens before other combinations. Then, they often use this set of memorized facts to derive solutions for problems involving number combinations that they do not already know at a recall level. *Derived facts* solutions are based on children's understanding of number relations and most children use derived facts before they learn all number facts at a recall level. Therefore derived facts play an important role in learning number facts since it is much easier for children to acquire number facts if they understand the relationships among number facts (Carpenter et al., 1999). For instance, understanding 5+6 is 1 more than 5+5 makes it easier for children to retain the number fact of 5+6.

Children build their invented strategies for multi-digit numbers on the methods that they use for adding and subtracting single-digit numbers (Fuson, Wearne, et al., 1997). Children's use of different strategies for multi-digit addition and subtraction problems are also related to their development of conceptual structures of multi-digit numbers. Understanding these conceptual structures provide additional insight into understanding of children's strategies for multi-digit problems. Therefore, I discuss these conceptual structures in the next section.

Children's Development of Conceptual Structures for Multi-digit Numbers

Fuson, Wearne, et al. (1997) have developed a framework for children's understanding of multi-digit English number words (such as fifty-four) and written number marks (54). The framework provides a sequential development, which consists of five levels of conceptual structures of two-digit numbers that children acquire. The framework is an extension of Fuson's (1990) theoretical analysis, and integrates the theoretical perspectives of four different projects that were designed to help students learn number concepts with understanding (Fuson, Wearne, et al., 1997). These projects are; (a) Cognitively Guided Instruction (Carpenter et al., 1989, 1996), (b) Conceptually Based Instruction (CBI) (Hiebert & Wearne, 1992, 1993, 1996), (c) the Problem Centered Mathematics Project (PCMP) (Murray & Olivier, 1989), and (d) the Supporting Ten-Structured Thinking Project (STSTP) (Fuson, Freivillig, & Burghardt, 1992; Fuson, Smith, et al., 1997). I will discuss CBI, PCMP, and STSTP in detail at the end of this chapter.

Fuson, Wearne, et al. (1997) named these conceptual structures the UDSSI triad model after the first letters of the names of the five conceptual structures, which are; (a) *Unitary conceptions*, (b) *Decade and ones conception*, (c) *Sequence-tens and ones conception*, (d) *Separate-tens and ones conception*, and (e) *Integrated sequence-separate tens* conception. Each conceptual structure can be explained as a triad of two-way relationships between number words (such as five), written number marks (5), and quantities (5 objects). A child may acquire more than one conceptual structure at a time and may alternate in using different conceptions in different situations. Rather than replacing conceptions, children add new conceptions to the old ones (Fuson, Wearne, et al., 1997).

Unitary conceptions include both single-digit conceptions and multi-digit conceptions. The *unitary single-digit* conception requires children to understand the relations between the number word (such as five), the number mark (5), and the quantity (five objects). Children build multi-digit conceptions from unitary single-digit conceptions. Therefore, children must have learned how to read and say the number words for single-digit numbers, write the corresponding number mark, and count the quantities for each number word and number mark before learning two-digit numbers. The learning of the *unitary single-digit* triad is often achieved by rote memorization since single-digit number words and corresponding number marks are arbitrary in most languages.

The *unitary multi-digit* conception is an extension of the unitary single-digit triad where the triad shows the relationship between the whole word, the whole mark, and the whole quantity. In this stage, children are not able to differentiate quantities into groupings, and number words and number marks into parts. For example, according to children at this level the 1 in 18 is not related to the teen in eighteen, and 18 is not separable into 10 and 8.

The *decade and ones* conception requires children to be able to separate the decade and the ones parts of a number word and begin to relate each part to which the quantity refers. For example, in fifty-three the fifty refers to 50 objects and three to three objects. When children first acquire the *decade and ones* conception, they might make a specific error of writing the number mark 53 as 503. However, children eventually learn either by rote or by understanding that 0 is not written, and that 503 is five hundred three, not fifty-three.

The *sequence-tens and ones* conception requires children to construct a ten structured version of the decade and ones conception. At this level, children are able to count by tens, see the groups of tens within a quantity, and choose to count these groups by tens (e.g., “ten, twenty, thirty, forty”).

The *separate-tens and ones* conception requires children to see the quantity as separate tens and ones. When children acquire the *separate-tens and ones* conception, they are able to see and count the groups rather than the objects in the groups (e.g., “one ten, two tens, three tens, four tens”). Children may also omit the word tens and count the groups of tens using single-digit numbers (e.g., “one, two, three, four tens”).

The *integrated sequence-separate tens* conception requires constructing both the *sequence-tens and ones* and *separate-tens and ones* conception and being able to use them interchangeably based on the problem structures. A child at this level is able to recognize immediately that 60 has six tens without counting by tens to 60 with keeping track of how many tens he counted or counting six tens to find out that they make sixty.

While children develop multi-digit conceptual structures, it is possible for some children to develop an incorrect conception, called a *concatenated single-digit* conception. If a child develops a *concatenated single-digit* conception, he constructs the triad relation between number word, number mark, and quantity as if the digits in the number were separated columns of single digits. For example he sees the five in 53 as five and connects it to five objects. Cobb and Wheatley (1988) found that even when children construct correctly one of the adequate multi-digit conceptions and are able to use it successfully to add or subtract numbers presented in a word problem, they might still develop a *concatenated single-digit* conception for the computation problem presented vertically and make an error.

Children’s construction of these conceptual structures depends on their experiences both in and out of school. Therefore, not all children construct all the conceptions (Verschaffel, Greer & Corte, 2007). On the other hand, students in the same classroom may construct one or more of these structures earlier than the other ones (Fuson, Smith, et al., 1997). Children’s construction of these conceptual structures of multi-digit numbers affects their use of different strategies for multi-digit addition and subtraction problems that I discuss in the next section.

Children’s Strategies of Multi-digit Addition and Subtraction Problems

Children’s strategies for multi-digit addition and subtraction problems are generalizations of, or more advanced methods of, the strategies that they use for single-digit addition and subtraction (Fuson, Wearne, et al., 1997). Unlike single-digit addition and subtraction strategies, multi-digit procedures depend much more on what is taught (Kilpatrick et al., 2001). For example, children in different countries learn different algorithms to add or subtract multi-digit numbers. Usually children are taught these algorithms since they are not able to invent those algorithms on their own. On the other hand, when given opportunities children can invent their own strategies for carrying out multi-digit computations (Carpenter et al., 1998), which are different from school-taught algorithms. Furthermore students who construct their own correct strategies have a positive disposition towards mathematics and approach mathematics with confidence (Kamii & Dominick, 1998). Carpenter et al. (1999) identified three different levels of strategies that children use to solve multi-digit addition and subtraction problems. These are; (a) *counting single units*, (b) *direct modeling with tens*, and (c) *invented algorithms*. Fuson, Wearne, et al. (1997) name children’s strategies for multi-digit addition and subtraction differently and categorize them into two levels which are; (a) *unitary methods*, and (b) *kinds of* *methods using tens*. The *unitary methods* and *counting single units* strategy are alike and are used by the children who use *direct modeling* with ones or *counting* by ones strategies. Fuson, Wearne, et al.'s category of *kinds of methods using tens* combines Carpenter et al.'s *direct modeling with tens* and *invented algorithms* categories. In the current study, Carpenter et al.'s framework will be used to classify students' strategies, since the study will explore the effect of CGI instruction on students' strategies.

Counting Single Units (Unitary)

Before students use base ten number concepts, they may solve problems involving two-digit numbers by counting by ones. Students who are at this level either use; (a) *direct modeling* with ones strategies by physically representing the two-digit numbers and following the action or relationship described in the problem or (b) *counting* strategies to solve the problem. In either case students count all the numbers by ones (Carpenter et al., 1999).

Direct Modeling with Tens

Students using the *direct modeling with tens* strategy physically represent the quantities using tens and ones by following the action or relationship described in the problem. After directly modeling the quantities, a student may count them by tens, by ones, or by a combination of tens and ones. Many students are able to construct multi-digit numbers and count the sets using knowledge of grouping of ten before they understand that they can break apart the tens within a particular representation. Therefore students modeling two digit numbers with base ten blocks might find it more difficult to solve problems involving the separating action, specifically when they need to trade a ten for ones like in the problem 64 -27. On the other hand some students may simply cover up some of the blocks on a ten-rod with their fingers to arrive at a solution without trading a ten-rod with ones (Carpenter et al., 1999).

Invented Algorithms

Students can invent their own algorithms to solve addition and subtraction problems. Invented algorithms are different from standard algorithms in an important way. Kamii and Livingston (1994) argue that when students are encouraged to do their own thinking for adding and subtracting numbers, they universally invent *from left-to-right* procedures by starting from the digit on the leftmost, which is the digit with the greatest place value. The underlying reason for that is; when we think about 278, for example, we think “200, 70, 8” not “8, 70, 200”. In fact, invented algorithms require students to think flexibly about numbers; to understand that numbers can be broken apart or put together in different ways (Kamii & Livingston, 1994). When they invent their own methods, students often do not use paper and pencil to carry out their invented algorithms; rather they do it in their head (Carpenter et al., 1999). Fuson, Wearne, et al. (1997) have classified six types of student-invented algorithms as; (a) *the decompose-tens and-ones method: Add or subtract everywhere and then regroup*; (b) *the decompose-tens and-ones method: regroup then add or subtract everywhere*, (c) *the decompose-tens and-ones method: alternate adding/subtracting and regrouping*, (d) *the begin-with-one-number method: begin with one and move up or down by tens and ones*, (e) *mixed methods: add or subtract tens, make sequence number with original ones, add/subtract other ones*, and (f) *change both number methods*. Carpenter et al. (1999) have identified three major types of invented strategies that are *incrementing, combining tens and ones,* and *compensating*. These three categories combine several categories that are presented separately by Fuson, Wearne, et al., (1997). Table four summarizes and gives examples of the three major types of invented strategies described by Carpenter et al. (1999), including explanations and examples from Fuson, Wearne, et al. (1997). The current study used CGI framework as a base to determine strategy groups, since it investigated the differences in students' use of strategies between the treatment and control groups, where the treatment was CGI professional development for teachers.

Table 4: Student Invented Algorithms

Student Invented Strategies

	Problem Types		
Strategies	**28 + 35**	**74 - 35**	**28 +□= 53**
Incrementing	Count on/add on tens, then ones 28, 38, 48, 58, 59, 60, 61, 62, 63 28 + 30→58 + 5→63	Countdown/subtract tens, then ones 74, 64, 54, 44, 43, 42, 41, 40, 39 74 - 30→44 - 5→39	Count on/add up tens, then ones 28, 38, 48, 49, 50,51, 52, 53: 35 added 28 + 20→48 + 5→53 : 25 added
	Count on/add on to make a ten, count on/add on tens, then the rest of ones 28, 29, 30, 50, 60, 61, 62, 63 28 + 2→30 + 30→ 60 + 3→63	Countdown/subtract to make a ten, countdown/subtract tens, then the rest of the ones 74, 73, 72, 71, 70, 60, 50, 40, 39 74 - 4→70 - 30 →40 - 1 →39	Count up/add up to make a ten, count up/add up tens, then the rest of ones 28, 29, 30, 40, 50, 51, 52, 53: 35 added 28 + 2→30 + 20 →50 + 3 →53: 25 added
	Count on/add on tens, add on ones, count on/add on other ones 20, 30, 40, 50, 58, 59, 60, 61, 62, 63 20 + 30→50 + 8→58 + 5→63	Countdown/subtract tens, add original ones, countdown/subtract other ones. 70, 60, 50, 40, 44, 43, 42, 41, 40,39 70 - 30 →40 + 4→ 44 - 5 →39	Count up/add up tens, add original ones, count up/add up other ones 20, 30, 40, 50, 58, 59, 60, 61, 62, 63: 25 added 20 + 30 →50 + 8 →58 + 5→ 63: 25 added
Combining Tens and Ones	Add tens, add ones, combine tens and ones 20 + 30 = 50, 8 + 5 = 13, 50 + 13 = 63	Subtract tens, subtract ones, combine totals 70 - 30 =40, 4 - 5 = -1, 40 -1 = 39	**
	Add ones, add tens, combine tens and ones 8 + 5 = 13, 20 + 30 = 50, 13 + 50 =63	Subtract ones, subtract tens, combine totals 4 - 5 = -1, 70 - 30 = 40, 40 - 1 =39	**
Compensation	Overshoot and come back 30 + 35 would be 65. 65 - 2 would be 63	Overshoot and come back 74 - 40 would be 34, 34 + 5 would be 39.	If it were 30, 30 + 23 would be 53, but it is 28, so add 2 more, it would be 25.
	Move some from one number to the other to make a tens number 28 + 2 →30, 35 - 2→33, 30 + 33→63	Make subtracted numbers a tens numbers, change other to maintain the difference 74 -35 = 79 - 40 = 39	Make initial number a tens number, change other to maintain difference 28 +□ = 53 is the same as 30 +□= 55, so □=25

Note: Adapted from both Children's Mathematics: Cognitively Guided Instruction (p.23), by T. P. Carpenter, E. Fennema, M. Loef Franke, and S. B. Empson, 1999, Portsmouth, NH: Heinemann. Copyright by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi and Suzan B. Empson, and "Children's Conceptual Structures for Multidigit Numbers and Methods of Multidigit Addition and Subtraction," by K. Fuson, D. Wearne, J.C. Hiebert, H. G. Murray, P. G. Human, A.I. Olivier, T. P. Carpenter, E. Fennema, *26*, p. 147-148.

School Taught Algorithms

"An algorithm is a step-by-step process that guarantees the correct solution to a given problem, provided the steps are executed correctly" (Barnett, 1998, p. 69). Usiskin (1998) lists the reasons for teaching algorithms as well as the dangers inherent in them. He states that we teach algorithms because they are powerful, reliable, fast, and instructive. Algorithms are powerful because they can be applied to classes of problems. When we know a particular algorithm we can apply it not only to one task, but also to all tasks of a particular kind. They are reliable because when done correctly they yield the correct answer all the time. They are fast because they provide a direct routine to the answer, and they are instructive because some algorithms are based on important mathematical ideas although they may not be seen easily, such as the regrouping action in addition that applies to the ideas of place value (Usiskin, 1998).

On the other hand, Usiskin (1998) argues that the properties that make algorithms important may also generate dangers. For example, since they are reliable when done correctly, students often blindly accept the answers without checking the reasonableness of their answers. Another danger is the overzealous application of algorithms, which is a tendency for students to over apply them even if the task could easily be done mentally. For example, a child may attempt to use a standard algorithm to calculate 28 + 32, which can be easily done mentally. Another danger of algorithms is the belief that algorithms train the mind. Although algorithms provide mental images, there is no evidence that these images transfer to broader abilities such as problem solving and creative thinking. In fact, evidence shows that "difficult algorithms seem to take students minds off the bigger picture and keep more important mathematics from being taught" (Usiskin, 1998, p. 16). Kamii and Dominic (1998) state that algorithms are efficient for adults who already knew that the four in 45 means 40. However, they do not enhance place value understanding of children who are still trying to make sense of the place value concept. Historically, the use of algorithms at the elementary and secondary levels has been emphasized in the teaching and learning of mathematics (Mingus & Grassl, 1998). The ongoing NCTM reform movements, however, de-emphasize the importance of algorithms and stress the importance of problem solving approaches, the conceptualization of mathematical processes, and real world applications of mathematics (Mingus & Grassl, 1998). The Common Core State Standards emphasize the use of strategies and algorithms that are based on place value and properties of operations until fourth grade and specify that students should “fluently add and subtract multi-digit whole numbers using the standard algorithm” in the fourth grade (CCSSM, p. 29). In addition, Reys and Thomas (2011) noted that the authors of CCSSM did not provide a definition for the standard algorithm. They argued that, “if the authors of CCSSM had a particular standard algorithm in mind, it was not made explicit nor is an argument offered for why a particular (standard) algorithm is expected” (p. 26). In fact there are many variations of algorithms that are used in the United States (Kilpatrick et al., 2001), and also in other countries (Fuson & Li, 2009). I will discuss several different algorithms that are used in the U.S. and in other countries in the following section.

Different Types of Addition and Subtraction Algorithms

Most students believe that algorithms are unique and need to be memorized. As a result, many of them believe that mathematics is a collection of rules that must be followed. However, if students understand that algorithms are not unique and different algorithms can be used to solve the same problem, they may start to think that mathematics is not a collection of rules, rather it is a way of making sense of the world (Sgroi, 1998). Most importantly, if students realize that mathematical procedures can be invented and are not unique, they may see themselves as future inventors of mathematics (Rubenstein, 1998). Exploring a variety of algorithms might help to lead to this desired outcome. There are many variations of algorithms that are used in the U.S. (Kilpatrick, et al., 2001), and also in other countries (Fuson & Li, 2009).

The Common U.S. Algorithm for Addition

When using the common U.S. algorithm for addition, students start with adding the numbers in the ones column. If the sum is equal to or larger than 10, students first regroup the ones into a ten, then they record the sum of the remaining ones in the ones place, and then place the regrouped ten above the top of the tens digit column. Students then add the numbers on the tens digit repeating the same regrouping procedure if the sum is equal to or larger than 10 and so on. Figure 2 illustrates the common U.S. algorithm for addition.

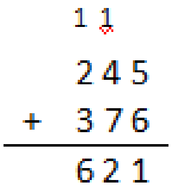
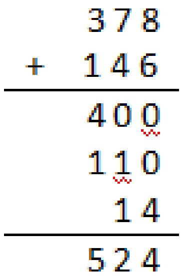


Figure 2: The common U.S. algorithm for addition

Teachers who use the conventional language for the addition algorithm would describe the regrouping process as *carry the 1* without connecting it to the regrouping principle on which the procedures of the standard algorithm are based.

Partial Sums Algorithm for Addition

Most students are able to develop different strategies that are effective to solve addition problems. For example, in solving 37 + 46, many students will mentally add 30 and 40 to get 70, then 6+7=13, and finally 70+13=83. However mental computations become difficult as the numbers get greater or contain decimals. The partial sums method, which emphasizes place value, can be used with large numbers, and it has been found to be useful by many teachers and students (Carrol & Porter, 1998). In this method numbers are first added by their place value. For example, to add 378 and 146, students first add the hundreds (300 +100) and continue from left to right, recording each partial sum. At the end they combine the partial sums. Figure 3 illustrates the partial sums algorithm for addition.



```
    378
+   146
-------
    400
    110
     14
-------
    524
```

Figure 3: The partial sums algorithm for addition

The common U.S. Algorithm for Subtraction

When using the common U.S. algorithm for subtraction, students subtract each digit of the subtrahend from the digit above it, starting from right to left. If the ones digit of the top number is less than the ones digit of the bottom number, students regroup one 10 from the tens digit as 10 ones, if the tens digit is other than 0. Then, they subtract one from the tens digit and add the 10 ones to the ones digit. Next they subtract the ones digit and then move on to the next digit, regrouping as needed, until every digit has been subtracted. Figure 4 illustrates the common U.S. subtraction algorithm.

$$\begin{array}{rccc} & & {}^{5} & {}^{1} \\ & 5 & \cancel{6} & 7 \\ - & 3 & 4 & 9 \\ \hline & 2 & 1 & 8 \end{array}$$

Figure 4: The common U.S. algorithm for subtraction

Teachers who use the conventional language for the subtraction algorithm would describe the regrouping process as *borrowing* from the next left digit, which hides the regrouping principle that underlies the procedure of the subtraction algorithm.

Partial Differences Algorithm for Subtraction

The partial differences method for subtraction is similar to the partial sums method for addition. When using this algorithm, students find the difference between two numbers in each column (Carrol & Porter, 1998). For example to subtract 476 from 832 students first subtract the hundreds (800-400), and then the tens (30-40), and continue from left to right, recording each partial difference. At the end they combine partial differences. Figure 5 illustrates the partial differences algorithm for subtraction.

$$\begin{array}{rccc} & 8 & 3 & 2 \\ - & 4 & 7 & 6 \\ \hline & 4 & 0 & 0 \\ & - & 4 & 0 \\ & - & & 4 \\ \hline & 3 & 5 & 6 \end{array}$$

Figure 5: Partial differences algorithm for subtraction

As it is seen from figure 2, the partial differences method may involve use of negative numbers, which may seem difficult for elementary school students. However many students use them with little difficulty, and some develop this method on their own. Students consider the negatives as having a deficit of that quantity rather than as positive and negative numbers (Carrol & Porter, 1998).

Europe – Latino Algorithm for Subtraction

Ron (1998) describes another alternative algorithm for subtraction, the Europe-Latino (E-L) algorithm, which is also known as the *add tens to both* or the *equal additions* method. This algorithm relies on the fact that the result of 583-47 is the same as 593-57. In this method both numbers are changed equally by adding a ten to each number. For example as it is seen in the example below, to subtract 47 from 583, students first add ten ones to the ones in the top number (the minuend), so the 3 becomes 13. Then they add a ten to the tens in the bottom number (the subtrahend), so the 4 tens become 5 tens. The difference between the adjusted subtrahend and the adjusted minuend is then typically determined by counting up, that is the child thinks from 7 to 13 is 6, and from 5 to 8 is 3. Figure 6 illustrates the Europe-Latino algorithm for subtraction.

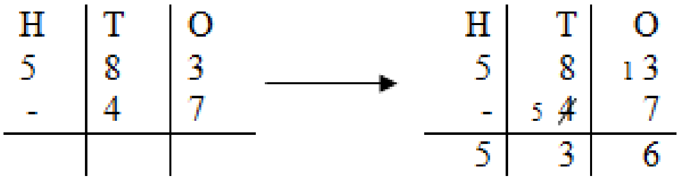


Figure 6: Europe-Latino algorithm for subtraction

Each algorithm has its advantages and disadvantages. Hence it is important for educators to think about which algorithms to teach and reasons for teaching those (Kilpatrick et al., 2001).

Next, I will discuss the differences between using invented algorithms and the common U.S. standard algorithms since they are the most prevalent algorithms that children learn in U.S. schools.

Differences between Standard Algorithms and Invented Algorithms

The differences between standard algorithms and invented algorithms were clearly put forward by Plunkett (1979). He pointed out that standard algorithms have the advantage of providing a routine that will work for any numbers, can be taught to, and carried out by someone who has no understanding of what is happening. The disadvantages are that; they do not correspond to how people think about numbers, and they do not encourage students to think about the numbers involved in problems. Rather, they encourage a belief that mathematics is arbitrary.

Learning the standard algorithm for addition with understanding poses three difficulties for students (Kilpatrick et al., 2001). First, the procedure moves from right to left in contrast to reading and in contrast to most invented algorithms. Second, placing the “carried” 1’ s above the top number can be a source of confusion since it changes the numbers while it does not change the sum. Third, while adding numbers in a given column children may forget to add the extra 1 (the ten or the hundred).

The procedure for the U.S. method of subtraction also poses several difficulties. It moves from right to left and involves alternating between two major steps. Step one involves regrouping when the digit in the top position is lesser than the same digit in the bottom number. Step two involves subtracting after the top number has been “fixed”. Alternating between these two steps poses three potential difficulties for children. The first difficulty is to learn this alternation and understand the reasons for it. The second is to remember to alternate the steps. Third is the possibility that alternation may cause children to generate a very common subtracting error, which is subtracting the lesser top digit from a greater bottom digit (Kilpatrick et al., 2001).

MacIntosh (1998) explains the distinction between standard algorithms and invented algorithms by an example. He states that when a group of students is asked what is 36 + 79, a number of students, who are from a classroom in which standard algorithms have been heavily emphasized, “will screw up their eyes, raise their hands as though writing in the air in front of them, and say, 6 and 9 are 15; put the 5 down and carry the 1; 3 and 7 are 10, and 1 more is 11. The answer is 115” (p. 45). On the other hand invented algorithms are flexible, adaptable to suit the numbers and almost always require understanding. When students use invented algorithms, we will expect to hear any or all of the following:

“3 and 7 are 10; 6 and 9 are 15; that’s 115.”

“30 and 70 are 100, 6 and 9 are 15; that’s 115.”

“36 and 80 are 116; less 1 is 115.”

“36 and 70 are 106; and 9 is 115.”

“79 and 6 are 85, and 30 is 115.”

“79 and 21 are 100; 36 less 21 is 15; 100 and 15 are 115.” (p. 45).

Student invented strategies are built on the foundational number concepts and on the fundamental properties of the number system; like the commutative, associative, and distributive (for multiplication) properties, and these are quite visible when one examines students’ strategies. Although standard algorithms are also built on number concepts, they are not quite visible for children to understand their conceptual underpinnings (Kilpatrick et al., 2001). When students learn standard algorithms without understanding, the reasoning behind them like why the “ones” are being “carried,” is often unclear which consequently causes students to develop some flawed procedures (Carroll & Porter, 1998), which result in systematic errors (Kilpatrick, Martin, & Schifter, 2003).

Students’ errors are not always of the same type. Some errors in procedures can be associated with students’ carelessness or overloaded working memory (Lemaire, Abdi, & Fayol, 1996), and some others can be due to the faulty or “buggy” algorithms students use (Brown & Burton, 1978). Brown and Burton (1978) have identified the most frequently occurring bugs in their study, where they interviewed 1,325 students. The descriptions and examples of these errors can be found in Table 5.

Table 5: Common Subtractions Bugs

Category	Common Subtraction Bugs
Smaller From Larger	Student subtracts the smaller number in a column from the larger number regardless of which one is on top. (324 – 117 = 213)
Borrow From Zero	When borrowing from a column whose top digit is 0, student writes 9 but does not continue to borrow from the column to the left of the zero. (502 – 347 = 255)
Borrow Across Zero	When the student needs to borrow from a column whose top digit is 0, he skips that column and borrows from the next one. (407 – 229 = 128 or 407 – 229 = 108) Note: This bug must be combined with either bug 5 or 6)
Stops Borrow at Zero	The student borrows from zero incorrectly and adds 10 correctly to the top digit of the current column. (406 – 348 = 148 or 406 – 348 = 108) Note: This bug must be combined with either bug 5 or 6)
0 – N = N	Whenever the top digit in a column is 0, the student writes the bottom digit as the answer. (205 – 183 = 182)
0 – N = 0	Whenever there is a 0 on top, the digit 0 is written as the answer. (205 – 112 = 103)
N – 0 = 0	Whenever there is a 0 on the bottom, 0 is written as the answer. (324 – 102 = 202)
Don’t Decrement Zero	When borrowing from a column in which the top digit is 0, the student rewrites the zero as 10, but does not change the 10 to 9 when incrementing the active column. (403 – 268 = 145)
Zero Instead Of Borrow	The student writes 0 as the answer in any column in which the bottom digit is larger than the top. (446 – 129 = 320)
Borrow From Bottom Instead of Zero	If the top digit in the in the column being borrowed from is 0, the student borrows from the bottom digit instead. (303 – 168 = 255 or 303 – 168 = 105 Note: This bug must be combined with either bug 5 or 6.

Note: Descriptions and examples of Brown and Burton’s (1978) common subtraction bugs. Adapted from Advances in Instructional Psychology (p. 45), ed. By R. Glaser, 1987, Hillsdale: NJ, Lawrence Erlbaum Associates. Copyright by Lawrence Erlbaum Associates. The ‘borrow’ language was used in the original table and was not changed on this table.

Review of Research Related to Invented Algorithms and Standard Algorithms

Studies examining student invented strategies have revealed that students who use invented algorithms have better understandings of the concepts and perform better than those who use standard algorithms. For example, Carpenter et al. (1998) found that students who were given time to master invented strategies before being introduced to the algorithm, demonstrated better knowledge of base ten number concepts than students who first learned the algorithms. Students who used invented strategies were able to transfer their knowledge to new situations and were more successful solving extension problems.

Kamii and Dominic (1998) investigated the effects of teaching computational algorithms by interviewing second, third, and fourth graders in 12 classes and reported that those who had not been taught any algorithms produced significantly more correct answers. In the case of errors, the incorrect answers of those who had not been taught algorithms were much more reasonable than those found in the classes where the emphasis was on algorithms. They concluded that algorithms hinder children's development of number sense and place value understanding.

Many children, who correctly carry out the algorithms procedurally, do not conceptually understand the reasons underpinning the procedures (Cobb & Wheatley, 1988). On the other hand, Fuson and Briars (1990) found that most of the students who practiced addition and subtraction with base ten blocks were able solve addition and subtraction problems correctly without using base ten blocks. These students also demonstrated meaningful addition and subtractions concepts such as identifying the traded one as a ten in both addition and subtraction problems.

Fuson et al. (1992) provided a more detailed description of the use of base ten blocks in another study. They chose 26 students among the highest achieving students of three second grade classroom to examine how easy it was for children to construct a relationship among number words, written multi-digit marks, and base ten blocks while exploring addition with the blocks, number marks, and maintaining these relationships. They assigned children to groups according to their initial knowledge levels i.e. high, medium, and low groups based on their pretest performances. An adult experimenter was assigned to each group to monitor the students' learning, collect data, and intervene if the groups were making wrong connections and not able to notice that. Each group was provided with base ten materials and different activities including linking the blocks, written marks, finding the ten-for-one equivalency, and up to 4 digit addition problems that would require trading in one, two, or three places. Their first conclusion was that it was fairly straightforward for children to use the features of the blocks to carry out correct blocks addition. The second conclusion was that second grade students could easily link the quantitative features of the blocks with written marks and English words.

Romberg and Collis (1985) concluded that children who have the capacity to reason about quantitative problems often do not use algorithmic procedures even though they know how to use them. On the other hand, children whose capacity to reason about quantitative problems is suspicious and who have not acquired other skills like direct modeling and counting may use the standard algorithm, but often make errors. Carraher and Schliemann (1985) did an error analysis to evaluate the relation between errors and children's use of counting strategies and school-taught procedures. They interviewed 50 Brazilian children ranging from seven to 13 years of age. They found that use of school-taught procedures was associated with the highest percentages of wrong answers, especially for subtraction tasks. Two kinds of errors were identified in school-taught routines. The most frequent error (in half the cases) was misinterpreting the rule “you can’t subtract the larger number from smaller number” to mean “subtract the smaller digit from the larger, which led some children to conclude that 25 is the result of 21 – 6.

Hiebert and Wearne (1996) analyzed the relations between children’s understanding of multi-digit numbers, their computational skill, and how instruction influenced these relations. They followed about 70 children over the first three years of schooling while they were learning about place value, multi-digit addition, and subtraction in two different instructional environments in which teachers used either conventional textbook instruction or alternative instruction. In the study, alternative instruction was described as an instruction that encouraged students to invent their own procedures and to make sense of procedures presented by others. Seventy children were interviewed and asked to solve tasks that were designed to assess their understanding of base-ten number system as well as their skills in adding and subtracting multi-digit numbers. They compared the students who received alternative instruction with those who received conventional instruction based on the tasks that assess their understanding. The differences were not significant between the two groups at the end of the first and second grade; however the difference was significant at the end of the third grade favoring the alternative instruction group. To measure the relationship between understanding and skill, they identified students at each interview who demonstrated substantial understanding (understanders) and who did not (nonunderstanders). The comparisons between the two groups showed that, before instruction on a task, students in the understanders group gradually improved their performance on the task by inventing procedures, whereas students in the nonunderstanders group did not.

They concluded that understanding and computational skills were closely related, and alternative instruction appeared to facilitate higher levels of understanding and skill.

Murray and Olivier (1989) analyzed the data that consisted of 147 interviews with third grade students who had at least nine months of intensive instruction on place value and the standard algorithm for addition. The problems used in the interviewing process were context free addition problems of increasing size. During interviews children were encouraged to use any strategy of their preference and were then asked to explain their strategy. They found that children used the standard algorithm infrequently; rather they used untaught informal computational strategies. Based on their findings, they formulated a theoretical framework that describes four levels of understanding of two digit numbers, which is shown in Table 6.

Table 6: Description of Children's Levels of Understanding of Two-digit Numbers

Levels of Understanding of Multi-digit numbers	Description
1st Level	A child has not yet acquired the numerocities of two digit numbers. May use counting all strategy to arrive at an answer.
2nd Level	A child has acquired the numerocities of two digit numbers, and may use counting on strategies to arrive at an answer.
3rd Level	A child can see multi-digit numbers as composite units of decade and ones.
4th Level	A child can see multi-digit numbers as groups of tens and some ones.

Murray and Olivier (1989) suggested that level 4 understanding is a prerequisite to execute the standard algorithm meaningfully. In general when level 1 and 2 students have difficulty in computation with larger numbers, teachers seem to "help" children by introducing the standard algorithm. However, researchers argued that even if the teachers try to build a conceptual basis for the algorithms (level 4), such efforts would be ill fated if level 2 and level 3 are bypassed. They concluded that superficial facility in executing the algorithm might hide serious deficiencies. In the next section I will discuss the aforementioned projects that were designed to help children learn number concepts with understanding.

Conceptually Based Instruction

Conceptually Based Instruction (CBI) is built on the notion of constructing connections between representations of mathematical ideas. Such instruction supports students' efforts to build relationships between physically, pictorially, verbally, and symbolically represented quantities and actions on quantities (Hiebert & Carpenter, 1992). Instruction that focuses on helping students construct connections provides one form of teaching for understanding.

In their study, Hiebert and Wearne (1992) were interested in the link between instruction, understanding, and performance. They compared the effects of CBI with the effects of conventional textbook instruction on children’s understanding of place value and their performances of multi-digit addition and subtraction with regrouping. CBI was provided in four first grade classrooms and conventional textbook-based instruction was provided in two first grade classrooms. Four principles guided the development of the conceptually based instruction. First, physical, pictorial, verbal, and symbolic representations were used as tools for demonstrating, recording, and communicating about quantities. Second, students were given enough opportunities to practice and become familiar with the use of representations after they were introduced to the students. Third, representations were used as a tool to solve problems, and fourth, class discussions focused on how to use the representation as well as their similarities and differences. Base ten blocks and unifix cubes were used as physical representations. The lessons began with posing problems to find the number of objects in sets consisting of 50-100 objects.

Class discussion and strategies began with counting by ones, and shifted to more efficient ways of counting such as by twos, by fives, and eventually by tens. Discussion about two-digit numbers frequently included the two ways of interpreting the number. Two digit addition and subtraction without regrouping were presented with join and separate word problems. Different representations were used to solve the problems and class discussion included presentation and explanation of solution strategies by the students and teacher. Researchers found that students who received conceptually based instruction performed significantly better on items measuring understanding of place value, two-digit addition and subtraction with regrouping, and they used strategies related to the tens and ones structure of the number system more often.

The Problem Centered Mathematics Project

The Problem Centered Mathematics Project focused on the mathematics curriculum in first through third grade and was interested in building on children's informal knowledge as well as studying and facilitating the development of their conceptual and procedural knowledge (Olivier, Murray, & Human, 1990). In their study Olivier, Murray, and Human (1990) developed an experimental curriculum based on the constructivist approach to be implemented in the treatment classrooms. Standard algorithms were not taught in these classrooms, and the teachers' role was to present all mathematical activity with a problem solving approach and challenge students to solve problems using their own strategies. Students were also expected to demonstrate and explain their methods both verbally and in a written form. Students were provided with loose counters and two sets of numeral cards in multiples of ten and one. For example, to represent the number 34, students needed to take the "30" card and place the "4" card over the zero of 30. Researchers concluded that a vast majority of students in treatment classrooms rapidly progressed to level 3 strategies and outperformed the students in control classrooms in all aspects of computation and word problems. Treatment group students also showed higher qualitative understanding of number and computational strategies. They identified different types of strategies used by the students, which are: (a) *accumulation*, (b) *iterative*, and (c) *replacement* strategies. In the CGI framework, accumulation falls into the combining tens and ones category, the iterative strategies fall into the incrementing category, and replacement falls into the compensating category of strategies.

The Supporting Ten-Structured Thinking Project

The Supporting Ten-Structured Thinking Project aimed to support first grade students' thinking of two-digit quantities as tens and ones (Fuson, Smith, et al., 1997). In their study researchers used the UDSSI triad model, developed by Fuson, Wearne, et al. (1997), to describe children's conceptual structures and to guide instructional design work. They sought to describe and then compare the learning of the children as it compares with that of East Asian and U.S. samples. They had two experimental classes; one was a Spanish speaking first grade class with 17 students, and the other was an English speaking class with the number of students ranging from 24 to 28. Researchers built teaching and learning activities in order to help children see objects grouped into tens and relate these ten-groupings and remaining ones to number words and number marks. They used penny frames, base-ten blocks, and methods such as children's drawing of quantities organized by ten to help children construct these conceptual structures. Children were assessed on various tasks that examined their thinking, whether unitarily or with tens and ones. The students from both classes demonstrated tens-and-ones thinking, and their performance looked more like that of east Asian children. Most children in the project were able to add and subtract involving regrouping and explain their regrouping. Their performance was considerably higher than that reported for U.S. children receiving traditional and reform instruction.

All four projects were similar in the sense that they were designed to improve children’s conceptual understanding of number concepts and operations. They all took a problem solving approach to teaching multi-digit concepts and operations. In all projects the teacher’s role was more like a facilitator of students’ learning, rather than being the transmitter of knowledge. Teachers did not introduce students to the standard algorithms, but rather encouraged and expected their students to invent their own strategies. The intent was to create a learning environment, in which children became active participants of the learning process and constructed their own understanding. In all of the projects excluding CGI, either researchers or other staff facilitated the classroom learning and teaching. In CGI, teachers attended CGI workshops and then facilitated the learning in their own classrooms.

Existing studies have shown that students who use invented algorithms have a better understanding of place value concepts and number properties. However we do not know much about the impact of students’ use of different strategies on their mathematics achievement as measured by a standardized test, which is generally used to compare students’ mathematics achievement at state, national, and international levels. The current study provides additional insight into the understanding of the impact of the use of different strategies on students’ mathematics achievement as measured by a standardized test.

CHAPTER THREE: RESEARCH DESIGN AND METHODOLOGY

The purpose of this study was to explore the effect of teachers' attending CGI professional developments on their students' problem solving strategies, and the effect of students' use of different problem solving strategies on their mathematics achievement. It is important to note that the study was conducted at the end of the first year of a two-year planned CGI professional development. Therefore the results of this study should be interpreted cautiously.

The current study is part of a larger cluster-randomized controlled trial. The chosen unit of randomization was the school from which teachers were invited to participate in the study. The schools that have at least three consenting teachers per grade level in first and second grade were assigned to either treatment or control group at random. The school level randomization ensured the minimization of treatment diffusion and eliminated the possibility of cross-classification of students who might transfer from treatment to control or from control to treatment classes within the same school. The following research questions were analyzed in this study;

1. Are there statistically significant differences in the number of first grade students in different strategy groups between treatment and control groups?
2. Are there statistically significant differences in the mathematics achievements (as measured by the Iowa Test of Basic Skills) of first grade students between different strategy groups controlling for students' prior mathematics achievement (as measured by student pretest)?

3. Are there statistically significant differences in the number of second grade students in different strategy groups between treatment and control groups?
4. Are there statistically significant differences in the mathematics achievements (as measured by the Iowa Test of Basic Skills) of second grade students between different strategy groups controlling for students' prior mathematics achievement (as measured by student pretest)?

For the current study, first and second grade students were investigated separately since research shows that older children have more advanced problem solving strategies than younger children (Canobi, Reeve, & Pattison, 2003; Carpenter & Moser, 1984). First and second grade students' strategies to solve single-digit and multi-digit problems were classified according to the strategy groups that were determined based on the CGI framework of strategies. For single-digit problems, the strategy groups were identical for both grade levels. However, strategies to solve multi-digit problems were classified in a different way for first and second grade students. The reason for the different classification is that the students in this study might have learned the procedures of standard algorithms in second grade if their teachers followed their textbook, which introduces both invented algorithms and standard algorithms at the second grade level (Dixon, Larson, Leiva, & Adams, 2013).

Description of Strategy Groups

CGI framework of strategies was used to determine the strategy groups that were under analysis in the current study. In place of *direct modeling* and *direct modeling with tens* strategies, the strategy groups included *concrete modeling* and *concrete modeling with tens* strategies to include all students who represented all quantities with or without following the relationship described in the problem. To understand the distinction, consider this problem:

Tanya had 18 apples. Her mother gave her some more apples and now she has 22 apples. How many apples did her mother give to Tanya?

To solve this problem, if a child represents 18 and then adds on 18 until he got to 22 to get the answer, this child is said to be representing all quantities by directly modeling the relationship described in the problem. If a child represents 22 and takes 18 away to get the answer, this child is said to be representing all quantities without following the relationship described in the problem since there is no subtraction action described in the problem.

For single digit problems, the strategies were categorized as (a) *concrete modeling*, (b) *counting*, and (c) *derived facts/recall* strategies. The *concrete modeling* strategy group includes students who represented all quantities with ones either by following or not following the action or relationship described in the problem. Likewise, the *counting* strategy group includes students who counted by ones to arrive at an answer but without representing all quantities with physical objects. Students in this group may have used their fingers or any other objects to keep track of their counting. The *derived facts/recall* strategy group includes students who used number properties, relations, or recall to arrive at an answer.

At the first grade level, the strategy groups for solving multi-digit problems was initially proposed to be as *unitary*, *concrete modeling with tens*, *invented algorithms*, and a *mixed* category which would have been further classified as *lower mixed* and *higher mixed* strategy groups. However, the preliminary analysis of data suggested different strategy groups labeled as: (a) *other, (b) unitary, (c) concrete modeling with tens,* and (d) *invented algorithms*, which is discussed in detail in chapter four. The *other* strategy group includes the students who did not use any aforementioned multi-digit strategies but used an *other* strategy, which is unidentifiable. On average, the other strategies yielded a false response for 95% of the time. For most of the time, a strategy was coded as *other* strategy if the students used apparent guess such as picking one of the numbers given in the problem as a response. At very rare cases (on average 5% of the time), the *other* strategy yielded a correct response and at those times the strategy used was not identifiable by the interviewer. The *unitary* group includes students who used *concrete modeling* with ones or *counting* by ones strategies. The *concrete modeling with tens* group includes students who modeled all quantities with tens and counted by tens or by tens and ones. The *invented algorithm* group includes students who used combining tens and ones, incrementing, or compensating strategies. The initial analysis of data suggested that no *mixed* group to be formed, which is discussed in detail in chapter four.

For second grade students, the strategy groups were initially proposed to be: (a) *unitary*, (b) *concrete modeling with tens*, (c) *invented algorithms*, (d) *standard algorithms*, and (e) *mixed* strategies. However, the preliminary analysis of data showed that, about half of the students who used *standard algorithms* did not use any of the more advanced strategies (*concrete modeling with tens* or *invented algorithms*) whereas the other half used either of them at least one time. Therefore strategy groups include: (a) *unitary*, (b) *concrete modeling with tens*, (c) *invented algorithms*, (d) *lower standard algorithm*, and (e) *higher standard algorithm*. The definition of *unitary, concrete modeling with tens*, and *invented algorithms* groups are identical to those that were described for first grade. The *lower standard algorithms* group includes the students who used standard algorithms and at least one *unitary* strategy, but no *concrete modeling with tens* or *invented algorithm* strategies. The *higher standard algorithm* group includes the students who used standard algorithms and at least one *concrete modeling with tens* or *invented algorithms* strategy. Tables 7 and 8 summarize the strategy groups and their descriptions for first and second grade, respectively.

Table 7: Strategy Groups for First Grade

	Strategy Groups	Descriptions
Single-Digit Numbers	Concrete Modeling	Students who represent all quantities with ones and count by ones
	Counting	Students who count by ones to arrive at an answer but without representing all quantities with physical objects.
	Derived Facts/Recall	Students who use number properties, relations, or recall
Multiple-Digit Numbers	Unitary	Students who use concrete modeling or counting by ones strategies.
	Concrete Modeling with Tens	Students who represent all quantities with tens and ones, and count by tens or by ten and ones.
	Invented Algorithms	Students who use combining tens and ones, incrementing, or compensating strategies.
	Other	Students who use unidentifiable strategy

Table 8: Strategy Groups for Second Grade

	Strategy Groups	Descriptions
Single-Digit Numbers	Concrete Modeling	The same as in first grade.
	Counting	The same as in first grade.
	Derived Facts/Recall	The same as in first grade.
Multi-Digit Numbers	Unitary	The same as in first grade
	Concrete Modeling with Tens	The same as in first grade
	Invented Algorithm	The same as in first grade
	Lower Standard Algorithm	Students who use standard algorithms, and at least one unitary but no concrete modeling or invented algorithms.
	Higher Standard Algorithm	Students who use standard algorithms, and at least one concrete modeling with tens or invented algorithms.

Criteria for Classification of Students into Strategy Groups

Carpenter and Moser (1984) classified students into level one that refers to the direct modeling strategy, if they used no more than one counting strategy in solving problems. Students were classified into level two, which refers to the transition phase between direct modeling and counting strategies, if they used counting strategies for two or more problems but fewer than 75% of the questions for which they did not use derived facts. They classified students into level three, which refers to the counting strategy phase, if the students used counting strategies for at least 75% of the problems. They did this classification based on six single-digit addition and six single-digit subtraction problems, a total of 12 questions. In this study, a lower percentage criterion was used for some of the classifications, since there was a fewer number of problems available involving single-digit and multi-digit numbers on the instrument used in data collection.

The current study used addition and subtraction problems together to classify students into each strategy group. Siegler (1988) stated that individual differences in strategy choices would be most closely related in addition and subtraction since they are both numerical tasks and children use similar strategies to solve them. There were six problems involving single-digit numbers in the first grade and second grade interviews used in data collection for this study with which to classify students into strategy groups. For multi-digit problems, the first grade interview had six problems and the second grade interview had seven problems, which were used in the classification of students into strategy groups. Tables 9 summarize the problems that were used in the classification process for each grade level.

Table 9: Single-Digit and Multi-Digit Problems for First and Second Grade

	Single-Digit Problems		**Multi-Digit Problems**	
First Grade Problems	Join Result Unknown:	4 + 9 =?	Join Result Unknown:	18 + 13 = ?
	Compare Difference Unknown:	15 – 8=?	Join Change Unknown:	17 + ? = 26
	Computation Problem:	6 + 5 =?	Join Result Unknown:	49 + 56 = ?
	Computation Problem:	15 – 7 =?	Computation Problem:	46 + 17 = ?
	Computation Problem:	4 + 8 =?	Computation Problem:	100 – 3 = ?
	Computation Problem:	5 + ?= 13	Computation Problem:	41 – 39 = ?
Second Grade Problems	Same as in First Grade		Join Result Unknown:	18 + 13 = ?
			Join Change Unknown:	17 + ? = 26
			Join Result Unknown:	49 + 56 = ?
			Separate Change Unknown:	42 -? = 36
			Computation Problem:	63 – 17 = ?
			Computation Problem:	100 – 3 = ?
			Computation Problem:	201 – 199 =?

Initially it was proposed to classify first and second grade students into the *concrete modeling* strategy group for single-digit problems if they use that specific strategy for at least 67% (four out of six) of the problems. However, initial analysis of data suggested that a 50% criteria be used, which is discussed in detail in chapter four. Likewise students were categorized into the *counting* strategy group if they used counting strategies for at least 50% of the problems. Students were classified into the *derived facts/recall* group if they used that specific strategy for at least 50% of the questions. With this classification students were classified into the most advanced strategy group that they used for at least 50% of the problems.

There were six problems involving multi-digit numbers that could be used in the classification of students into strategy groups at the first grade level. Initially, it was proposed to classify students into the *unitary* group if they used *concrete modeling* by ones or *counting* by ones strategies for at least 83% (five out of six) of the problems, and to classify students into other strategy groups (*concrete modeling with tens,* and *invented algorithms)* if they used those strategies for at least 67% (four out of six) of the problems. However, initial analysis of data suggested that a 50% criterion be used for the classification of first grade students into multi-digit strategy groups, which is discussed in detail in chapter four. With this classification students were classified into the most advanced strategy group that they used for at least 50% of the problems.

For second grade students, there were seven problems involving multi-digit numbers that could be used to classify students into strategy groups. Initially, it was proposed to classify second grade students into multi-digit strategy groups if they used any of those strategies for at least 72% (five out of seven) of the problems. However, initial analysis of data suggested a 42% criterion (three out of seven problems) be used, which is discussed in detail in chapter four. With this classification students were classified into the most advanced strategy group that they were able to use for at least three problems. Tables 10 and 11 summarize the descriptions of the strategy groups and the criteria used for classifying students into the strategy groups for both grade levels, respectively.

Table 10: Strategy Description and Classification Criteria for First Grade

	Strategy Groups	Description of Strategy Groups for First Grade	Criteria for First Grade
Single Digit Numbers	Concrete Modeling	If a student represents all quantities and count by ones.	If students use concrete modeling strategy for at least 50% (three out of six) of the problems.
	Counting	If a student counts by ones without representing all the quantities.	If students use counting strategies for at least 50% of the questions
	Derived Facts/Recall	If a student uses number properties or relations.	If students use derived fact/recall strategies for at least 50% of the problems.
Multi-digit Numbers	Unitary	If a student represents all quantities with ones or if a student uses any of the counting by ones strategies.	If students use direct modeling or counting strategies for at least 50% (three out of six) of the problems.
	Concrete Modeling with Tens	If a student represents all quantities with tens.	If students represents all quantities with tens for at least 50% of the problems.
	Invented Algorithms	If a student uses combining tens and ones, incrementing or compensating strategies.	If students use invented algorithm strategies for at least 50% of the problems.
	Other	If a student uses unidentifiable strategy	If students use unidentifiable strategies for at least 50% of the problems.

Table 11: Strategy Description and Classification Criteria for Second Grade

	Strategy Groups	Description of Strategy Groups for Second Grade	Criteria for Second Grade
Single Digit Numbers	Concrete Modeling	The same as the first grade description.	The same as the first grade criterion.
	Counting	The same as the first grade description.	The same as first grade criterion.
	Derived Facts/Recall	The same as the first grade description.	The same as the first grade criterion.
Multi Digit Numbers	Unitary	If a student represents all quantities with ones or tens but count only by ones.	If students use direct modeling or counting by ones strategies for three or more problems.
	Concrete Modeling with Tens	If a student represents all quantities with tens	If students represent all quantities with tens for three or more problems.
	Invented Algorithm	If a student uses combining tens and ones, incrementing or compensating strategies.	If students use invented algorithms for three or more problems.
	Lower Standard Algorithm	Students who use the procedures of standard algorithms, and at least one unitary but no concrete modeling or invented algorithms.	If students use standard algorithms for three or more problems, and at least one unitary but no concrete modeling with tens or invented algorithms.
	Higher Standard Algorithm	Students who use standard algorithms, and at least one concrete modeling with tens or invented algorithms.	If students use standard algorithm for three or more problems, and at least one concrete modeling with tens or invented algorithms.

Population and Sample

The current study is a part of a larger CGI study and used a subsample of it. The author of the current study conducted student interviews and administered the ITBS as part of the data collection. Institutional Review Board (IRB) approval was attained by the researchers from two universities and can be seen in Appendix A and B. All public elementary schools with three to nine teachers at the first and second grade level and within one of the two school districts of a region located in the southeastern U.S. were eligible to participate in the CGI study. Therefore, the population for the current study is all elementary schools in the two school districts located in the southeast of the United States. To determine the participant schools, first school principals were contacted via email by the researchers of the larger CGI study. Schools were given priority to participate in the study if all first and second grade teachers volunteered to participate. Otherwise schools were chosen on a first come, first served basis. Table 12 shows descriptive characteristics of the first and second grade students in the two school districts combined based on the data provided by the State Department of Education (citation not provided to protect the anonymity of the districts involved).

Table 12: Descriptive Characteristics of Students

1st Grade	White	Black or African American	Hispanic or Latino	Asian	Native Hawaiian or Other Pacific Islander	American Indian/Alaska Native	Two or more Races	Total
Female	1070	1880	2487	134	28	8	152	5759
Male	1229	2064	2735	157	27	8	147	6367
Total	2299	3944	5222	291	55	16	299	12126
2nd Grade	**White**	**Black or African American**	**Hispanic or Latino**	**Asian**	**Native Hawaiian or Other Pacific Islander**	**American Indian/Alaska Native**	**Two or more Races**	**Total**
Female	1099	1869	2583	142	18	12	138	5861
Male	1174	2047	2737	170	22	11	136	6297
Total	2273	3916	5320	312	40	23	274	12158

Twenty-two elementary schools participated in the CGI study. The schools were randomly assigned to treatment (n=11) and control (n=11) groups. Randomization of schools occurred in the following way:

Block random assignment of schools to condition: The planned design for the study was a multisite cluster randomized-controlled trial with randomization occurring at the school level with schools blocked on district and school proportion free/reduced-price lunch (FRL). For the blocking procedure, the project methodologist ranked schools on proportion FRL and formed within-district matched-pairs. For each matched pair, one was randomly assigned to treatment, the other control. For each of the two participating districts, there were an odd number of schools. For the one unmatched school in each district, a coin-toss simulation was run to determine condition for that school (Schoen, LaVenia, Tazaz, et al., 2014).

In the first grade level, there were 47 teachers in the treatment group and 50 teachers in the control group at the end of the first year of the CGI study. Likewise, in the second grade level, 46 and 44 teachers were in the treatment and control groups, respectively. Teachers obtained parental consent forms by sending the forms home with the students. A total of 732 second grade students and 744 first grade students in treatment groups and 730 second grade students and 723 first grade students in control groups participated in the larger study. Four students were selected randomly from each teacher's classroom to participate in the student interviews. The sampling procedure for student interviews occurred in the following way:

A stratified random sampling procedure was used to identify two boys and two girls from the classroom of each participating teacher. This sampling procedure was designed to result in one student from each of the following four categories: (a) one boy with above-average (classroom) pretest achievement, (b) one girl with above-average pretest achievement, (c) one boy with below-average pretest achievement, and (d) one girl with below-average pretest achievement. In most cases, four students were sampled for interview, and each sampled student had an alternate student. The alternate was sampled at random from the same gender and pretest strata as the initially sampled student. Alternates were called upon when the initially sampled student was absent or otherwise unavailable to be interviewed at the time of testing. There were rare instances where there were no students from a given stratum to sample from, where being, the target sample of four initial students and four alternates could not be achieved for that given class (Schoen et al., 2015). From the second grade level 286 students, and from the first grade level 336 students were interviewed. Therefore the sample for the current study consisted of 336 first grade and 286 second grade students who participated in the student interviews.

Intervention

Teachers in the treatment group attended a four-day CGI workshop in the summer of 2013 and four follow-up days arranged throughout the 2013-2014 academic year. Teachers in the control group in one district were invited to a two-day professional development session for the district program called Bridge to STEM during June 2013 and September 2013. In the other district, administrators preferred to be a strict business-as-usual condition for the control group teachers in their district and the study did not provide professional development for those teachers. Teachers received a stipend for each day they attended the workshops (Schoen, LaVenia, Tazaz, et al., 2014).

In the summer workshops, treatment teachers viewed videos of students solving problems, learned about the taxonomy of problem types, and practiced writing different types of problems. They studied the book *Children’s mathematics: Cognitively Guided Instruction* (first edition) over the course of the workshop sessions (Schoen, LaVenia, Tazaz, et al., 2014).

They learned about children’s solution strategies, and how they are connected to the different problem types. Additionally, they extended their knowledge about properties of arithmetic operations by examining students’ invented strategies, and they also learned about students’ understanding of the equal sign. Teachers also went to a school site and interviewed students to gain additional insight about what they had learned in the professional development. In the follow up workshops, which occurred in the fall of 2013 and in the spring of 2014, teachers extended their knowledge of students’ thinking and strategies to multi-digit numbers and had an opportunity to watch the instruction of an expert CGI teacher in a real classroom.

Instrumentation

The current study used the data obtained from three different measures of student achievement, which were: (a) a student pretest and (b) student interviews developed by the researchers in three universities involved in the replication study and (c) a student posttest as measured by the Iowa Test of Basic Skills (ITBS) (Hoover, Dunbar, & Frisbie, 2001). The student pretest was used as a covariate to control for initial differences in students’ mathematics achievement. Student interviews were used to classify students into strategy groups. Interviewers entered students’ major strategies and their counting strategy (if any) along with other information necessary for the larger study. This study, however, used only the data entered for major strategies and counting strategies to classify students into strategy groups. These data were turned into the quantitative data by coding students’ major strategies and their counting strategies (if any) by the researchers at one of the universities involved in the CGI study. A student posttest, the ITBS, was used to compare students’ mathematics achievement.

Development of the Instruments

There are two researcher-developed instruments in this study. These are student pretest and student interview instruments. The student pretest instrument was developed with the collaboration of researchers at three research universities located in the southeastern U.S. The research team consisted of experts in mathematics, mathematics education, educational psychology, and educational measurement. The measures developed by Carpenter et al. (1989) were reviewed in the development of the pretests. After the research team prepared a draft of a set of items, they sent the draft to the advisory board members of the CGI study for review and feedback. The advisory board consists of the researchers who are experts in the CGI research. The research team revised the test items based on the feedback provided by the advisory board (Schoen, LaVenia, Farina, et al., 2014). Both first and second grade student pretest instruments include a total of 20 mathematics problems including counting problems, word problems, and computation problems. Table 13 shows the distribution of each type of problems in the pretest instrument.

Table 13: Number of test items in the pretest instrument

Problem Types	**Number of Test Items for both First and Second Grade**
Counting	3
Word Problems	7
Computation	10
Total	20

Similar to the student pretest instrument, the student interview instrument was also developed by the researchers of the larger CGI study. The student interview instrument has four sections, which are counting and number screening, word problems, computation, and equality. Similar to the process of development of the pretest instrument, the advisory board members provided their feedback on a draft of items, and the items were revised based on the feedback provided by the advisory board members. After development of the complete draft of the student interview instrument, a pilot study was conducted with 34 students who were not in the CGI study. The results of the pilot study led researchers to revise; (a) the set of items, (b) the verbal script for the interview, (c) the instructions for pacing, and (d) the data recording system (Schoen et al., 2015). The research team also developed a coding instrument that enabled interviewers to code students' strategies in real time (Schoen et al., 2015). Table 14 shows the distribution of each type of problem in the student interview instrument.

Table 14: Number of test items in the student interview instrument

Problem Types	First Grade Interview	Second Grade Interview
Counting and Number Screening	6	6
Word Problems	7	8
Computation	8	8
Equality	8	8
Total	29	30

The current study used only word problems and computation problems involving single-digit and multi-digit numbers from the interview instrument. Counting and number screening, equations, one multiplication word problem, one division word problem, and one computation problem were not used in the current study. The reason not to include one computation problem is due to the fact that the item was designed to measure students' thinking for number relations, and students did not need to use a strategy to solve that specific computation problem.

The third measure of student achievement that was used in the current study is the Iowa Test of Basic Skills (ITBS), which is a written and standardized test of student achievement. The reason for using the ITBS as a student posttest was to obtain valid, reliable, and policy-relevant data. For the CGI study students were administered the *Math Problems and Math Computation* sections of the ITBS. Table 15 shows the number of problems for different problem types in the *Math Problems* section of the ITBS for level 7 (first grade) and for level 8 (second grade).

Table 15: Number of test items in ITBS

Problem Types	**Level 7**	**Level 8**
Addition and Subtraction	14	13
Multiplication and Division	3	6
Multi-step	1	5
Model Equations	3	-
Other	9	6
Total	30	30

The *Math Computation* section of ITBS has two sections. The first section includes multiple-choice addition and subtraction problems, which are presented verbally. In the second section of the ITBS students work on their own and have limited time (six minutes in first grade and eight minutes in second grade) to solve the addition and subtraction problems that are presented with numerals and symbols either in horizontal or vertical form. There are 16 and 17 problems in the second section of the ITBS for level 7 and level 8, respectively. In level 7, there are seven problems presented horizontally whereas nine problems are presented vertically. In level 8, 10 of the problems are presented in horizontal form and 10 of the problems are presented in vertical form.

Reliability and Validity

Reliability refers to the measure of consistency over time and over similar samples, and an instrument is said to be reliable if it yields similar data from similar respondents over time. (Cohen, Manion, & Morrison, 2007). Validity refers to the extent to which measures indicate what they are supposed to be measuring (Check & Schutt, 2012). Regardless of the research design, researchers strive to minimize invalidity and maximize validity (Cohen et al., 2007). There are three types of student outcome measures that were used in this study. These are: (a) a student pretest that was developed by the researchers in three universities, (b) a student interview that was developed by the researchers in three universities, and (c) the ITBS.

The first measure of student outcome, the student pretest was compared to the Discovery Education Assessment (DEA) to test the content validity of the pretest items. The Cronbach's alpha reliability of the DEA was reported to be .83 at the second grade level (Smith & Kurz, 2008). The reliability estimate of the grade one assessment of DEA was not reported. For both first and second grade, the correlation between the DEA overall scale score and the counting and word problems sections of the student pretest was greater than .4 which indicates moderate convergent validity between the measures of student mathematics achievement (Schoen et al., 2015).

The second measure of student outcome, the student interview, was developed to investigate students' solution strategies for addition and subtraction problems. To develop student interview protocol, the researchers working on the larger study reviewed measures developed by Carpenter et al. (1989), which has a reported Cronbach's alpha reliability of .83 and .66 for the computation and word problem sections, respectively.

To calculate inter-rater reliability of student interviews, the percentage agreement method was used. For the current study, about 13% of the total sample (79 out of 622) was rated by two independent raters to calculate inter-rater reliability. The percent agreement between the two raters for the major strategy was 82.7% (Schoen et al., 2015). The percentage agreement method is a commonly used procedure, which is conceptually simple and easily computed (Drew, Hardman, & Hosp, 2008). In the literature it is common to use a portion of data to compute inter-rater reliability. There are published research studies in which only 10% to 15% of the total sample was rated by two independent raters and this sub-sample is utilized to derive the inter-rater reliability estimate (Fan & Chen, 1999).

The third measure of the student outcome is the ITBS, which is a standardized test used to measure student achievement. In the current study, depending on their grade level, students were administered the level 7 (first grade) or level 8 (second grade) test forms of ITBS. For the ITBS, the internal consistency estimates of subtests across test forms are reported to be in the .80s and .90s according to the Kuder-Richardson Formula 20 (Spies, Carlson, & Geisinger, 2010).

Data Collection

Three different measures of student achievement were used in this study. This section discusses the data collection procedures for each kind of measure. The first measure of student achievement is the student pretest, which was administered to the students in the regular classroom setting by their classroom teachers at the beginning of the 2013-2014 academic years. Teachers were provided with pretest materials, a testing administration guide, student testing booklets, and parental consent forms. The administration of the student pretest took place in a time frame from August through September in 2013. Pretest materials were picked up from the schools by CGI project staff during the last two weeks of September 2013.

The second measure of student outcomes, which is the student interview, took place in a time frame from April 2014 through the end of May 2014. The project directors recruited a total of 14 interviewers, including the author of the current study, for the interview process. The trainings for interviewers occurred in three main phases. At the first phase, the interviewers attended a two-day workshop where they received detailed instruction about interviewing protocol and learned about different problem types and students' solution strategies as defined in the CGI framework. In addition they learned about how to ask follow-up questions which aimed to make students' thinking and the strategy that they used more salient (Schoen, LaVenia, Tazaz, et al., 2014).

At the second phase, interviewers went to a school site to have a field experience where more experienced interviewers conducted the interview with real students, and less experienced interviewers observed. The school chosen for this purpose was a private school whose data could not be used in the research study. After each of these interviews, the groups of interviewers discussed what they observed to have a common understanding of identifying students' strategies. Following the field experience, an additional daylong workshop took place where the team watched and coded the strategies of a chosen student together (Schoen, LaVenia, Tazaz, et al., 2014).

At the third phase, interviewers started with real data collection where they conducted interviews in pairs (interviewer and observer). After each interview, the pair compared their notes and came to an agreement on how to code the student's strategies. Interviewing in pairs lasted two weeks and ended with one last day of classroom training to discuss the experiences and resolve any discrepancies between the interviewers (Schoen, LaVenia, Tazaz, et al., 2014).

Each interviewer was provided with a laptop and a camera to videotape the interviews. Additionally they were provided with a coding instrument developed by the researchers of the larger CGI study to code students' strategies. Interviewers also took notes on the coding instrument to clarify how exactly the student used a specific strategy. A semi-structured interview format was used for student interviews. Initially interviewers read from the interviewer's script to inform students about the interview process. Then interviews started with counting and number screening and continued with word problems, computation problems, and equality problems. Each word problem was read to the student in its entirety and was reread as many times as the student needed so that remembering the details would not cause any problem. Children were provided with snap cubes, base ten blocks, and paper and pencil. The interviewers also let students know that they were allowed to use their fingers if they wanted to, since children may hide the use of a particular strategy if they think it is not valued or not allowed according to socio-mathematical classroom norms (Yackel & Cobb, 1996). However, they were not required to use any of the manipulatives.

The problems in the word problems section were ordered from easier to more difficult ones. Therefore, not to cause any frustration for children, interviewers were given the discretion to terminate the word problem section if a student was not able to solve three consecutive problems successfully. For scoring purposes the remaining of the word problems were coded as *mercy* indicating that the word problem section was terminated. However, the computation and equality problems sections were not terminated even if a child was not able to solve those problems successfully. Students' strategies were coded regardless of students obtaining a right or wrong answer. On average the interview was designed to last about 45 minutes (Schoen, LaVenia, Tazaz, et al., 2014).

The third measure of student outcome was the ITBS. The project staff, that was assigned to conduct the student interviews, also administered the ITBS in May of the 2013-2014 academic years. The testing team attended a one-day classroom training about the test administration process. The team was instructed to strictly follow the scripts provided in the test administration booklet. The teachers were also present in the classroom during the testing time to take care of any unpredictable issues. On average the ITBS test lasted about an hour.

Data Analysis

First Research Question

The first research question was: Are there statistically significant differences in the number of first grade students in different strategy groups between treatment and control groups?

To answer this research question, single-digit problem strategies and multi-digit problem strategies were analyzed separately. First grade students who participated in the student interviews were classified into *concrete modeling, counting,* and *derived facts/recall* strategy groups for single-digit problem strategies. Then, Chi-square analysis was performed to find differences in the number of students in different strategy groups between treatment and control groups. Likewise, first grade students were classified into *other, unitary, concrete modeling with tens,* and *invented algorithms* strategy groups for multi-digit problem strategies. Again Chi-square analysis was performed to find differences in the number of students in different strategy groups between treatment and control groups.

Second Research Question

The second research question was: Are there statistically significant differences in the mathematics achievement (as measured by the Iowa Test of Basic Skills) of first grade students between different strategy groups, controlling for students' prior mathematics achievement (as measured by student pretest)?

This research question investigated the differences in the mathematics achievement of first grade students who were classified into different strategy groups for single-digit problems and for multi-digit problems. The analysis was conducted separately for single-digit and multi-digit strategies. The mathematics achievement of students in *concrete modeling, counting, and derived facts/recall* strategy groups, was compared using MANCOVA. Likewise, the mathematics achievement of students in *other, unitary, concrete modeling with tens,* and *invented algorithms* strategy groups was compared using MANCOVA. The Iowa Test of Basic Skills was used to measure the mathematics achievement of the students, and the student pretest was used as a covariate.

Third Research Question

The third research question was: Are there statistically significant differences in the number of second grade students in different strategy groups between treatment and control groups?

The analysis for this question was similar to the analysis of the first research question. To answer this research question, second grade students who participated in the student interviews were classified into *concrete modeling, counting, and derived facts/recall* strategy groups for single-digit problems. Then, Chi-square analysis was performed to find differences in the number of students in different strategy groups between treatment and control groups. Likewise second grade students were classified into *unitary, concrete modeling with tens, invented algorithms, lower standard algorithms,* and *higher standard algorithms* strategy groups for multi-digit problem strategies. Again Chi-square analysis was performed to find whether there were significant differences in the number of students in different strategy groups between treatment and control groups.

Fourth Research Question

The fourth research question was: Are there statistically significant differences in the mathematics achievements (as measured by the Iowa Test of Basic Skills) of second grade students between different strategy groups, controlling for students' prior mathematics achievement (as measured by student pretest)?

The analysis for this research question was similar to the analysis of the second research question. This research question investigated the differences in the mathematics achievement of students in different strategy groups (for single-digit problems and for multi-digit problems) at the second grade level. The analysis was conducted separately for single-digit strategies, and multi-digit strategies. The mathematics achievement of students in *concrete modeling, counting,* and *derived facts/recall* strategy groups were compared using MANCOVA. Likewise, the mathematics achievement of students in the *unitary, concrete modeling with tens, invented* *algorithms*, *lower standard algorithms*, and *higher standard algorithms* strategy groups were compared using MANCOVA. The Iowa Test of Basic Skills was used to measure the mathematics achievement of the students, and the student pretest was used as covariate. Table 16 summarizes each research question, dependent and independent variables, and statistical procedures used in data analysis.

Table 16: Research Questions, Variables, and Statistical Procedures

Research Questions	Independent Variables	Dependent Variables	Statistical Procedure
1. Are there statistically significant differences in the numbers of first grade students in different strategy groups between treatment and control groups?	Condition	Strategy Group	CHI-SQUARE
2. Are there statistically significant differences in the mathematics achievements (measured by ITBS) of first grade students between different strategy groups, controlling for students' prior mathematics achievement?	Strategy group	ITBS (*Math Problems and Math Computation*)	MANCOVA
3. Are there statistically significant differences in the numbers of second grade students in different strategy groups between treatment and control groups?	Condition	Strategy Group	CHI-SQUARE
4. Are there statistically significant differences in the mathematics achievements (measured by ITBS) of second grade students between different strategy groups, controlling for students' prior mathematics achievement?	Strategy group	ITBS (*Math Problems and Math Computation*)	MANCOVA

CHAPTER FOUR: DATA ANALYSIS

Introduction

The purpose of this study was to explore the effect of teachers' attending CGI professional developments on their students' problem solving strategies, and the effect of students' use of different problem solving strategies on their mathematics achievement. It is important to note that the study was conducted at the end of the first year of a two-year planned CGI professional development. Therefore the results of this study should be interpreted cautiously.

First, the study analyzed the differences in students' use of strategies between treatment and control groups. The treatment was CGI professional development, and the teachers in the treatment group attended CGI workshops whereas the teachers in the control group did not. The students, both in the classes of treatment teachers (treatment students) and in the classes of control teachers (control students) were classified into the strategy groups according to their use of strategies. Student interviews were used to identify the strategies used by the students and to classify them into the strategy groups. Next, the study analyzed the differences in the mathematics achievement of students between different strategy groups. A student posttest, which was ITBS (*Math Problems* and *Math Computation*), was used to compare students' mathematics achievement. A student pretest was used as a covariate.

This chapter explains the methods used to classify students into strategy groups and the statistical analyses used to answer each research question. The first part displays sample demographics separately for first and second grade students. The second part explains how the strategy groups were determined and how students were classified into the strategy groups based on the selected criteria. The third part presents the results of statistical analysis used to answer each research question.

Demographics of Participants

The current study was a part of a larger study and the researcher used a subsample of it. The sample for this study consisted of both first and second grade students. There were 336 first grade students from 21 elementary schools and 286-second grade students from 22 elementary schools in this study. All the schools were located in the southeastern United States and spanned over two counties.

First Grade Students

There were 175 first grade students in the control group and 161 students in the treatment group. Among those, 158 were females and 149 were males. The gender was not indicated for 29 students. The breakdown for the ethnicity percentages is listed in Table 17. The percentage breakdown illustrates that there was a larger percentage of Hispanic students (36%) compared to any other ethnic/racial group. Twenty-eight of the students were White, and 21% were African American. The rest of the ethnicities made up approximately 15% of the sample.

Table 17: First Grade - Race / Ethnicity

		Frequency	Percent
Valid	Missing	29	8.6
	AsianPacific Islander/	16	4.8
	Black	72	21.4
	Hispanic	121	36.0
	Multiracial	5	1.5
	White	93	27.7
	Total	336	100.0

The distribution of Free and Reduced Lunch (FRL) status and English Language Learners (ELL) status are presented in tables 18 and 19. More than 50% of first graders were qualified for free and reduced lunch, and 72% of first graders were not qualified for ELL. Free and Reduced Lunch and ELL status were missing for about 9% of the students.

Table 18: First Grade - Free and Reduced Lunch Status

		Frequency	Percent
Valid	not qualified for FRL	109	32.4
	qualified for FRL	198	58.9
	Total	307	91.4
Missing	missing	29	8.6
Total		336	100.0

Table 19: First Grade - English Language Learners Status

		Frequency	Percent
Valid	not qualified for ELL	241	71.7
	qualified for ELL	66	19.6
	Total	307	91.4
Missing	missing	29	8.6
Total		336	100.0

Second Grade Students

There were 286 second grade students in this study. Of these students, 144 were in the control group and 142 were in the treatment group. There were 134 females and 125 males. For 27 students gender was not indicated. The breakdown for the ethnicity percentages is listed in Table 20. The percentage breakdown illustrates that 38% of the students were White, 28% were Hispanic, and 14% were African American. The rest of the ethnicities made up approximately 10% of the sample, and ethnicity was not indicated for about 10% of the students.

Table 20: Second Grade - Race / Ethnicity

		Frequency	Percent
Valid	Asian Pacific Islander /	17	5.9
	Black	41	14.3
	Hispanic	81	28.3
	American Indian Alaskan Native /	3	1.0
	Multiracial	8	2.8
	White	108	37.8
	Total	258	90.2
Missing	missing	28	9.8
Total		286	100.0

The distribution of Free and Reduced Lunch (FRL) status and English Language Learners (ELL) status are presented in tables 21 and 22. The percentages of students who qualified for FRL and who did not were about the same, and a majority of students (72%) were not qualified for ELL.

Table 21: Second Grade - Free and Reduced Lunch Status

		Frequency	Percent
Valid	not qualified for FRL	127	44.4
	qualified for FRL	132	46.2
	Total	259	90.6
Missing	missing	27	9.4
Total		286	100.0

Table 22: Second Grade - English Language Learners Status

		Frequency	Percent
Valid	not qualified for ELL	206	72.0
	qualified for ELL	53	18.5
	Total	259	90.6
Missing	missing	27	9.4
Total		286	100.0

Strategy Groups and Classification of Students

Item Analysis – First Grade

Item analysis was conducted for the items used to classify first grade students into strategy groups, which is based on 336 students. For 12 items (including both single-digit and multi-digit problems), the cronbach's alpha was 0.711. Cronbach's alpha did not increase with deletion of any of the items; therefore none of the items were dropped from the analysis. Tables 23 and 24 displays reliability statistics, and item-total statistics, respectively.

Table 23: First Grade - Reliability Statistics

Reliability Statistics	
Cronbach's Alpha	N of Items
.711	12

Table 24: First Grade - Item-Total Statistics

Item-Total Statistics				
	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
WP6_correct	6.25	10.554	.228	.706
WP7_correct	6.76	10.024	.330	.694
WP9_correct	6.65	9.972	.345	.693
WP10_correct	6.72	9.923	.373	.689
WP12_correct	6.80	9.997	.356	.692
RT1_correct	6.16	10.891	.254	.707
RT2_correct	6.42	10.459	.212	.708
RT3_correct	6.19	10.728	.278	.704
RT4_correct	6.64	10.009	.315	.696
RT6_correct	6.40	7.990	.547	.655
RT7_correct	6.89	8.578	.428	.681
RT14_correct	6.30	7.896	.506	.666

Item difficulty level showed that there were three items that had low difficulty level and two items that had high difficulty level. The three low-level items were among single-digit problems, and the two high-level items were among multi-digit problems. Table 25 shows item level difficulty.

Table 25: First Grade - Item Difficulty Level

	Item Difficulty Level		
Item Number	No. Correct Answers	% Correct	Difficulty Level
WP6 - JRU (4+9)	281	83.6	Low
WP7 - CDU (15-8)	115	34.2	Medium
WP9 - JRU - (18+13)	154	45.8	Medium
WP10 - JDU- (26-17)	131	39	Medium
WP12 - JRU - (49 + 56)	99	29.5	High
RT1 - (6+5)	319	94.9	Low
RT2 - (15-7)	232	69	Medium
RT3 - (4+8)	308	91.7	Low
RT4 - (46+17)	153	45.5	Medium
RT6 - (100-3)	213	63.4	Medium
RT7 - (41-39)	50	14.9	High
RT14 – (5+_=13)	238	70.8	Medium

Item Analysis – Second Grade

Item analysis was conducted also for the items used to classify second grade students into strategy groups, which was based on 286 students. For 13 items (including both single-digit and multi-digit problems), the cronbach's alpha was 0.781. All the items were kept in the analysis since Cronbach's alpha was sufficiently large, and deletion of any item would increase it only by 0.002. Tables 26 and 27 displays reliability statistics, and item-total statistics, respectively.

Table 26: Second Grade - Reliability Statistics

Reliability Statistics	
Cronbach's Alpha	N of Items
.781	13

Table 27: Second Grade - Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
WP6_correct	9.10	20.751	.210	.782
WP7_correct	9.40	20.192	.205	.782
WP9_correct	9.29	20.201	.201	.783
WP10_correct	9.45	19.863	.244	.780
WP12_correct	9.34	20.133	.234	.780
WP13_correct	9.48	19.815	.282	.777
RT1_correct	9.02	18.982	.566	.760
RT2_correct	9.10	18.782	.506	.761
RT3_correct	9.04	18.991	.535	.761
RT4_correct	9.41	16.930	.576	.748
RT6_correct	9.13	16.081	.687	.734
RT7_correct	9.64	14.464	.670	.735
RT14_correct	8.92	15.264	.488	.770

Item difficulty level showed that there were six items that had low difficulty level and one item that had high difficulty level. The five of the six low-level items were among single-digit problems, and only one was among multi-digit problems, which involved a single-digit subtrahend. The only high-level item was among multi-digit problems. Table 28 shows item level difficulty for the second grade problems.

Table 28: Second Grade - Item Difficulty Level

	Item Difficulty Level		
Item Number	No. Correct Answers	% Correct	Difficulty Level
WP6 - JRU (4+9)	265	92.7	Low
WP7 - CDU (15-8)	179	62.6	Medium
WP9 - JRU - (28+43)	204	71.3	Medium
WP10 - JDU- (26-17)	158	55.2	Medium
WP12 - JRU - (49 + 56)	198	69.2	Medium
WP13 - SDU - (42-36)	156	54.5	Medium
RT1 - (6+5)	280	97.9	Low
RT2 - (15-7)	256	89.5	Low
RT3 - (4+8)	275	96.2	Low
RT4 - (63-17)	161	56.3	Medium
RT6 - (100-3)	232	81.1	Low
RT7 - (201-199)	72	25.2	High
RT14 – (5+_=13)	252	88.1	Low

Single-Digit Strategies

The single-digit problems were the same for both first and second grade levels. Therefore, the designation of single-digit strategy groups was also the same for both grade levels. There were 6 problems involving single-digit numbers that could be used to classify students into strategy groups. Item level analysis of strategies showed that a majority of the first graders (67% or more) used either *concrete modeling* or *counting* strategies for all but one of the six items. For that particular one item (RT1) 20% of the first graders used *concrete modeling*, 40% used *counting*, 37% used *derived facts* or recall strategies. Table 29 illustrates the frequencies and percentages of each strategy used by the first graders for each problem.

Table 29: First Grade - Frequencies and Percentages of Strategies Used

	Concrete Modeling	Counting	Derived Facts /Recall	Standard Algorith	Other	Total
WP 6	178 53%	108 32.1%	39 11.6%	0 0%	11 3.3%	336 100%
WP 7	152 45.2%	98 29.2%	23 6.9%	1 .3%	62 18.5%	336 100%
RT1	67 19.9%	135 40.2%	123 36.6%	0 0%	11 3.3%	336 100%
RT2	159 47.3%	120 35.7%	38 11.4%	3 .9%	16 4.8%	336 100%
RT3	93 27.7%	175 52.1%	55 16.3%	0 0%	13 3.9%	336 100%
RT14	62 18.5%	162 48.2%	62 18.5%	1 .3%	45 13.4%	332 98.8%

The most frequent strategy used by the second graders was the *counting* strategy. Second graders used *derived facts/recall* strategies more often than first graders, and they used the concrete modeling strategy less often. A majority of the second grade students (69% or more) either used *counting* or *derived facts/recall* strategies for most problems. Table 30 illustrates the frequencies and percentages of each strategy used by the second graders for each problem.

Table 30: Second Grade - Frequencies and Percentages of Strategies Used

	Concrete Modeling	Counting	Derived Facts/Recall	Standard Algorithm	Other	Total
WP 6	95	106	76	5	4	286
	33.2%	37.1%	26.5%	1.7%	1.4%	100%
WP 7	89	100	43	33	21	286
	31.1%	35%	15%	11.5%	7.3%	100%
RT1	20	104	156	0	5	285
	7%	36.4%	54.6%	0%	1.7%	99.7%
RT2	61	120	77	18	9	285
	21.3%	42%	26.9%	6.3%	3.1%	99.7%
RT3	19	156	104	3	3	285
	6.6%	54.5%	36.4%	1%	1%	99.7%
RT14	23	147	81	11	16	278
	8%	51.4%	28.3%	3.8%	5.6%	97.2%

Strategy groups for single digit problems included *concrete modeling, counting,* and *derived facts/recall* strategies. Initially a 67% (four out of six problems) criterion was used to classify students into strategy groups. However, 123 of the 336 first grade students could not be classified with this criterion. Therefore the criterion was changed to 50% (three out of six problems). First, students who used *derived facts/recall* strategies for at least 50% of the problems were classified into the *derived facts/recall* strategy group. Of the remaining students, those who used *counting* strategies for at least 50% of the problems were classified into the *counting* strategy group. Finally, the remaining students were classified into the *concrete modeling* strategy group if they used that specific strategy for at least 50% of the problems.

In this classification, students were classified into the most advanced strategy group that they used for at least three problems. For example, if a student used three *derived facts/recall* strategies and three *counting* strategies, that student was classified into the *derived facts/recall* strategy group. This way of classification is justified because use of *concrete modeling – counting – derived facts/recall* strategies show a progression in students' development of number sense (Carpenter et al., 1999). As a result of this classification, only 27 students at the first grade level and 19 students at the second grade level were not classified into any strategy group. Tables 31 and 32 show the numbers of students in each strategy group for each grade level, respectively.

Table 31: First Grade - Numbers of Students in Single-Digit Strategy Groups

STRATEGY

	Strategy	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Concrete Modeling	113	33.6	36.6	36.6
	Counting	152	45.2	49.2	85.8
	Derived Facts/Recall	44	13.1	14.2	100.0
	Total	309	92.0	100.0	
Missing	.	27	8.0		
Total		336	100.0		

Table 32: Second Grade - Numbers of Students in Single-Digit Strategy Groups

STRATEGY

	Strategy	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Concrete Modeling	34	11.9	12.7	12.7
	Counting	142	49.7	53.2	65.9
	Derived Facts/Recall	91	31.8	34.1	100.0
	Total	267	93.4	100.0	
Missing	.	19	6.6		
Total		286	100.0		

Multi-Digit Strategies – First Grade

There were six problems involving multi-digit numbers that could be used to classify students into multi-digit strategy groups in the first grade level. Item level analysis of strategies showed that the most common strategy used for multi-digit problems by the first graders was the *unitary* (*concrete modeling* or *counting*) strategies. The next most common strategy was the *other* strategy, which indicates that the strategy used could not be identified. The reason for the frequent use of the *other* strategy is reasonable since the curriculum focuses on single-digit numbers in the first grade level. The *invented algorithm* strategy was the third most frequently used strategy and *concrete modeling with tens* was the fourth. Use of the *standard algorithm* was the least most common strategy used by the first graders for multi-digit problems. Table 33 shows the frequencies of the strategies used for multi-digit problems in the first grade level.

Table 33: First Grade - Frequencies and Percentages of Strategies Used

	Unitary	Concrete Modeling with Tens	Invented	Standard Algorithm	Other	Total
WP 9	217	22	46	14	37	336
	64.6%	6.5%	13.7%	4.2%	11%	100%
WP 10	211	14	15	6	90	336
	51.3%	4.2%	4.5%	1.8%	26.8	100%
WP12	72	58	65	20	121	336
	21.5%	17.3%	19.3%	6%%	36%	100%
RT4	166	42	68	20	40	336
	49.4%	12.5%	20.2%	6%	11.9	100%
RT6	245	22	23	2	41	333
	72.9%	6.5%	6.9%	.6%	12.2	99.1%
RT7	139	23	57	31	83	333
	41.4	6.8%	17%	9.2%	24.7	99.1%

As proposed, initially a criterion of at least 67% criterion (four out of six problems) was used to classify students into the strategy groups (*unitary, concrete modeling with tens, invented algorithms, lower mixed and higher mixed* strategy groups). However, there were 106 of 336 students that could not be classified into any strategy groups. In addition, there were only 11 students in the *concrete modeling with tens* strategy group who used that strategy for at least 67% of the problems, and 14 students in the *higher mixed* (*invented* and *concrete modeling with tens*) strategy group. Therefore, the strategy groups and classification criterion were changed. Strategy groups included the three major strategy groups (*unitary*, *concrete modeling with tens*, and *invented algorithms*) and an "*other*" strategy group. The criterion was determined to be 50% (at least three out of six problems).

The classification of students into the strategy groups was accomplished in the following order. First, students who used *invented algorithms* for at least 50% of the problems were classified into the *invented algorithms* strategy group. There were 49 students in the *invented algorithms* strategy group. Of the remaining students those who used a *concrete modeling with tens* strategy for at least 50% of the problems were classified into the *concrete modeling with tens* strategy group. There were 22 students in this strategy group. Next, the students who used *unitary* strategies (*concrete modeling* or *counting* strategies) for at least 50% of the problems were classified into the *unitary* strategy group. There were 199 students in this strategy group. Finally, students who used *other* strategies for at least 50% of the problems were classified into the *other* strategy group. There were 45 students in the *other* strategy group. As a result of this classification there were only 21 students who could not be classified into any of the strategy groups and no mixed strategy groups were formed. Table 34 displays the frequencies of students in each strategy group.

Table 34: First Grade - Numbers of Students in Multi-Digit Strategy Groups

STRATEGY

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Other	45	13.4	14.3	14.3
	Unitary	199	59.2	63.2	77.5
	Concrete Modeling with Tens	22	6.5	7.0	84.4
	Invented Algorithms	49	14.6	15.6	100.0
	Total	315	93.8	100.0	
Missing	.	21	6.3		
Total		336	100.0		

The *concrete modeling with tens* strategy was identified using the following procedure. Each student's strategy and counting method (e.g. by ones, twos, tens or tens-and-ones) was analyzed for each multi-digit problem. If a student used a concrete modeling strategy and counted by tens or by tens-and-ones for a particular problem, then the strategy used was recoded as the *concrete modeling with tens* strategy. There were several instances where the strategy used was a *counting* strategy and the students counted by tens or tens-and-ones. In these cases, the strategy was recoded as an *invented algorithms* strategy since counting by tens or tens-and-ones without physically modeling the quantities is similar to an *invented algorithms* strategy. Table 35 displays the numbers of recoded strategies for each multi-digit problem in the first grade.

Table 35: First Grade - Number of Recoded Strategies

	Concrete modeling with tens	Invented algorithm
WP 9	22	1
WP10	14	1
WP12	58	9
RT4	42	6
RT6	22	1
RT7	23	3

Multi-Digit Strategies – Second Grade

There were seven questions involving multi digit numbers that could be used to classify students into strategy groups in the second grade level. Item level analysis of strategies showed that the most commonly used strategy for multi digit problems by second graders was the *standard algorithm*. *Unitary*, *invented algorithms*, and *concrete modeling with tens* strategies were the next most common strategies, respectively. Table 36 displays the frequencies of each strategy used for each multi-digit problem in the second grade.

Table 36: Second Grade - Frequencies and Percentages of Multi-digit Strategies Used

	Unitary	Concrete Modeling with Tens	Invented	Standard Algorithm	Other	Total
WP 9	41 14.3%	38 13.3%	43 15%	150 52.4%	14 4.9%	286 100%
WP 10	131 45.8%	11 3.8%	24 8.4%	96 33.6%	24 8.4%	286 100%
WP12	24 8.4%	38 13.3%	43 15%	151 52.8%	30 10.5%	286 100%
WP13	88 30.8%	6 2.1%	26 9.1%	117 40.9%	49 17.1	286 100%
RT4	83 29%	23 8%	31 10.8%	131 45.8%	16 5.6%	284 99.3%
RT6	173 60.5%	16 5.6%	47 16.4%	22 7.7%	20 7%	283 99%
RT7	34 11.9%	5 1.7%	30 10.5%	156 54.5%	56 19.6%	281 98.3%

Strategy groups included *unitary, concrete modeling with tens, invented algorithms*, and *standard algorithm* strategy groups. Instead of the *mixed* category which proposed initially the *standard algorithm* strategy group was further split in two groups as the *lower standard algorithm* group and the *higher standard algorithm* group because preliminary analysis of data showed that 56 students in the *standard algorithm* strategy group did not use any *invented algorithms* or *concrete modeling with tens* strategies whereas 45 of them used at least one of these strategies. The two-level *standard algorithm* strategy group was used to distinguish the *standard algorithm* students who used at least one *invented algorithm* or *concrete modeling with tens* strategies from the students did not use either of these strategies.

Initially a criterion of at least four out of seven problems (57%) was chosen to classify students into strategy groups. However there were only nine students who used a *concrete* *modeling with tens* strategy for at least 57% of the problems, and 76 of 286 students could not be classified into any strategy groups. Therefore the criterion was changed to be at least three out of seven problems (42%). A three or more problems criterion was justified because Carpenter and Moser (1984) classified students into their level-two strategy group if students used counting strategies for two or more problems in their study where they used an instrument with 12 problems. After the criterion was revised, there were at least 22 students in each strategy group, and there were only 40 students who could not be classified into any strategy group.

The classification of students into strategy groups was accomplished in the following way. First, students who used *invented algorithms* for at least three problems were classified into the *invented algorithms* strategy group. There were 33 students in this group. Second, students who used a *concrete modeling with tens* strategy for at least three problems were classified into the *concrete modeling with tens* strategy group. There were 22 students in this group. Third, students who used a *unitary* strategy for at least three problems were classified into the *unitary* strategy group. There were 90 students in this group. Of the remaining students those who used a *standard algorithm* strategy for at least three of the problems and who used at least one *invented strategy* or *a concrete modeling strategy* were classified into the *higher standard algorithm* group. There were 45 students in this group. Then, students who used a standard algorithm for at least three of the problems and a unitary strategy (but no *invented* or *concrete modeling with tens* strategies) were classified into the *lower standard algorithm* group. There were 56 students in this group.

The aim of this classification was to classify students according to their proficiency in thinking and dealing with multi-digit numbers. Therefore, students were classified first into the *unitary, concrete modeling with tens*, and *invented algorithms* groups. These strategies are the strategies that are invented by students, and show a progression in their understanding of multi-digit numbers. On the other hand, students are not likely to invent the procedures of standard algorithm. A student who can only use unitary strategies or students who can actually use invented algorithms can be taught how to use the standard algorithm. Therefore, students who did not use any of the student invented strategies (*unitary, concrete modeling with tens,* or *invented algorithms*) consistently for at least three of the problems were classified into either the *lower standard algorithm* or the *higher standard algorithm group*, if they used standard algorithms consistently for at least three problems. Table 37 displays the frequencies of each strategy group.

Table 37: Second Grade - Numbers of Students in Multi-digit Strategy Groups

STRATEGY

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Unitary	90	31.5	36.6	36.6
	Lower Standard Algorithm	56	19.6	22.8	59.3
	Concrete Modeling with Tens	22	7.7	8.9	68.3
	Higher Standard Algorithm	45	15.7	18.3	86.6
	Invented Algorithms	33	11.5	13.4	100.0
	Total	246	86.0	100.0	
Missing	.	40	14.0		
Total		286	100.0		

For the identification of a *concrete modeling with tens* strategy the same procedure, as in the first grade, was followed. The strategy was recoded as *concrete modeling with tens* if students used a modeling strategy and counted by tens or tens-and-ones for a particular problem. If students used a *counting* strategy, and counted by tens or tens-and-ones for solving a particular problem, then that strategy was recoded as an *invented algorithms* strategy. Table 38 shows the numbers of strategies recoded as either *concrete modeling with tens* or *invented algorithms* for each problem.

Table 38: Second Grade - Number of Recoded Strategies

	Concrete modeling with tens	Invented algorithm
WP 9	38	3
WP10	11	0
WP12	38	2
WP13	6	2
RT4	23	2
RT6	16	0
RT7	5	0

Inter-rater reliability for the major strategy used was calculated using the percentage agreement method. Thirteen percent of the total number of student interviews (79 out of 623) were coded by two independent raters to check inter-rater reliability. The inter-rater reliability for the major strategies was 82.7% (Schoen et al., 2015). The author of this study calculated the inter-rater reliability for the "counting by" variable, which was a secondary variable under the two major strategy groups (*direct modeling strategy* and *counting strategy*). Raters first entered the major strategy used by a student to solve the problem. If the strategy was a *direct modeling* or *counting* strategy, then raters entered a "counting by" variable to indicate whether the student counted by ones, twos, or tens, etc. The "counting by" variable was used to identify the *concrete* *modeling with tens* strategies. The percentage agreement method was used to calculate the inter-rater reliability for the “counting by” variable. The percentage agreement for the “counting by” variable on average between the two raters for multi-digit problems was 84.1%.

Results of Statistical Analysis

Research Question One

The first research question was: Are there statistically significant differences in the numbers of first grade students in different strategy groups between treatment and control groups? To answer this research question single-digit, and multi-digit strategies were analyzed separately.

a. Differences in the numbers of first grade students in single-digit strategy groups between treatment and control.

Chi-square analysis was used to test whether numbers of first grade students in single-digit strategy groups were significantly different for treatment and control groups. The assumption of an expected cell frequency of at least five per cell was met. Results showed that the differences in the numbers of students in strategy groups were not significant between treatment and control with $\chi^2 = 2.075$, $p > .05$. Tables 39 displays the numbers of students in each strategy group for treatment and control, and table 40 shows the result of the chi square analysis.

Table 39: First Grade - Single-digit Strategy * Condition Cross-tabulation

Strategy * Condition Cross-tabulation

			Condition		
			Control	Treatment	Total
Strategy	Concrete Modeling	Count	57	56	113
		Expected Count	59.6	53.4	113.0
		% within Condition	35.0%	38.4%	36.6%
	Counting	Count	86	66	152
		Expected Count	80.2	71.8	152.0
		% within Condition	52.8%	45.2%	49.2%
	Derived Facts/Recall	Count	20	24	44
		Expected Count	23.2	20.8	44.0
		% within Condition	12.3%	16.4%	14.2%
Total		Count	163	146	309
		Expected Count	163.0	146.0	309.0
		% within Condition	100.0%	100.0%	100.0%

Table 40: First Grade - Single-digit Chi-Square Test

Chi-Square Tests

	Value	*df*	*p* (2-sided)
Pearson Chi-Square	2.075[a]	2	.354
Likelihood Ratio	2.076	2	.354
N of Valid Cases	309		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 20.79.

b. Differences in the numbers of first grade students in multi-digit strategy groups between treatment and control groups.

Chi-square analysis was used to test whether the numbers of first grade students in multi-digit strategy groups was significantly different between treatment and control groups. The assumption of an expected cell frequency of at least five per cell was met. Although a higher percentage of treatment group students were in more advanced strategy groups (*concrete modeling with tens* and *invented algorithms*), these differences were not statistically significant with $\chi^2 = 7.372$, $p > .05$. Tables 41 and 42 display the results of the statistical analysis.

Table 41: First Grade - Multi-digit Strategy * Condition Cross-tabulation

Strategy * Condition Cross-tabulation

			Condition		
			Control	Treatment	Total
Strategy	Other	Count	24	21	45
		Expected Count	23.6	21.4	45.0
		% within Condition	14.5%	14.0%	14.3%
	Unitary	Count	112	87	199
		Expected Count	104.2	94.8	199.0
		% within Condition	67.9%	58.0%	63.2%
	Concrete Modeling with tens	Count	6	16	22
		Expected Count	11.5	10.5	22.0
		% within Condition	3.6%	10.7%	7.0%
	Invented Algorithms	Count	23	26	49
		Expected Count	25.7	23.3	49.0
		% within Condition	13.9%	17.3%	15.6%
Total		Count	165	150	315
		Expected Count	165.0	150.0	315.0
		% within Condition	100.0%	100.0%	100.0%

Table 42: First Grade - Multi-digit Chi-square Test

Chi-Square Tests

	Value	*df*	*p* (2-sided)
Pearson Chi-Square	7.372[a]	3	.061
Likelihood Ratio	7.535	3	.057
N of Valid Cases	315		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 10.48.

Research Question Two

The second research question was: Are there statistically significant differences in the mathematics achievement (as measured by the ITBS) of first grade students between different strategy groups? To answer this research question single-digit and multi-digit strategies were analyzed separately.

a. Differences in the mathematics achievement of first graders between single-digit strategy groups.

Multivariate Analysis of Covariance (MANCOVA) analysis was used to test whether there were statistically significant differences in the mathematics achievement of first grade students between single-digit strategy groups, which are *concrete modeling, counting, and derived facts/recall*. MANCOVA is a multivariate extension of the analysis of covariance (ANCOVA) and tests whether there are statistically significant mean differences among groups after adjusting the dependent variable for differences on one or more covariates (Tabachnick & Fidell, 2013). In the analysis, the *Math Problems* (MP) and *Math Computation* (MC) scores of the ITBS were used as dependent variables, and strategy group was used as the grouping variable. The student pretest scores were used as covariate.

First, multivariate normality, homogeneity of variance, homogeneity of variance-covariance matrices, and linearity assumptions of the MANCOVA were checked (Tabachnick & Fidell, 2013). The Kolgorov-Smirnov (KS) test was used to check multivariate normality. Although the KS test was significant for the *concrete modeling* and *counting* strategy groups for the ITBS *Math Problems*, and it was significant for the *counting* and *derived facts/recall* strategies for the ITBS *Math Computation*, the data approximately followed the 45° line. In addition even with unequal group sample sizes, the MANCOVA is robust violating the normality assumption when cell sizes are greater than or equal to 20 (Mardia, 1971), which was the case in this analysis. Therefore a multivariate test was still conducted. Table 43 summarizes the KS test statistics and figure 7 shows the Q-Q plots of strategy groups for dependent variables. Q-Q plots show the quantiles of the theoretical normal distribution against quantiles of the sample distribution. Points that fall on or close to the diagonal line suggest evidence of normality (Lomax & Hahs-Vaughn, 2012).

Table 43: First Grade - Single-digit -The Kolgorov-Smirnow Test Statistics

		Kolmogorov-Smirnov[a]		
	Strategy	Statistic	*df*	*p*
SS_MP	Concrete Modeling	.090	106	.036
	Counting	.098	135	.003
	Derived Facts/Recall	.112	40	.200*
SS_MC	Concrete Modeling	.089	106	.040
	Counting	.101	135	.002
	Derived Facts/Recall	.116	40	.187

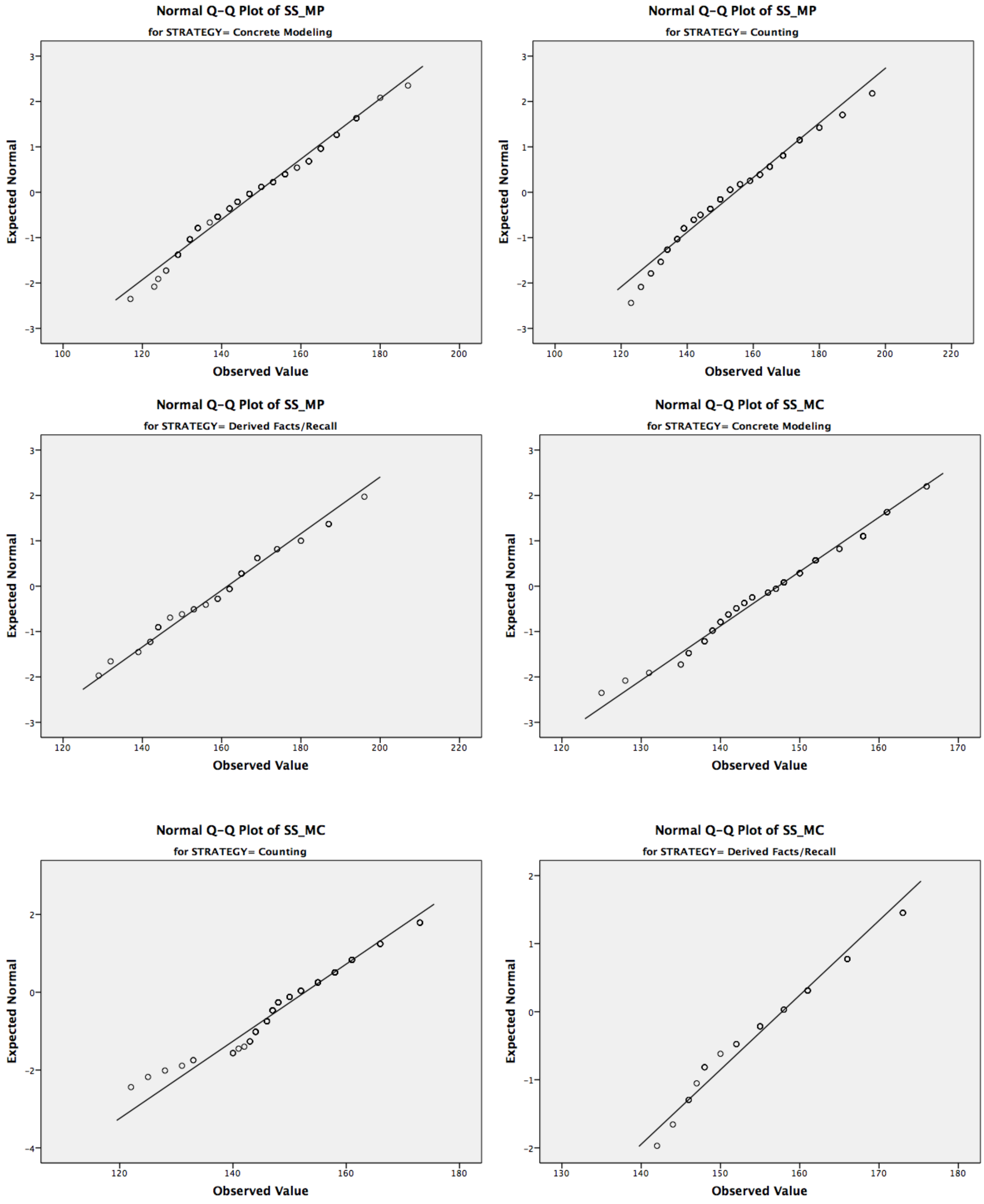


Figure 7: First Grade - Single-digit - Q-Q Plots

The homogeneity of variance assumption suggests that the variability in the dependent variable (DV) is expected to be about the same at all levels of the grouping variable, and the homogeneity of variance-covariance matrices assumption (equality of covariance matrices) suggest that variance-covariance matrices within each cell are sampled from the same population variance-covariance matrix and can reasonably be pooled to create a single estimate of error (Tabachnick & Fidell, 2013). Box's test reveals that the assumption of homogeneity of variance-covariance matrices was met with Box's M = 6.326 with $F_{(6,\ 123096.653)} = 1.04$, $p > .05$. Table 44 shows the results of Box’s test of equality of covariance matrices.

Table 44: First Grade - Single-digit - Box's Test

Box's Test of Equality of Covariance Matrices[a]	
Box's M	6.326
F	1.039
df_1	6
df_2	123096.653
p	**.397**

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + G1Pr_Math + STRATEGY

According to Levene’s test, the homogeneity of variance assumption was met with $p > 0.05$, which is shown in table 45.

Table 45: First Grade - Single-digit - Levene’s Test

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	*p*
SS_MP	.140	2	278	**.870**
SS_MC	.073	2	278	**.929**

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + G1Pr_Math + STRATEGY

Linearity assumption was checked through the analysis of scatter plot and correlations. The scatter plot showed a linear relationship between the dependent variables, and the correlation matrix showed a high but not perfect correlation between the two dependent variables. Therefore it was assumed that the linearity assumption was met. Figure 8 shows the scatter plot and table 46 shows the correlation matrix between the dependent variables.

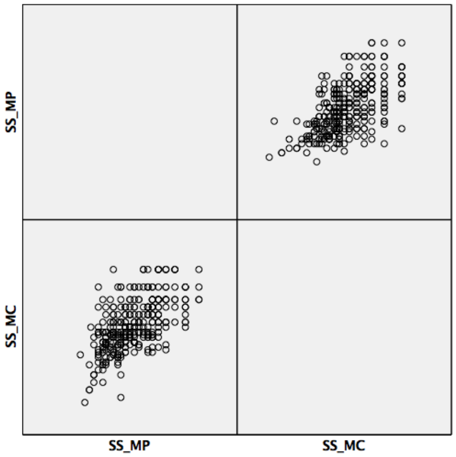


Figure 8: First Grade - Scatter Plot of DVs

Table 46: First Grade - Correlation Matrix between DVs

Correlations

		SS_MP	SS_MC
SS_MP	Pearson Correlation	1	.597**
	p (2-tailed)		<.001
	N	307	307
SS_MC	Pearson Correlation	.597**	1
	p (2-tailed)	<.001	
	N	307	307

**. Correlation is significant at the 0.01 level (2-tailed).

MANCOVA analysis: Single-Digit Strategies – First Grade

The mean scores and standard deviations for strategy groups are shown in table 47. The mean score for *derived facts/recall* strategy group was higher than the mean score for *counting* group, and the mean score for *counting* strategy group was higher than the *concrete modeling* strategy group for both ITBS *Math Problems* and *Math Computation* scores.

Table 47: First Grade - Single-digit - Descriptive Statistics

Descriptive Statistics

	Strategy	Mean	Std. Deviation	N
SS_MP	Concrete Modeling	148.99	15.037	106
	Counting	154.61	16.603	135
	Derived Facts/Recall	161.45	16.011	40
	Total	153.46	16.424	281
SS_MC	Concrete Modeling	147.31	8.346	106
	Counting	152.69	10.078	135
	Derived Facts/Recall	157.78	9.124	40
	Total	151.38	9.963	281

The statistical analysis showed that strategy group was statistically significant in determining the combined test results of the ITBS, controlling for student pretest score with $F_{(4,554)} = 4.631$, $p < .01$, and Pillai's Trace = .065. The summary of the statistical test results is given in Table 48.

Table 48: First Grade - Single-digit - Multivariate Tests

Multivariate Tests[a]

Effect		Value	*F*	Hypothesis df	Error *df*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power[d]
Intercept	Pillai's Trace	.996	37138.547[b]	2.000	276.000	<.001	.996	74277.094	1.000
	Wilks' Lambda	.004	37138.547[b]	2.000	276.000	<.000	.996	74277.094	1.000
	Hotelling's Trace	269.120	37138.547[b]	2.000	276.000	<.000	.996	74277.094	1.000
	Roy's Largest Root	269.120	37138.547[b]	2.000	276.000	<.000	.996	74277.094	1.000
G1Pr_Math	Pillai's Trace	.384	85.920[b]	2.000	276.000	<.000	.384	171.839	1.000
	Wilks' Lambda	.616	85.920[b]	2.000	276.000	<.000	.384	171.839	1.000
	Hotelling's Trace	.623	85.920[b]	2.000	276.000	<.000	.384	171.839	1.000
	Roy's Largest Root	.623	85.920[b]	2.000	276.000	<.000	.384	171.839	1.000
Strategy	Pillai's Trace	.065	4.631	4.000	554.000	<.001	.032	18.523	.948
	Wilks' Lambda	.935	4.693[b]	4.000	552.000	<.001	.033	18.772	.950
	Hotelling's Trace	.069	4.755	4.000	550.000	<.001	.033	19.018	.953
	Roy's Largest Root	.069	9.553[c]	2.000	277.000	<.000	.065	19.105	.980

a. Design: Intercept + G1Pr_Math + STRATEGY
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level.
d. Computed using alpha =

The test of between-subject effects indicated that strategy group was a significant factor on the ITBS *Math Computation* with $F_{1\ (2, 277)}$ =9.546, $p < .01$, $\eta^2 = 0.064$ but not significant on the ITBS *Math Problems* with $F_{2\ (2,277)}$ =1.212, $p > .05$, $\eta^2 = 0.009$. The summary of the results between-subject effects is provided in Table 49.

Table 49: First Grade - Single-digit - Between Subject Effects

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Corrected Model	SS_MP	29717.019[a]	3	9905.673	59.898	.000	.393	179.695	1.000
	SS_MC	9079.031[b]	3	3026.344	44.792	.000	.327	134.375	1.000
Intercept	SS_MP	4717940.387	1	4717940.387	28528.763	.000	.990	28528.763	1.000
	SS_MC	4653993.875	1	4653993.875	68881.891	.000	.996	68881.891	1.000
G1Pr_Math	SS_MP	24868.245	1	24868.245	150.375	.000	.352	150.375	1.000
	SS_MC	5457.174	1	5457.174	80.769	.000	.226	80.769	1.000
Strategy	**SS_MP**	400.732	2	200.366	1.212	**.299**	.009	2.423	.264
	SS_MC	1289.903	2	644.951	9.546	**.000**	.064	19.091	.980
Error	SS_MP	45808.838	277	165.375					
	SS_MC	18715.460	277	67.565					
Total	SS_MP	6693295.000	281						
	SS_MC	6467533.000	281						
Corrected Total	SS_MP	75525.858	280						
	SS_MC	27794.491	280						

a. R Squared = .393 (Adjusted R Squared = .387)
b. R Squared = .327 (Adjusted R Squared = .319)
c. Computed using alpha =

Pairwise comparisons showed that students classified into the *concrete modeling* strategy group had a significantly lower mean score with *p*< .05 for the *Math Computation* of the ITBS than the students classified into the *counting* or *derived facts/recall* strategy groups. Although the mean score of the students classified into the *derived facts/recall strategy* group was higher than the students in *counting* strategy group, this difference was not statically significant with *p*>.05. Table 50 presents the results of pairwise comparison statistics, and figure nine shows the profile plot for estimated marginal means of the ITBS *Math Computation* for the three strategy groups.

Table 50: First Grade - Single-digit - Pairwise Comparisons

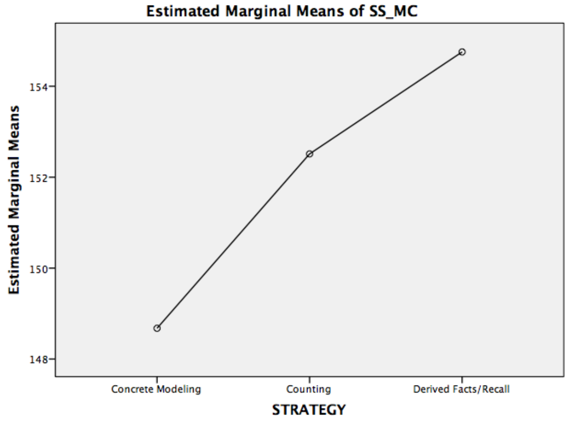
Pairwise Comparisons

Dependent Variable	(I) STRATEGY	(J) STRATEGY	Mean Difference (I-J)	Std. Error	p^b	95% Confidence Interval for Difference[b]	
						Lower Bound	Upper Bound
SS_MP	Concrete Modeling	Counting	-2.324	1.690	.170	-5.652	1.003
		Derived Facts/Recall	-3.097	2.505	.217	-8.030	1.835
	Counting	Concrete Modeling	2.324	1.690	.170	-1.003	5.652
		Derived Facts/Recall	-.773	2.367	.744	-5.433	3.887
	Derived Facts/Recall	Concrete Modeling	3.097	2.505	.217	-1.835	8.030
		Counting	.773	2.367	.744	-3.887	5.433
SS_MC	Concrete Modeling	Counting	-3.835*	1.080	**.000**	-5.962	-1.708
		Derived Facts/Recall	-6.078*	1.601	**.000**	-9.231	-2.926
	Counting	Concrete Modeling	3.835*	1.080	**.000**	1.708	5.962
		Derived Facts/Recall	-2.243	1.513	.139	-5.222	.736
	Derived Facts/Recall	Concrete Modeling	6.078*	1.601	**.000**	2.926	9.231
		Counting	2.243	1.513	.139	-.736	5.222

Based on estimated marginal means

*. The mean difference is significant at the

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).



Covariates appearing in the model are evaluated at the following values: G1Pr_Math = .09391

Figure 9: First Grade - Single-digit - Estimated Marginal Means

b. Differences in mathematics achievement of first grade students between multi-digit strategy groups

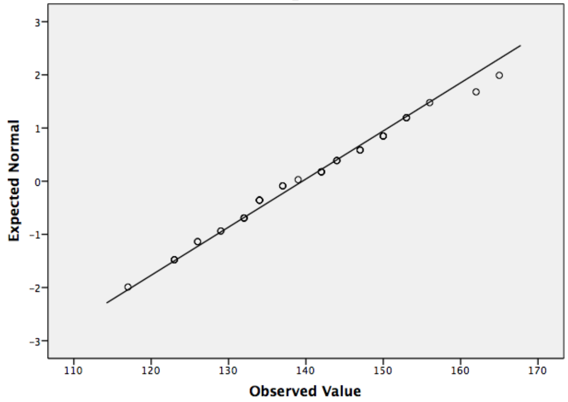
Multivariate Analysis of Covariance (MANCOVA) analysis was used to test whether there were statistically significant differences in the mathematics achievement of first grade students between multi-digit strategy groups. The assumptions of MANCOVA (multivariate normality, homogeneity or variance, homogeneity of variance-covariance matrices, and linearity) were checked prior to initiating the analysis. Kolmogorov-Smirnov (KS) test was used to check multivariate normality. The KS test was not significant for all strategy groups on both sections of the ITBS except *unitary* strategy group. In addition, the data approximately followed the 45^o line. Therefore, the multivariate test was still conducted. Additionally, MANCOVA is robust to violation of normality when cell sizes are greater than or equal to 20, which was the case in this analysis. Table 51 summarizes the KS test statistics and figure 10 shows the Q-Q plots of strategy groups for each dependent variable.

Table 51: First Grade - Multi-digit - Normality Test

		Kolmogorov-Smirnov[a]		
	STRATEGY	Statistic	*df*	*p*
SS_MP	Other	.121	42	.131
	Unitary	.094	181	**.001**
	Concrete Modeling with Tens	.184	20	.073
	Invented Algorithms	.123	44	.092
SS_MC	Other	.081	42	.200*
	Unitary	.089	181	**.001**
	Concrete Modeling with Tens	.186	20	.067
	Invented Algorithms	.110	44	.200*

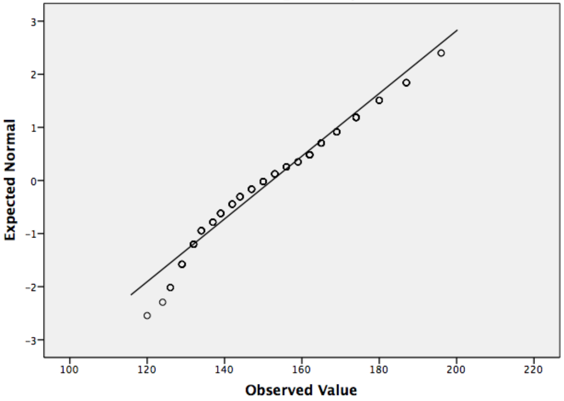
Normal Q-Q Plot of SS_MP

for STRATEGY_A= Other



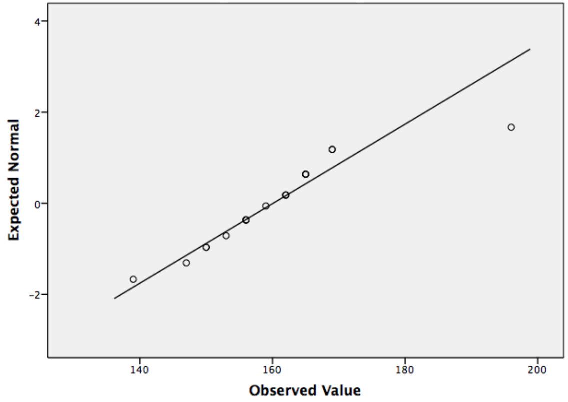
Normal Q-Q Plot of SS_MP

for STRATEGY_A= Unitary



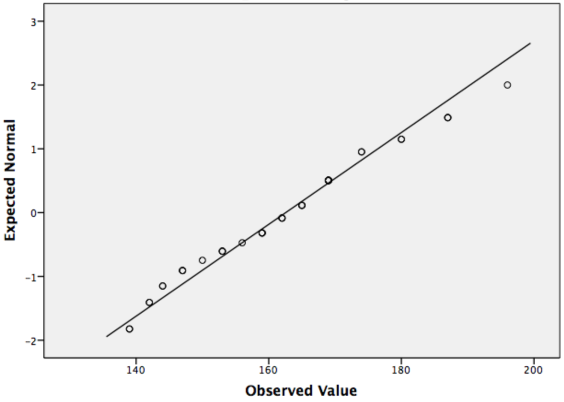
Normal Q-Q Plot of SS_MP

for STRATEGY_A= Concrete Modeling with Tens



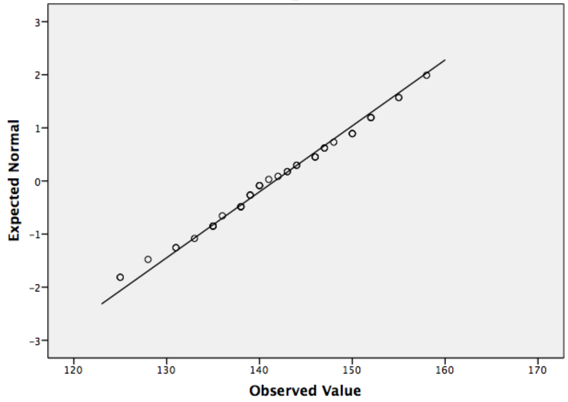
Normal Q-Q Plot of SS_MP

for STRATEGY_A= Invented Algorithms



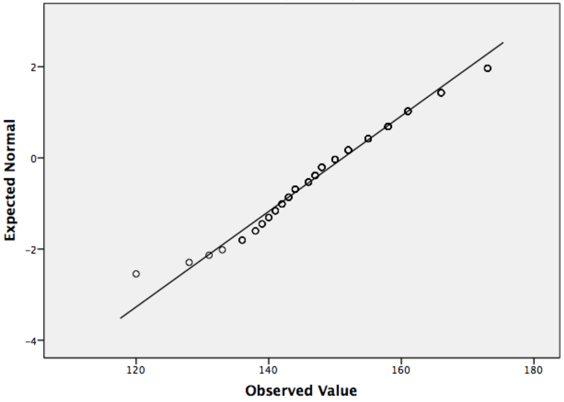
Normal Q-Q Plot of SS_MC

for STRATEGY_A= Other



Normal Q-Q Plot of SS_MC

for STRATEGY_A= Unitary



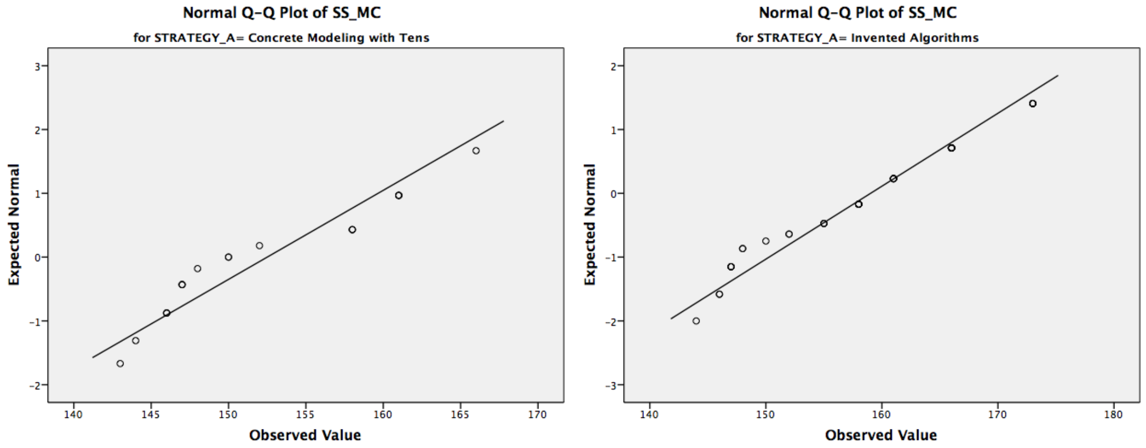


Figure 10: First Grade - Multi-digit - Q-Q Plots

Box's test revealed that the homogeneity of variance-covariance matrices assumption was not met with Box's M = 22.474 with $F_{(9,\ 41428.672)} = 2.435$, $p < .05$. Pillai's test statistics was chosen for the analysis since it is more robust to the violations of homogeneity of the variance-covariance matrices (Tabachnick & Fidell, 2013). Table 52 shows the results of Box's test of equality of covariance matrices.

Table 52: First Grade - Multi-digit - Box's Test

Box's Test of Equality of Covariance Matrices[a]

Box's M	22.474
F	2.435
df_1	9
df_2	41428.672
p	.009

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.

a. Design: Intercept + G1Pr_Math +STRATEGY_A

According to Levene's test, the homogeneity of variance assumption was met for ITBS *Math Computation* with $p > 0.05$ and not met for ITBS *Math Problems* with $p < 0.05$. Table 53 displays the results of Levene's test.

Table 53: First Grade - Multi-digit - Levene's Test

Levene's Test of Equality of Error Variances[a]

	F	df_1	df_2	*p*
SS_MP	5.986	3	283	.001
SS_MC	.721	3	283	.540

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + G1Pr_Math + STRATEGY_A

Linearity between dependent variables was already checked in the previous analysis. It was found that there was a linear relationship between the dependent variables, and the correlation matrix showed a high but not perfect correlation between the dependent variables. Therefore it was concluded that the linearity assumption was met.

MANCOVA analysis: First Grade Multi-digit Strategies

The mean score for *invented algorithms* group was higher than the mean score for *concrete modeling with tens* group, the mean score for *concrete modeling with tens* group was higher than *unitary* strategy group, and the mean score for unitary strategy group was higher than the mean score for *other* strategy group for both ITBS *Math Problems* and *Math Computation*. Table 54 displays the descriptive statistics for strategy groups for each dependent variable.

Table 54: First Grade - Multi-digit - Descriptive Statistics

Descriptive Statistics

	STRATEGY	Mean	Std. Deviation	N
SS_MP	Other	139.548	11.0480	42
	Unitary	152.144	16.8902	181
	Concrete Modeling with Tens	160.100	11.4566	20
	Invented Algorithms	162.818	13.8418	44
	Total	152.491	16.7244	287
SS_MC	Other	141.643	8.0511	42
	Unitary	151.155	9.5399	181
	Concrete Modeling with Tens	152.500	7.1635	20
	Invented Algorithms	159.068	8.6465	44
	Total	151.070	10.2108	287

The statistical analysis showed that strategy group was significant in determining the combined test results of the ITBS when controlling for student pretest score with $F_{(6,564)} = 5.807$, $p < .01$, and Pillai's Trace = .116. The summary of the statistical test results is given in Table 55.

Table 55: First Grade - Multi-digit - Multivariate Tests

Multivariate Tests^a

Effect		Value	*F*	Hypothesis *df*	Error df	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power^d
Intercept	Pillai's Trace	.995	27980.627^b	2.000	281.000	.000	.995	55961.254	1.000
	Wilks' Lambda	.005	27980.627^b	2.000	281.000	.000	.995	55961.254	1.000
	Hotelling's Trace	199.150	27980.627^b	2.000	281.000	.000	.995	55961.254	1.000
	Roy's Largest Root	199.150	27980.627^b	2.000	281.000	.000	.995	55961.254	1.000
G1Pr_Math	Pillai's Trace	.335	70.934^b	2.000	281.000	.000	.335	141.868	1.000
	Wilks' Lambda	.665	70.934^b	2.000	281.000	.000	.335	141.868	1.000
	Hotelling's Trace	.505	70.934^b	2.000	281.000	.000	.335	141.868	1.000
	Roy's Largest Root	.505	70.934^b	2.000	281.000	.000	.335	141.868	1.000
Strategy	Pillai's Trace	.116	5.807	6.000	564.000	.000	.058	34.844	.998
	Wilks' Lambda	.885	5.898^b	6.000	562.000	.000	.059	35.386	.998
	Hotelling's Trace	.128	5.987	6.000	560.000	.000	.060	35.924	.998
	Roy's Largest Root	.114	10.750^c	3.000	282.000	.000	.103	32.251	.999

a. Design: Intercept + G1Pr_Math + Strategy

b. Exact statistic

c. The statistic is an upper bound on F that yields a lower bound on the significance level.

d. Computed using alpha =

The test between-subject effects indicated that strategy group was a significant factor on both the ITBS *Math Problems* with $F_{1\,(3,\,282)}$ =4.140, $p < .05$, $\eta 2 = .042$ and the ITBS *Math Computation* with $F_{2\,(3,\,282)}$ =10.395, $p < .01$, $\eta 2 = 1$. The summary of the results between-subject effects is provided in Table 56.

Table 56: First Grade – Multi-digit – Between-Subject Effects Test

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power^{c}
Corrected Model	SS_MP	33472.105^{a}	4	8368.026	50.722	.000	.418	202.889	1.000
	SS_MC	10963.502^{b}	4	2740.875	40.993	.000	.368	163.972	1.000
Intercept	SS_MP	3539825.324	1	3539825.324	21456.427	.000	.987	21456.427	1.000
	SS_MC	3463926.430	1	3463926.430	51807.045	.000	.995	51807.045	1.000
G1Pr_Math	SS_MP	20563.392	1	20563.392	124.644	.000	.307	124.644	1.000
	SS_MC	4374.002	1	4374.002	65.418	.000	.188	65.418	1.000
Strategy	SS_MP	2049.241	3	683.080	4.140	.007	.042	12.421	.848
	SS_MC	2085.069	3	695.023	10.395	.000	.100	31.185	.999
Error	SS_MP	46523.624	282	164.977					
	SS_MC	18855.105	282	66.862					
Total	SS_MP	6753777.000	287						
	SS_MC	6579747.000	287						
Corrected Total	SS_MP	79995.728	286						
	SS_MC	29818.606	286						

a. R Squared = .418 (Adjusted R Squared = .410)
b. R Squared = .368 (Adjusted R Squared = .359)
c. Computed using alpha =

Pairwise comparisons showed that students classified into the *other* strategy group had a significantly lower mean score than any of the students classified into the other strategy groups (*unitary*, *concrete modeling with tens*, and *invented algorithms*) for the ITBS *Math Problems* and *Math Computation* with $p< .05$. The *invented algorithms* group had a significantly higher mean score on the ITBS *Math Computation* than the *unitary* group. The mean differences on the ITBS *Math Problems* between the *unitary, concrete modeling with tens* and *invented algorithms* strategy groups were not statistically significant. Although not significant, the *invented algorithm* group had higher mean score than the *concrete modeling with tens* strategy group on the ITBS *Math Computation* and the *concrete modeling with tens* group had higher mean score than the *invented algorithm*s strategy group on the ITBS *Math Problems*. Table 57 presents the results of pairwise comparison statistics and Figure 11 shows the profile plots of estimated marginal means of the ITBS problem solving and counting scores for the strategy groups.

Table 57: First Grade - Multi-digit - Pairwise Comparisons

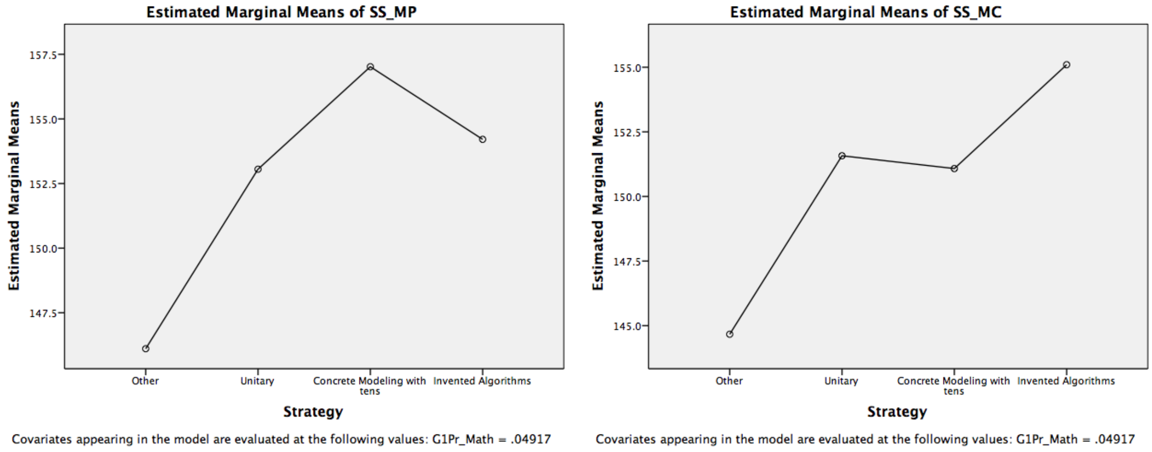
Pairwise Comparisons

Dependent Variable	(I) Strategy	(J) Strategy	Mean Difference (I-J)	Std. Error	p^b	95% Confidence Interval for Difference^b	
						Lower Bound	Upper Bound
SS_MP	Other	Unitary	-6.946*	2.257	.002	-11.390	-2.503
		Concrete Modeling with tens	-10.911*	3.595	.003	-17.987	-3.835
		Invented Algorithms	-8.101*	3.086	.009	-14.176	-2.027
	Unitary	Other	6.946*	2.257	.002	2.503	11.390
		Concrete Modeling with tens	-3.965	3.048	.194	-9.964	2.034
		Invented Algorithms	-1.155	2.321	.619	-5.724	3.414
	Concrete Modeling with tens	Other	10.911*	3.595	.003	3.835	17.987
		Unitary	3.965	3.048	.194	-2.034	9.964
		Invented Algorithms	2.810	3.499	.423	-4.078	9.698
	Invented Algorithms	Other	8.101*	3.086	.009	2.027	14.176
		Unitary	1.155	2.321	.619	-3.414	5.724
		Concrete Modeling with tens	-2.810	3.499	.423	-9.698	4.078
SS_MC	Other	Unitary	-6.906*	1.437	.000	-9.735	-4.077
		Concrete Modeling with tens	-6.411*	2.289	.005	-10.915	-1.906
		Invented Algorithms	-10.429*	1.965	.000	-14.296	-6.562
	Unitary	Other	6.906*	1.437	.000	4.077	9.735
		Concrete Modeling with tens	.496	1.940	.799	-3.324	4.315
		Invented Algorithms	-3.523*	1.478	.018	-6.432	-.614
	Concrete Modeling with tens	Other	6.411*	2.289	.005	1.906	10.915
		Unitary	-.496	1.940	.799	-4.315	3.324
		Invented Algorithms	-4.019	2.228	.072	-8.403	.366
	Invented Algorithms	Other	10.429*	1.965	.000	6.562	14.296
		Unitary	3.523*	1.478	.018	.614	6.432
		Concrete Modeling with tens	4.019	2.228	.072	-.366	8.403

Based on estimated marginal means

*. The mean difference is significant at the

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).



Covariates appearing in the model are evaluated at the following values: G1Pr_Math = .04917

Covariates appearing in the model are evaluated at the following values: G1Pr_Math = .04917

Figure 11: First Grade - Multi-digit - Estimated Marginal Means

Research Question Three

The third research question was: Are there statistically significant differences in the numbers of second grade students in different strategy groups between treatment and control groups? To answer this research question single-digit, and multi-digit strategies were analyzed separately.

a. Differences in the numbers of second grade students in single-digit strategies between treatment and control groups.

Chi-square analysis was used to test whether the numbers of second grade students in single-digit strategy groups were significantly different for treatment and control groups. The assumption of an expected cell frequency of at least five per cell was met. Nine percent of the control group students and 17% of the treatment group students were in the *concrete modeling* strategy group. Sixty-three percent of the control group students and 44% of the treatment group students were in the *counting* strategy group. Twenty-eight percent of the control group students and 40% of the treatment group students were in the *derived facts/recall* strategy group. The differences in the numbers of students in strategy groups were significant with $\chi^2 = 10.171$, $p < 0.05$. Tables 58 and 59 summarize the results of the statistical analysis.

Table 58: Second Grade - Single-digit Strategy * Condition Cross-tabulation

STRATEGY * Condition Cross-tabulation

			Condition		
			Control	Treatment	Total
STRATEGY	Concrete Modeling	Count	12	22	34
		Expected Count	17.1	16.9	34.0
		% within Condition	9.0%	16.5%	12.7%
	Counting	Count	84	58	142
		Expected Count	71.3	70.7	142.0
		% within Condition	62.7%	43.6%	53.2%
	Derived FactsRecall/	Count	38	53	91
		Expected Count	45.7	45.3	91.0
		% within Condition	28.4%	39.8%	34.1%
Total		Count	134	133	267
		Expected Count	134.0	133.0	267.0
		% within Condition	100.0%	100.0%	100.0%

Table 59: Second Grade - Single-digit - Chi-square Tests

Chi-Square Tests

	Value	*df*	*p* (2-sided)
Pearson Chi-Square	10.171[a]	2	.006
Likelihood Ratio	10.253	2	.006
N of Valid Cases	267		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 16.94.

b. Differences in the number of second grade students in multi-digit strategy groups between treatment and control groups.

Chi-square analysis was used to test whether the numbers of second grade students in multi-digit strategy groups were significantly different for treatment and control groups. The assumption of an expected cell frequency of at least five per cell was met. The differences in the numbers of students in multi-digit strategy groups were not significant with $\chi^2 = 3.83$, $p > .05$. Tables 60 and 61 summarize the results of the statistical analysis.

Table 60: Second Grade - Multi-digit Strategy * Condition Cross-tabulation

STRATEGY * Condition Cross-tabulation

			Condition		
			Control	Treatment	Total
STRATEGY	Unitary	Count	37	53	90
		Expected Count	44.3	45.7	90.0
		% within Condition	30.6%	42.4%	36.6%
	Lower Standard Algorithm	Count	30	26	56
		Expected Count	27.5	28.5	56.0
		% within Condition	24.8%	20.8%	22.8%
	Concrete Modeling with Tens	Count	12	10	22
		Expected Count	10.8	11.2	22.0
		% within Condition	9.9%	8.0%	8.9%
	Higher Standard Algorithm	Count	25	20	45
		Expected Count	22.1	22.9	45.0
		% within Condition	20.7%	16.0%	18.3%
	Invented Algorithms	Count	17	16	33
		Expected Count	16.2	16.8	33.0
		% within Condition	14.0%	12.8%	13.4%
Total		Count	121	125	246
		Expected Count	121.0	125.0	246.0
		% within Condition	100.0%	100.0%	100.0%

Table 61: Second Grade - Multi-digit - Chi-square Test

Chi-Square Tests

	Value	*df*	*p* (2-sided)
Pearson Chi-Square	3.834[a]	4	.429
Likelihood Ratio	3.850	4	.427
N of Valid Cases	246		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 10.82.

Research Question Four

The fourth research question was: Is there a statistically significant difference in the mathematics achievements of second grade students between different strategy groups? To answer this research question single-digit, and multi-digit strategies were analyzed separately.

a. Differences in the mathematics achievement of students between single-digit strategy groups (*concrete modeling, counting,* and *derived facts/recall*)

Multivariate Analysis of Covariance (MANCOVA) was used to test whether there were statistically significant differences in the mathematics achievement of second grade students between single-digit strategy groups. First, the assumptions of MANCOVA (multivariate normality, homogeneity of variance, homogeneity of variance-covariance matrices, and linearity) were checked. The Kolmogorov-Smirnov (KS) test was used to check multivariate normality. Although the KS test was significant for several strategy groups (*counting* and *derived facts/recall* with ITBS *Math Problems* and *concrete modeling* and *counting* with ITBS *Math Computation*), the data approximately followed the 45^{o} line. Additionally, MANCOVA is robust violating normality assumption when cell sizes are greater than or equal to 20 (Mardia, 1971), which is the case in this analysis. Therefore, a multivariate test was still conducted. Table 62 summarizes the KS test statistics and figure 12 shows the Q-Q plots of dependent variables for strategy groups.

Table 62: Second Grade - Single-digit – Normality Test

		Kolmogorov-Smirnov[a]		
	STRATEGY	Statistic	*df*	*p*
SS_MP	Concrete Modeling	.088	33	.200*
	Counting	.095	134	.005
	Dervied FactsRecall/	.132	84	.001
SS_MC	Concrete Modeling	.173	33	.013
	Counting	.090	134	.009
	Dervied FactsRecall/	.093	84	.067

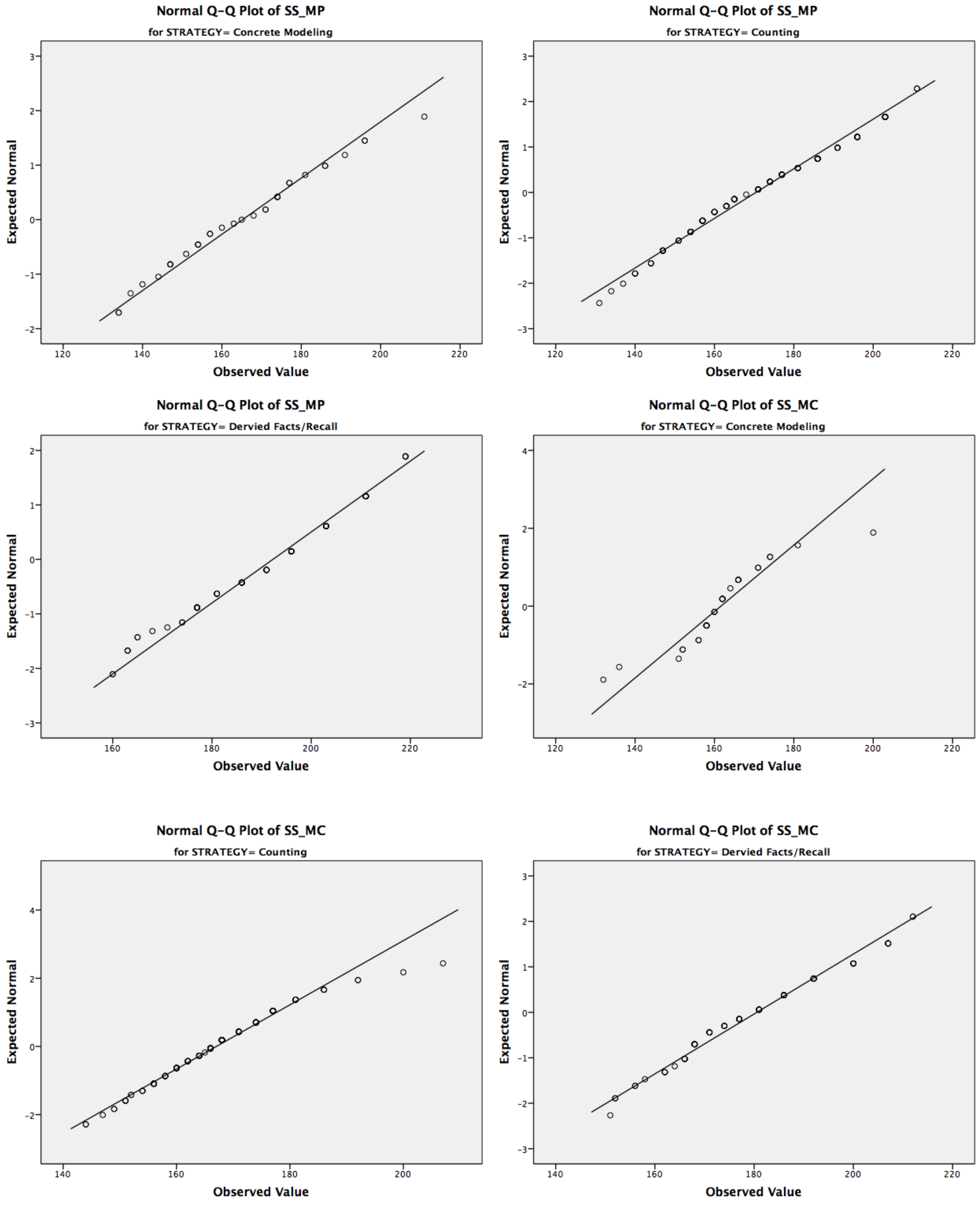


Figure 12: Second Grade - Single-digit - Q-Q Plots

The homoscedasticity assumption requires the population covariance matrices to be equal for the dependent variables for each group. Box's test revealed that the assumption of equality of covariance matrices across the cells was not met with Box's M = 24.505 with $F_{(6,\ 78852.341)} = 4.015$, $p < .01$. Pillai’s test statistics was chosen for the analysis since it is more robust to the violation of the homogeneity of covariance matrices. Table 63 shows the results of Box’s test of equality.

Table 63: Second Grade - Single-digit - Box’s Test

Box's Test of Equality of Covariance Matrices[a]	
Box's M	24.505
F	4.015
df_1	6
df_2	78852.341
p	.001

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + G2Pr_Math + STRATEGY

According to Levene’s test, homogeneity of variance assumption was met with $p > 0.05$ for the ITBS problem solving score and not met with $p < 0.05$ for the ITBS counting score. Table 64 displays the Levene’s test statistics.

Table 64: Second Grade - Single-digit - Levene’s Test

Levene's Test of Equality of Error Variances[a]

	F	df_1	df_2	*p*
SS_MP	1.185	2	247	.308
SS_MC	13.255	2	247	.000

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + G2Pr_Math + STRATEGY

Linearity assumption was checked through the analysis of scatter plot and correlations. The scatter plot showed a linear relationship between dependent variables, and the correlation matrix showed a high but not perfect correlation between the two dependent variables. Therefore it was assumed that the linearity assumption was met. Figure 14 shows the scatter plot and table 65 shows the correlation matrix between the dependent variables.

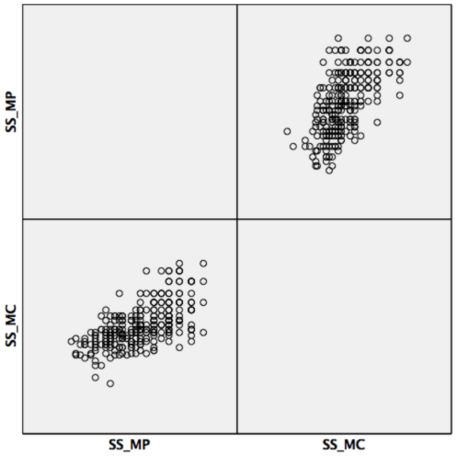


Figure 13: Second Grade -Scatter Plot of DV's

Table 65: Second Grade - Correlation Matrix between DV's

Correlations

		SS_MP	SS_MC
SS_MP	Pearson Correlation	1	.621**
	p (2-tailed)		<.001
	N	270	270
SS_MC	Pearson Correlation	.621**	1
	p (2-tailed)	<.001	
	N	270	270

**. Correlation is significant at the 0.01 level (2-tailed).

MANCOVA analysis: Second Grade - Single-Digit Strategies

The mean for the *derived facts/recall* strategy group was higher than the mean for the *counting* strategy group, and the mean for the *counting* strategy group was higher than the mean for the *concrete modeling* strategy group for both the ITBS *Math Problems* (MP) and *Math Computation* (MC). Table 66 displays the descriptive statistics for strategy groups for each dependent variable.

Table 66: Second Grade - Descriptive Statistics for Single-digit Strategy Groups

Descriptive Statistics

	STRATEGY	Mean	Std. Deviation	N
SS_MP	Concrete Modeling	165.21	19.368	33
	Counting	170.47	18.302	134
	Derived FactsRecall/	192.05	15.307	83
	Total	176.94	20.522	250
SS_MC	Concrete Modeling	161.64	11.720	33
	Counting	167.02	10.626	134
	Derived FactsRecall/	180.48	15.258	83
	Total	170.78	14.307	250

The statistical analysis showed that strategy group was significant in determining the combined test results of the ITBS when controlling for student pretest score with $F_{(4,492)} = 9.898$, $p < .01$, and Pillai's Trace = .149. The summary of the statistical results is given in Table 67.

Table 67: Second Grade - Single-digit - Multivariate Tests

Multivariate Tests[a]

Effect		Value	*F*	Hypothesis *df*	Error *df*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power[d]
Intercept	Pillai's Trace	.996	27627.881[b]	2.000	245.000	.000	.996	55255.762	1.000
	Wilks' Lambda	.004	27627.881[b]	2.000	245.000	.000	.996	55255.762	1.000
	Hotelling's Trace	225.534	27627.881[b]	2.000	245.000	.000	.996	55255.762	1.000
	Roy's Largest Root	225.534	27627.881[b]	2.000	245.000	.000	.996	55255.762	1.000
G2Pr_Math	Pillai's Trace	.452	101.060[b]	2.000	245.000	.000	.452	202.119	1.000
	Wilks' Lambda	.548	101.060[b]	2.000	245.000	.000	.452	202.119	1.000
	Hotelling's Trace	.825	101.060[b]	2.000	245.000	.000	.452	202.119	1.000
	Roy's Largest Root	.825	101.060[b]	2.000	245.000	.000	.452	202.119	1.000
STRATEGY	Pillai's Trace	.149	9.898	4.000	492.000	.000	.074	39.592	1.000
	Wilks' Lambda	.852	10.222[b]	4.000	490.000	.000	.077	40.888	1.000
	Hotelling's Trace	.173	10.544	4.000	488.000	.000	.080	42.177	1.000
	Roy's Largest Root	.167	20.525[c]	2.000	246.000	.000	.143	41.049	1.000

a. Design: Intercept + G2Pr_Math + STRATEGY

b. Exact statistic

c. The statistic is an upper bound on F that yields a lower bound on the significance level.

d. Computed using alpha =

The tests of between-subject effects indicated that strategy group was a significant factor on both the ITBS *Math Problems* with $F_{1\,(2, 246)} = 13.24$, $p < 0.01$, $\eta 2 = 0.097$ and the ITBS *Math Computation* with $F_{2\,(2,246)} = 14.0$, $p < 0.01$, $\eta 2 = 0.102$. The summary of the results of between-subject effects is displayed in Table 68.

Table 68: Second Grade - Single-digit - Between Subjects Effects

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Corrected Model	SS_MP	62434.138[a]	3	20811.379	120.666	.000	.595	361.997	1.000
	SS_MC	19110.342[b]	3	6370.114	49.191	.000	.375	147.572	1.000
Intercept	SS_MP	5591743.530	1	5591743.530	32421.281	.000	.992	32421.281	1.000
	SS_MC	5139118.272	1	5139118.272	39684.862	.000	.994	39684.862	1.000
G2Pr_Math	SS_MP	33340.741	1	33340.741	193.312	.000	.440	193.312	1.000
	SS_MC	6646.735	1	6646.735	51.327	.000	.173	51.327	1.000
STRATEGY	SS_MP	4566.224	2	2283.112	13.238	**.000**	.097	26.475	.997
	SS_MC	3627.199	2	1813.600	14.005	**.000**	.102	28.010	.998
Error	SS_MP	42427.962	246	172.471					
	SS_MC	31856.558	246	129.498					
Total	SS_MP	7931803.000	250						
	SS_MC	7342419.000	250						
Corrected Total	SS_MP	104862.100	249						
	SS_MC	50966.900	249						

a. R Squared = .595 (Adjusted R Squared = .590)
b. R Squared = .375 (Adjusted R Squared = .367)
c. Computed using alpha =

Pairwise comparisons showed that the *invented algorithms* group scored significantly higher on both the ITBS MP and MC with $p < 0.01$ than the *counting* strategy group and the *concrete modeling* group. Although the differences were not significant, the *counting* strategy group scored higher on the ITBS MC than the *concrete modeling* group, whereas the *concrete modeling* group scored higher on the ITBS MP than the *counting* strategy group. Table 69 presents the results of the pairwise comparison statistics.

Table 69: Second Grade - Single-digit - Pairwise Comparisons

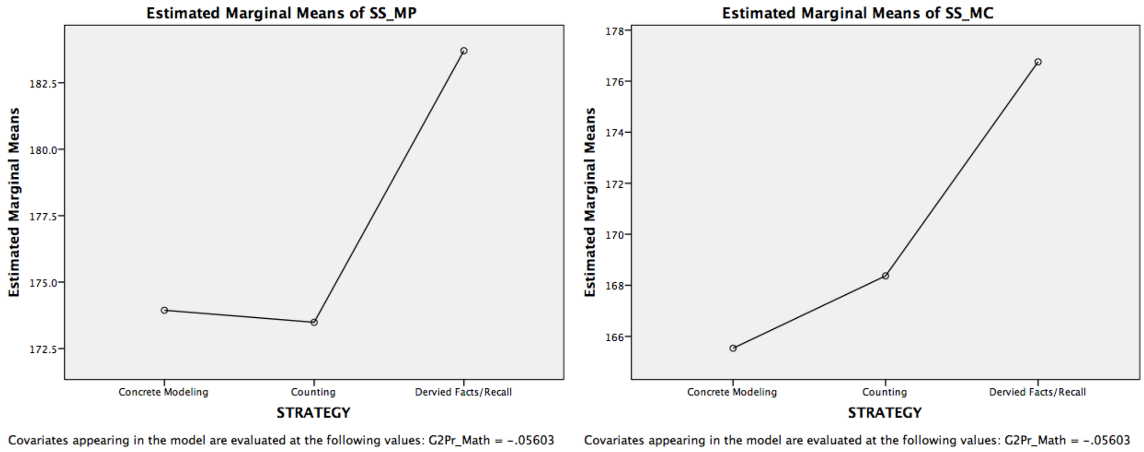
Pairwise Comparisons

Dependent Variable	(I) STRATEGY	(J) STRATEGY	Mean Difference (I-J)	Std. Error	p^b	95% Confidence Interval for Difference^b Lower Bound	95% Confidence Interval for Difference^b Upper Bound
SS_MP	Concrete Modeling	Counting	.453	2.585	.861	-4.639	5.545
		Derived FactsRecall/	-9.764*	2.969	.001	-15.611	-3.917
	Counting	Concrete Modeling	-.453	2.585	.861	-5.545	4.639
		Derived FactsRecall/	-10.217*	2.008	.000	-14.172	-6.261
	Derived FactsRecall/	Concrete Modeling	9.764*	2.969	**.001**	3.917	15.611
		Counting	10.217*	2.008	**.000**	6.261	14.172
SS_MC	Concrete Modeling	Counting	-2.836	2.240	.207	-7.248	1.576
		Derived FactsRecall/	-11.223*	2.572	.000	-16.289	-6.156
	Counting	Concrete Modeling	2.836	2.240	.207	-1.576	7.248
		Derived FactsRecall/	-8.387*	1.740	.000	-11.814	-4.959
	Derived FactsRecall/	Concrete Modeling	11.223*	2.572	**.000**	6.156	16.289
		Counting	8.387*	1.740	**.000**	4.959	11.814

Based on estimated marginal means

*. The mean difference is significant at the

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).



Covariates appearing in the model are evaluated at the following values: G2Pr_Math = -.05603

Covariates appearing in the model are evaluated at the following values: G2Pr_Math = -.05603

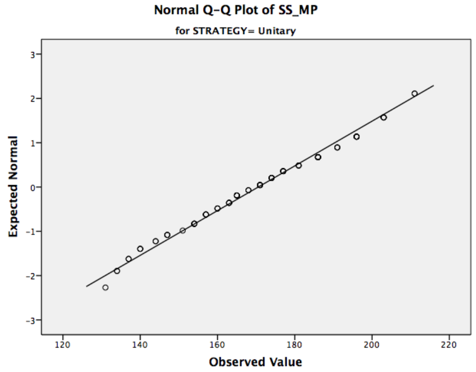
Figure 14: Second Grade - Single-digit - Estimated Marginal Means

b. Differences in the mathematics achievement of second grade students between multi-digit strategy groups

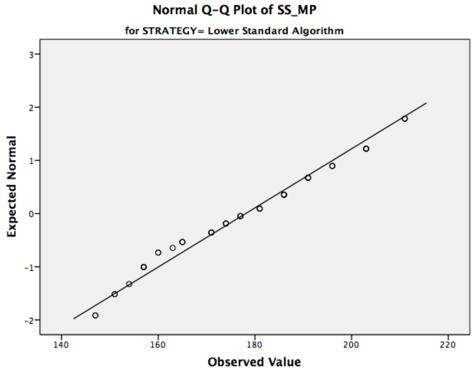
Multivariate Analysis of Covariance (MANCOVA) was used to test whether there were statistically significant differences in the mathematics achievement of second grade students between strategy groups. First, the assumptions of MANCOVA (multivariate normality, homogeneity of variance, homogeneity of variance-covariance matrices, and linearity) were checked. The Kolmogorov-Smirnov (KS) test was used to check multivariate normality. The KS test was non-significant for all strategy groups except for the *invented algorithms* group with the ITBS *Math Problems* and it was non-significant for all but the *unitary* and *higher standard algorithm* groups with the ITBS *Math Computation*. Although the KS test was significant for a few strategy groups, the data approximately followed the 45^{o} line. Additionally, MANCOVA is robust violating normality assumption when cell sizes are greater than or equal to 20 (Mardia, 1971), which was the case in this analysis. Therefore a multivariate test was still conducted. Table 70 summarizes the KS test statistics and figure 15 shows the Q-Q plots of strategy groups for dependent variables.

Table 70: Second Grade - Multi-digit - Normality Test

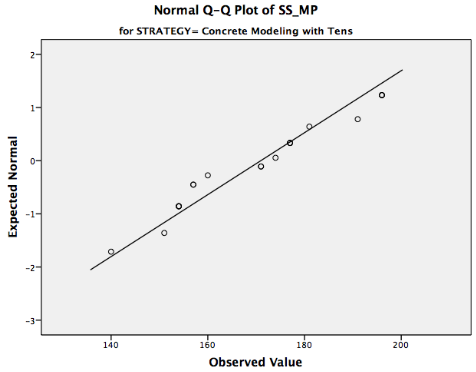
		Kolmogorov-Smirnov^{a}		
	STRATEGY	Statistic	*df*	*p*
SS_MP	Unitary	.076	85	$.200^{*}$
	Lower Standard Algorithm	.104	53	$.200^{*}$
	Concrete Modeling with Tens	.156	22	.179
	Higher Standard Algorithm	.085	41	$.200^{*}$
	Invented Algorithms	.257	32	**.000**
SS_MC	Unitary	.110	85	**.012**
	Lower Standard Algorithm	.115	53	.080
	Concrete Modeling with Tens	.138	22	$.200^{*}$
	Higher Standard Algorithm	.180	41	**.002**
	Invented Algorithms	.115	32	$.200^{*}$


Normal Q-Q Plot of SS_MP

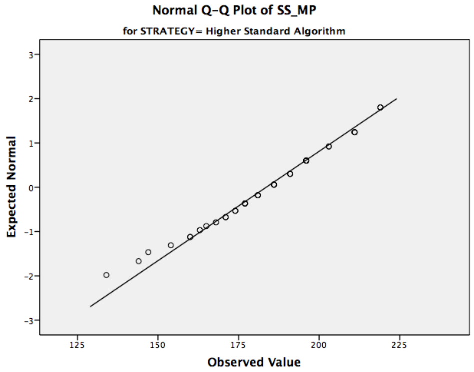
for STRATEGY= Unitary


Normal Q-Q Plot of SS_MP

for STRATEGY= Lower Standard Algorithm


Normal Q-Q Plot of SS_MP

for STRATEGY= Concrete Modeling with Tens


Normal Q-Q Plot of SS_MP

for STRATEGY= Higher Standard Algorithm

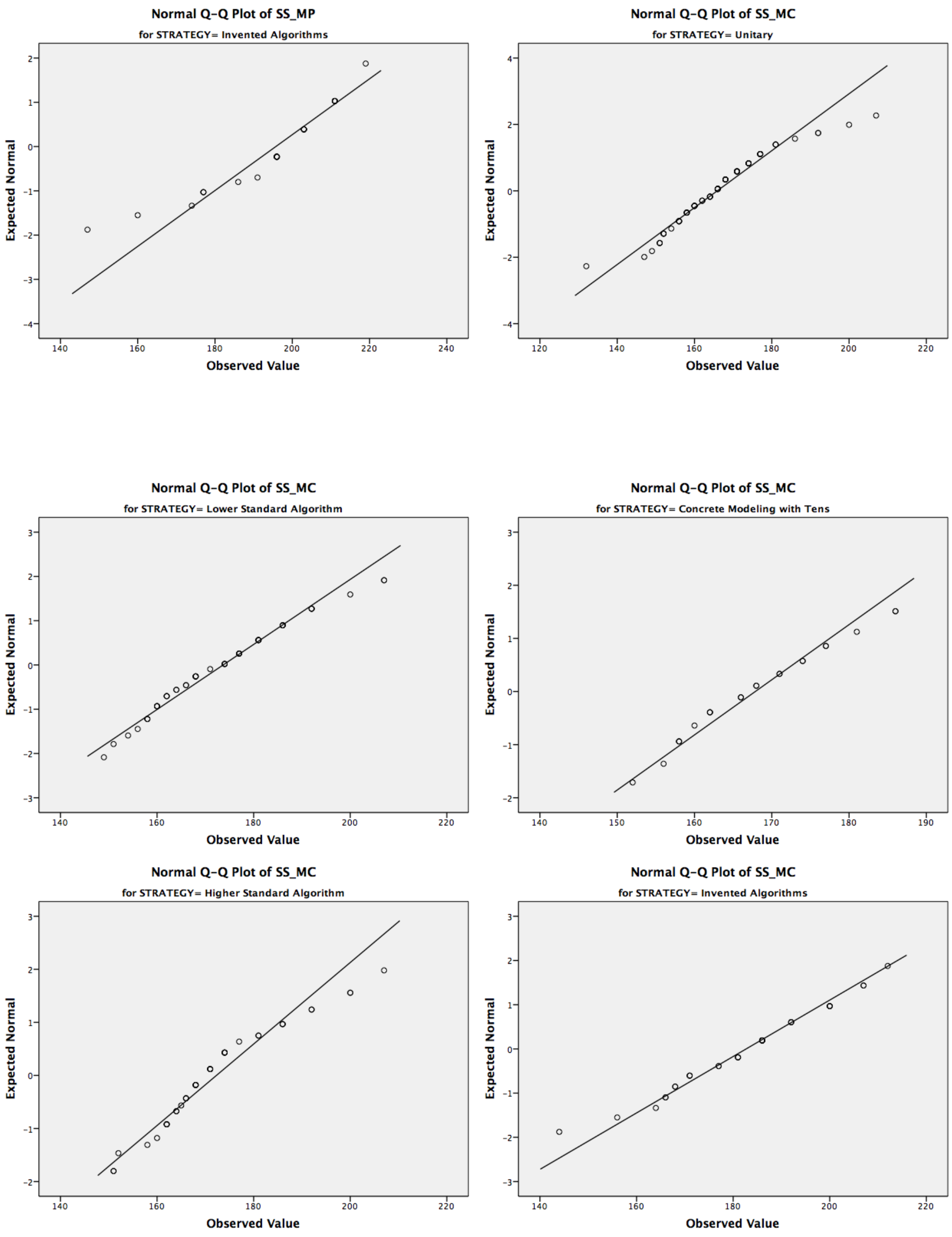


Figure 15: Second Grade - Multi-digit - Q-Q Plots

The homogeneity of variance-covariance matrices assumption was checked using Box's test which revealed that the assumption of equality of covariance matrices across the cells was met with Box's M = 14.730 with $F_{(12,\ 81349.8)} = 1.198$, $p > .05$. Table 71 shows the results of Box’s test of equality.

Table 71: Second Grade - Multi-digit - Box's Test

Box's Test of Equality of Covariance Matrices[a]	
Box's M	14.730
F	1.198
df_1	12
df_2	81349.786
p	.277

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + G2Pr_Math + STRATEGY

The Levene's test showed homogeneity of variance assumption was with $p>.05$. Table 72 shows the results of Levene's test.

Table 72: Second Grade - Multi-digit - Levene's Test

Levene's Test of Equality of Error Variances[a]

	F	*df1*	*df2*	*p*
SS_MP	1.094	4	227	.360
SS_MC	1.304	4	227	.269

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + G2Pr_Math + STRATEGY

Linearity assumption was checked through scatter plots and correlations. The scatter Plot showed a linear relationship between the DV's and correlations showed that there was a high correlation (but not perfect) between the DV's. These results suggested that the linearity assumption was met. Figure 16 shows scatter plot and table 73 shows the correlation matrix between DV's.

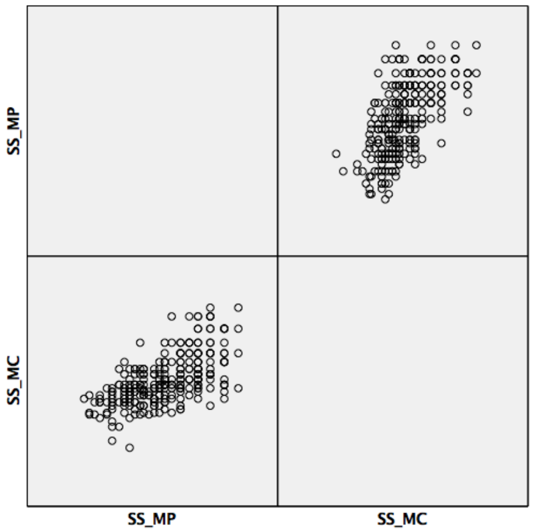


Figure 16: Second Grade - Scatter Plot of DV's

Table 73: Second Grade - Correlations between DV's

Correlations			
		SS_MP	SS_MC
SS_MP	Pearson Correlation	1	.621**
	p (2-tailed)		<.001
	N	270	270
SS_MC	Pearson Correlation	.621**	1
	p (2-tailed)	<.001	
	N	270	270

**. Correlation is significant at the 0.01 level (2-tailed).

MANCOVA Analysis: Second Grade Multi-digit Strategies

Table 74 displays the mean scores and standard deviations for multi-digit strategy groups. For the ITBS *Math Problems*, the mean scores were from highest to lowest for *invented algorithms*, *higher standard algorithm*, *concrete modeling with tens*, *lower standard algorithm*, and *unitary* groups, respectively. For the ITBS *Math Computation*, the *invented algorithm* group had the highest mean score, and the mean scores for *higher* and *lower standard algorithm* group were about the same.

Table 74: Second Grade - Descriptive Statistics for Multi-digit Strategies

Descriptive Statistics				
	STRATEGY	Mean	Std. Deviation	N
SS_MP	Unitary	170.553	19.8065	85
	Lower Standard Algorithm	177.442	17.5470	52
	Concrete Modeling with Tens	170.955	17.1589	22
	Higher Standard Algorithm	183.561	20.2498	41
	Invented Algorithms	195.719	15.8465	32
	Total	177.905	20.4029	232
SS_MC	Unitary	165.882	11.6644	85
	Lower Standard Algorithm	173.442	13.6287	52
	Concrete Modeling with Tens	167.864	9.6328	22
	Higher Standard Algorithm	172.293	13.0158	41
	Invented Algorithms	182.687	15.6647	32
	Total	171.216	13.8663	232

The statistical analysis showed that strategy group was significant in determining the combined test results of the ITBS when controlling for student pretest score with $F_{(8,452)} = 5.125$, $p < .01$, and Pillai's Trace = .166). The summary of the statistical result is given in Table 75.

Table 75: Second Grade - Multi-digit Strategies - Multivariate Tests

Multivariate Tests^{a}

Effect		Value	*F*	Hypothesis *df*	Error *df*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power^{d}
Intercept	Pillai's Trace	.996	29093.835^{b}	2.000	225.000	.000	.996	58187.671	1.000
	Wilks' Lambda	.004	29093.835^{b}	2.000	225.000	.000	.996	58187.671	1.000
	Hotelling's Trace	258.612	29093.835^{b}	2.000	225.000	.000	.996	58187.671	1.000
	Roy's Largest Root	258.612	29093.835^{b}	2.000	225.000	.000	.996	58187.671	1.000
G2Pr_Math	Pillai's Trace	.502	113.436^{b}	2.000	225.000	.000	.502	226.872	1.000
	Wilks' Lambda	.498	113.436^{b}	2.000	225.000	.000	.502	226.872	1.000
	Hotelling's Trace	1.008	113.436^{b}	2.000	225.000	.000	.502	226.872	1.000
	Roy's Largest Root	1.008	113.436^{b}	2.000	225.000	.000	.502	226.872	1.000
STRATEGY	Pillai's Trace	.166	5.125	8.000	452.000	**.000**	.083	41.004	.999
	Wilks' Lambda	.837	5.225^{b}	8.000	450.000	.000	.085	41.801	.999
	Hotelling's Trace	.190	5.324	8.000	448.000	.000	.087	42.592	.999
	Roy's Largest Root	.164	9.273^{c}	4.000	226.000	.000	.141	37.094	1.000

a. Design: Intercept + G2Pr_Math + STRATEGY

b. Exact statistic

c. The statistic is an upper bound on F that yields a lower bound on the significance level.

d. Computed using alpha =

The test between-subject effects indicated that strategy group was a significant factor both on the ITBS *Math Problems* with $F_{1\ (4,\ 226)}$ =7.364, $p < .01$, η2 = 0.115 and on the ITBS *Math Counting* with $F_{2\ (4,226)}$ =5.855, $p < .01$, η2 = 0.094. The summary of the result of between-subject effects is provided in Table 76.

Table 76: Second Grade - Multi-digit - Between-Subject Effects

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Corrected Model	SS_MP	55925.579[a]	5	11185.116	62.828	.000	.582	314.139	1.000
	SS_MC	15039.038[b]	5	3007.808	23.140	.000	.339	115.700	1.000
Intercept	SS_MP	6060016.063	1	6060016.063	34039.674	.000	.993	34039.674	1.000
	SS_MC	5602012.666	1	5602012.666	43098.000	.000	.995	43098.000	1.000
G2Pr_Math	SS_MP	38791.025	1	38791.025	217.893	.000	.491	217.893	1.000
	SS_MC	7857.418	1	7857.418	60.450	.000	.211	60.450	1.000
STRATEGY	SS_MP	5244.252	4	1311.063	**7.364**	**.000**	.115	29.457	.996
	SS_MC	3044.289	4	761.072	**5.855**	**.000**	.094	23.421	.982
Error	SS_MP	40234.335	226	178.028					
	SS_MC	29376.186	226	129.983					
Total	SS_MP	7439018.000	232						
	SS_MC	6845438.000	232						
Corrected Total	SS_MP	96159.914	231						
	SS_MC	44415.224	231						

a. R Squared = .582 (Adjusted R Squared = .572)
b. R Squared = .339 (Adjusted R Squared = .324)
c. Computed using alpha =

Pairwise comparisons showed that the *invented algorithms* group scored significantly higher than any other strategy groups on both the ITBS *Math Problems* and *Math Computation*. The *higher standard algorithm* group scored significantly higher than the *lower standard algorithm* and *unitary* groups on the ITBS *Math Problems*. The *unitary, lower standard algorithm* and *concrete modeling* groups did not differ significantly from each other. Table 77 presents the results of pairwise comparison statistics, and figure 17 shows estimated marginal means of the *Math Problems* and *Math Computation* for each strategy group.

Table 77: Second Grade - Multi-digit - Pairwise Comparisons

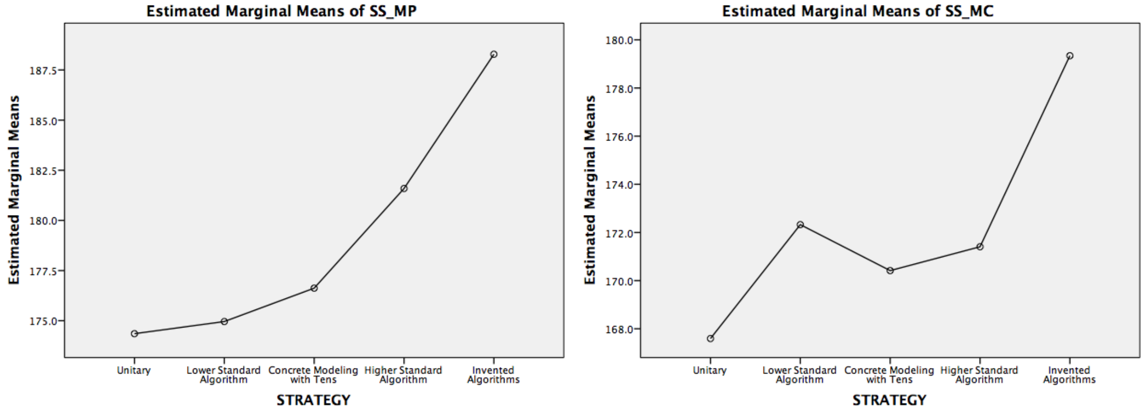
Pairwise Comparisons

Dependent Variable	(I) STRATEGY	(J) STRATEGY	Mean Difference (I-J)	Std. Error	p^b	95% Confidence Interval	
						Lower Bound	Upper Bound
SS_MP	Unitary	Lower Standard Algorithm	-.605	2.387	.800	-5.309	4.099
		Concrete Modeling with	-2.269	3.194	.478	-8.563	4.025
		Higher Standard Algorithm	-7.237*	2.567	.005	-12.295	-2.178
		Invented Algorithms	-13.928*	2.870	.000	-19.583	-8.272
	Lower Standard Algorithm	Unitary	.605	2.387	.800	-4.099	5.309
		Concrete Modeling with	-1.664	3.438	.629	-8.439	5.111
		Higher Standard Algorithm	-6.631*	2.787	.018	-12.123	-1.140
		Invented Algorithms	-13.323*	3.017	.000	-19.267	-7.378
	Concrete Modeling with Tens	Unitary	2.269	3.194	.478	-4.025	8.563
		Lower Standard Algorithm	1.664	3.438	.629	-5.111	8.439
		Higher Standard Algorithm	-4.967	3.564	.165	-11.990	2.056
		Invented Algorithms	-11.658*	3.801	.002	-19.147	-4.169
	Higher Standard Algorithm	Unitary	7.237*	2.567	.005	2.178	12.295
		Lower Standard Algorithm	6.631*	2.787	.018	1.140	12.123
		Concrete Modeling with	4.967	3.564	.165	-2.056	11.990
		Invented Algorithms	-6.691*	3.169	.036	-12.936	-.447
	Invented Algorithms	Unitary	13.928*	2.870	**.000**	8.272	19.583
		Lower Standard Algorithm	13.323*	3.017	**.000**	7.378	19.267
		Concrete Modeling with	11.658*	3.801	**.002**	4.169	19.147
		Higher Standard Algorithm	6.691*	3.169	**.036**	.447	12.936
SS_MC	Unitary	Lower Standard Algorithm	-4.732*	2.040	**.021**	-8.751	-.712
		Concrete Modeling with	-2.822	2.729	.302	-8.200	2.556
		Higher Standard Algorithm	-3.813	2.193	.084	-8.135	.509
		Invented Algorithms	-11.747*	2.452	.000	-16.580	-6.915
	Lower Standard Algorithm	Unitary	4.732*	2.040	.021	.712	8.751
		Concrete Modeling with	1.910	2.938	.516	-3.879	7.699
		Higher Standard Algorithm	.919	2.381	.700	-3.774	5.611
		Invented Algorithms	-7.016*	2.578	.007	-12.095	-1.936
	Concrete Modeling with Tens	Unitary	2.822	2.729	.302	-2.556	8.200
		Lower Standard Algorithm	-1.910	2.938	.516	-7.699	3.879
		Higher Standard Algorithm	-.991	3.045	.745	-6.992	5.010
		Invented Algorithms	-8.925*	3.247	.006	-15.325	-2.526
	Higher Standard Algorithm	Unitary	3.813	2.193	.084	-.509	8.135
		Lower Standard Algorithm	-.919	2.381	.700	-5.611	3.774
		Concrete Modeling with	.991	3.045	.745	-5.010	6.992
		Invented Algorithms	-7.934*	2.708	.004	-13.270	-2.599
	Invented Algorithms	Unitary	11.747*	2.452	**.000**	6.915	16.580
		Lower Standard Algorithm	7.016*	2.578	**.007**	1.936	12.095
		Concrete Modeling with	8.925*	3.247	**.006**	2.526	15.325
		Higher Standard Algorithm	7.934*	2.708	**.004**	2.599	13.270

Based on estimated marginal means

*. The mean difference is significant at the

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).



Covariates appearing in the model are evaluated at the following values: G2Pr_Math = .00461

Covariates appearing in the model are evaluated at the following values: G2Pr_Math = .00461

Figure 17: Second Grade - Multi-digit - Estimated Marginal Means

Summary

In summary, there was no statistically significant difference in the number of students who were classified into *concrete modeling, counting*, and *derived facts/recall* between treatment and control groups at the first grade level. There was also no statistically significant difference in the number of first grade students who were classified into the *other, unitary, concrete modeling with tens*, and *invented algorithms* strategies between treatment and control groups.

When differences in first grade students' mathematics achievement between single-digit strategy groups were investigated, it was found that the differences on the ITBS *Math Problems* section were not significant between strategy groups. However on the *Math Computation* section, the students in *derived facts/recall* and *counting* strategy groups had significantly higher mean scores than students in the *concrete modeling* group.

For multi-digit strategies, the first grade students in the *other* strategy group had a significantly lower mean score on both the ITBS *Math Problems* and *Math Computation* sections than all the other multi-digit strategy groups (*unitary, concrete modeling with tens*, and *invented algorithms*). On the *Math Problems* section, the differences between *unitary, concrete modeling with tens,* and *invented algorithms* were not statistically significant. However on the *Math Computation* section, the students in the *invented algorithms* group had significantly higher mean score than the students in the *unitary* strategy group. The differences between *concrete modeling with tens,* and *invented algorithms* groups were not statistically significant on the *Math Computation* section.

At the second grade level, there were statistically significant differences in the numbers of students in single-digit strategy groups (*concrete modeling, counting, and derived facts/recall*) between treatment and control. Forty percent of treatment students were in the *derived facts/recall* strategy group whereas only 28% percent of control students were in this strategy group. Forty-four percent of treatment students were in the *counting* strategy group whereas 63% of control students were in this strategy group. A greater percentage of treatment students (17%), and a lower percentage of control students (nine percent) were in the *concrete modeling* strategy group. These differences were significant at alpha of 0.05. For multi-digit strategies, there were no statistically significant differences in the number of second grade students in multi-digit strategy groups between treatment and control groups.

In terms of the differences in second grade students' mathematics achievement between single-digit strategy groups, the students in the *derived facts/recall* strategy groups scored significantly higher on both the ITBS *Math Problems* and *Math Computation* sections than the students in the *counting* or *concrete modeling* strategy groups. Differences in the students' mathematics achievement on both the ITBS *Math Problems* and *Math Computation* sections between *counting* and *concrete modeling* strategy groups were not statistically significant.

For multi-digit strategies, students in the *invented algorithms* group scored significantly higher on both the ITBS *Math Problems* and *Math Computation* sections than students in any other strategy groups (*unitary, lower standard algorithm*, *concrete modeling with tens*, and *higher standard algorithm*). The students in the *higher standard algorithm* group scored significantly higher on the ITBS *Math Problems* than the students in the *unitary* and the *lower standard algorithm* groups. The students in the *lower standard algorithm* group scored significantly higher on the ITBS *Math Computation* section than students in the *unitary* strategy group. The differences between other strategy groups were not statistically significant.

CHAPTER FIVE: SUMMARY, DISCUSSION, AND RECOMMENDATIONS

Introduction

Existing research on students' use of different strategies have concluded that instruction has an effect on students' actual use of strategies (Carpenter, Hiebert, & Moser, 1983; Villasenor & Kepner, 1993; Fuson, Smith, & Lo Cicero, 1997), as well as on students' ability to use them flexibly (Blote et al., 2001; De Smedt et al., 2010;). Blote et al. (2001) concluded that students who initially learn to use one standard procedure continue to use the same procedure even after they are taught other procedures and become inflexible problem solvers with limited understanding. Additionally, Villasenor and Kepner (1993) found that students of CGI teachers used more advanced strategies than students of non-CGI teachers.

Peters et al. (2012) suggested that mathematics textbooks and lessons should include more word problems and external representations to stimulate children to make flexible strategy choices, rather than using a single strategy for all problems. They also suggested that more research is needed to evaluate the success of powerful instructional settings on students' use of strategies. This study aimed to fill this gap and provided additional insight into the understanding of the impact of teachers' attending CGI professional developments, which can be considered as powerful instruction, on students' use of strategies

The research about students' strategies indicated that students' use of invented algorithms has a positive effect on their understanding of place value concepts and number properties (Carpenter et al., 1998; Kamii & Domicik, 1998; Fuson and Briars, 1990). The lacking piece in the literature was the impact of students' use of strategies on their mathematics achievement as measured by a standardized test, which is generally used to compare students' mathematics achievement at the state, national, and international levels. In this study, students were classified into strategy groups according to their use of problem solving strategies. First, the numbers of students in strategy groups were compared between the treatment and control groups. Then, the mathematics achievement of students (as measured by the ITBS) in different strategy groups was compared. Therefore, the current study also shed light on the effect of students' use of strategies on their mathematics achievement as measured by a standardized test.

Summary and Discussion

The current study was a part of a larger cluster-randomized controlled trial and the researcher used a subsample of it. The purpose of this study was to investigate the effect of teachers' attending CGI professional developments on their students' use of problem solving strategies, and the effect of students' use of different strategies on their mathematics achievement. This study was conducted at the end of the first year of a two-year planned CGI professional development. Therefore the results of this study should be interpreted cautiously.

First, the study analyzed the differences in students' use of strategies between treatment and control groups. The treatment was CGI professional developments, and the teachers in the treatment group attended CGI workshops whereas the teachers in the control group did not. The students, both in the classes of treatment teachers (treatment students) and in the classes of control teachers (control students), were classified into strategy groups according to their use of strategies. Student interviews were used to identify the strategies used by the students and to classify them into the strategy groups. Next, the study analyzed the differences in the mathematics achievement of students between different strategy groups. A student posttest, which was ITBS (*Math Problems* and *Math Computation*), was used to compare students' mathematics achievement. A student pretest was used as a covariate.

The data were collected during the 2012 - 2013, and 2013 - 2014 school years from 22 elementary schools that were located in two school districts in the southeastern United States. Schools were randomly assigned to either treatment or control groups for which randomization occurred at the school level with schools blocked on district and school proportion free/reduced-price lunch (FRL).

The teachers in the treatment schools attended a four-day CGI professional development in the summer of 2013 and another four-day follow up workshop in the fall of 2013 and spring of 2014. The teachers in the control schools in one district were invited to a two-day professional development session for the district program called Bridge to STEM during June 2013 and September 2013. This program was not related to the activities of CGI professional development in any way. The other school district administrators preferred to be a strict business-as-usual condition for their teachers, and the study did not provide a professional development for those teachers.

Participants of this study included both first and second grade students. There were 336 first grade students, and 286 second grade students. The data from students were collected at three different points. First, students were administered the pretest by their teachers in the beginning of the 2013-2014 school year. Next, CGI project staff interviewed the students in the spring of 2014, where the students were asked to solve a variety of problems. Lastly, students were administered the ITBS in the spring of 2014 by the CGI project staff.

In general the interview process took about 45 to 60 minutes. The word problems and computation problems sections of the interview protocol were used in this study. There were six single-digit problems (word problems and computation) that could be used to classify students into strategy groups for both the first and second grade levels. There were six multi-digit problems for first grade and seven multi-digit problems for second grade, which could be used in classification of students into strategy groups. Students were classified into the single-digit strategy groups based on the most advanced strategy that they used for three or more of those problems. Likewise, students were also classified into the multi-digit strategy group based on the most advanced strategy that they used for three or more of those problems. The ITBS (*Math Problems* and *Math Computation*) was used to measure students' mathematics achievement and student pretest was used as a covariate in data analysis.

The first research question asked whether the treatment had an effect on first grade students' use of single-digit and multi-digit strategies. In order to address this research question, students were classified into single-digit strategy groups (*concrete modeling, counting, and derived facts/recall*) and multi-digit strategy groups (*other, unitary, concrete modeling with tens, and invented algorithms*) separately.

Chi-square analysis was used to investigate the differences in the number of treatment and control students in different strategy groups. Analysis was conducted separately for single digit and multi-digit strategy groups. Results showed that, there were not statistically significant differences in single-digit strategy groups between the treatment and control groups at the first grade level.

These results were consistent with the findings of Carpenter et al. (1989). When examining students' use of strategies, Carpenter et al., (1989) reported no differences between the students of CGI teachers, and the students of non-CGI teachers. On the other hand, Villasenor and Kepner (1993) reported that the students of CGI teachers used more advanced strategies than the students of non-CGI teachers. In their study, Villasenor and Kepner looked at how often students in both groups used a more advanced strategy and compared the treatment and control groups. The current study however classified students into the most advanced strategy group that they used for three or more problems. It should also be noted that at the time of the data collection, the treatment teachers had received only the first year of a two-year planned CGI professional development. Therefore the results of this study should be interpreted cautiously.

The statistical analysis for multi-digit strategies at the first grade level also yielded non-significant results between treatment and control groups. Although a greater percentage of treatment students used more advanced strategies (*derived facts/recall*, and *concrete modeling with tens*), these differences were not statistically different. Not having significant differences in the number of treatment and control group students in multi-digit strategy groups at the first grade level is reasonable, since in first grade instructional time focuses on developing an understanding of addition, subtraction, and strategies for addition and subtraction within 20 according to the Common Core State Standards (CCSSO, 2010).

The second research question looked at the impact of strategy groups (single-digit, and multi-digit separately) on students' mathematic achievement as measured by the ITBS controlling for students' prior achievement. Multivariate analysis of covariance was used to investigate the differences between strategy groups. The analysis was conducted separately for single-digit strategies and for multi-digit strategies. Results showed that the single-digit strategy group was a significant factor on the combined test scores of ITBS at the first grade level. Strategy group was a significant factor on the ITBS *Math Computation* score, but was not a significant factor on the *Math Problems* score. Students in the *derived facts/recall* and *counting* strategy groups had significantly higher mean scores on the ITBS *Math Computation* than the students in the *concrete modeling* group.

The results indicate that students' mathematics achievement increases as they progress toward using more advanced strategies. This result was what was expected and also consistent with the literature since the research has identified that children progress from using *concrete modeling* strategies to *counting* strategies, and from *counting* strategies to *derived facts/recall* strategies as their understanding of number sense increase (Carpenter et al., 1999). Based on these results, it can be recommended that first and second grade teachers should have a goal for all their students to progress to the most advanced strategies, which are *derived facts/recall*, which consecutively will increase their mathematics achievement.

For multi-digit strategies (*other, unitary, concrete modeling with tens, and invented*) at the first grade level, the results showed that strategy group was a significant factor on combined test results of the ITBS. It was significant both on the *Math Problems* section and the *Math Computation* section of the ITBS. The analysis showed that the *other* strategy group, which stands for the unidentifiable strategies, had a significantly lower mean score than the rest of the multi-digit strategy groups for both the *Math Problems*, and *Math Computation* sections of the ITBS. The *invented algorithms* group had a significantly higher mean score on the *Math*

Computation section than the *unitary* group, however the differences between *concrete modeling with tens*, and *invented algorithms* were not significant. Additionally, differences between *unitary, concrete modeling with tens*, and *invented algorithms* were not significant for the *Math Problems* section of the ITBS.

Based on these results, it is important to note that the *unitary* group students (the simplest multi-digit strategy group) had a significantly higher mean score than the *other* strategy group. It was also interesting that the difference between *unitary* and *invented algorithms* groups was not significant for the *Math Problems* section of the ITBS. Another interesting point is that, although not significant, the *concrete modeling with tens* group had a higher mean score than the *invented algorithms* group for the *Math Problems* section of the ITBS. These results can be interpreted as evidence to show the importance of modeling at early grades since Carpenter et al. (1993) stated that the most obvious signs of problem solving deficiencies in older students appear to have occurred due to the lack of attending to the obvious features of problem situations.

The third research question looked at the impact of the treatment on students' use of single-digit, and multi-digit strategies at the second grade level. To answer this research question, students were classified into single-digit, and multi-digit strategy groups, separately. Chi-square analysis was used to investigate the differences between the numbers of treatment and control students in strategy groups. Analysis was conducted separately for single digit and multi-digit strategies. Results showed that there was a significant difference in the numbers of treatment and control students in single-digit strategy groups. A majority of control students (63%) were in the *counting* strategy group, whereas only 28% were in *derived facts/recall* strategy group. On the other hand, the percentage of treatment students who were in the *derived* *facts/recall* strategy group (40%), and who were in *counting* strategy group (44%) was approximately the same. For the *concrete modeling* strategy group, 17% of treatment students, and nine percent of control students were in this strategy group.

The distribution of students in strategy groups indicated that treatment students showed more progression towards the most advanced strategy group (*derived facts/recall*) than control students, whereas a majority of control students were in the *counting* strategy group. This finding is consistent with the research stating that students of CGI teachers used more advanced strategies than students of non-CGI teachers (Villasenor & Kepner, 1993). Based on these results it can be concluded that the students in the classes of treatment teachers had more opportunities to use a variety of strategies (*concrete modeling, counting, and derived facts/recall*) in a more balanced way, whereas students seemed to use the counting strategies more often than other strategies in the classes of control teachers.

There might be several reasons for treatment students' having more progression towards *derived facts/recall* strategies. First of all, research has shown that CGI teachers can identify the problems that their students solve and the strategies that their students use more successfully than non-CGI teachers (Carpenter et. al., 1989). This might have enabled the treatment teachers in this study to better facilitate their students' progression towards the use of more advanced strategies. Secondly, Kazemi and Franke (2001) stated that knowing the sequence of how children develop problem-solving strategies enables teachers to pose problems that challenge their students' thinking.

For multi-digit problems in the second grade level, students were classified into: (a) *unitary*, (b) *concrete modeling with tens*, (c) *invented algorithms*, (d) *lower standard algorithms*, and (e) *higher standard algorithms* strategy groups. Classification of students into multi-digit strategy groups showed no statistical differences between treatment and control students at alpha level of 0.05.

There might be several reasons for not having significant differences in the use of multi-digit strategies between treatment and control students at the second grade level. One reason might be the fact that treatment teachers learned about multi-digit strategies during the professional developments throughout the fall of 2013, and the spring of 2014. The student interviews were also conducted in the spring of 2014. This might have given limited time to the treatment teachers to discuss and reinforce the use of student invented strategies with multi-digit numbers. Additionally, if the teachers in this study followed their textbook, which introduces both invented algorithms and the standard algorithms at the second grade level, students might have learned the standard algorithms and this might have interfered with students' use of their invented strategies. The analysis of the strategies used for each multi-digit problem showed that the most frequently used strategy for multi-digit problems was the standard algorithm at the second grade level.

The literature indicates that the changes in teachers' practices were related to the increased years of experience with CGI (Jacobs, Lamb, & Philipp, 2010). Therefore, it is important to provide teachers with the time that they need to understand and plan to implement the newly learned students' thinking of multi-digit strategies and the CGI principles into their instruction. Therefore, it is recommended to conduct a similar study at the end of the second year of the CGI study after teachers attending the two-year planned professional development and having more experiences with the use of CGI principles.

The fourth research question looked at the impact of strategy groups (single-digit and multi-digit) on students' mathematics achievement as measured by a standardized test at the second grade level. Multivariate analysis of covariance was used to answer this question. The analysis was performed separately for single-digit and multi-digit strategy groups. Results showed that the single-digit strategy group was a significant factor on the combined test scores of ITBS as well as on the *Math Problems*, and *Math Computation* sections. Students in the *derived facts/recall* strategy group had a significantly higher mean score than the students in the *counting* or *concrete modeling* groups. Although not significant, the *concrete modeling* group had a higher mean score than the *counting* strategy group on the *Math Problems* section, and the *counting* strategy group had a higher mean score on the Math Computation section of the ITBS. The higher mean score of the students in the *concrete modeling* group than students in the *counting* group on the *Math Problems* section of the ITBS shows again that modeling at the beginning might be crucial for students because "…some of the most compelling exhibitions of problem-solving deficiencies in older students appeared to have occurred because the students did not attend to what appear to be obvious features of problem situations (Carpenter et al., 1993, p. 428). Therefore, being able model the problems might have helped direct modelers to make sense of the problems on the *Math Problems* section of the ITBS.

These findings are consistent with the research which has identified children's progression from using *concrete modeling* to *counting*, and from *counting* to *derived facts/recall* strategies as they progress with their understanding of number sense (Carpenter et al., 1999). Students' level of understanding of number sense significantly affects their mathematics achievement. These results suggest again that it should be a goal for all first and second grade teachers to provide their students with opportunities to explore different strategies (from simplest to most advanced ones), and facilitate their students' progression towards the use of most advanced strategies (derived facts/recall) if they want to increase their students' mathematics achievement.

For multi-digit strategies, results indicated that strategy group was a statistically significant factor on combined test results of the ITBS, and it was a significant factor both on the ITBS *Math Problems* and *Math Computation* sections. Students in the *invented algorithms* group had a significantly higher mean score on both sections of the ITBS than any other strategy groups (*higher standard algorithm, concrete modeling with tens, lower standard algorithm*, and *unitary*). The *higher standard algorithm* group had a significantly higher mean score on the ITBS *Math Problems* section than the students in the *unitary* or *lower standard algorithms* group. Students in the *lower standard algorithm* group had a significantly higher mean score on the ITBS *Math Computation* section than the students in the *unitary* strategy group. The *concrete modeling with tens* group did not differ significantly from the *unitary, lower standard algorithm*, or *higher standard algorithm* groups either for the ITBS *Math Problems* or *Math Computation* sections.

These results support the findings of the literature, which revealed that students who use invented algorithms have better understandings of the concepts and perform better than those who use standard algorithms (Carpenter et al., 1998). The literature indicates that students who used invented strategies were able to transfer their knowledge to new situations and were more successful solving extension problems (Carpenter et al., 1998). The invention and application of invented algorithms involves facets of number sense like decomposition / re-composition and understanding of number properties (McIntosh, Reys, & Reys, 1992). Therefore, invented algorithms are built on the foundational number concepts and on the fundamental properties of the number system, like the commutative, associative, and distributive (for multiplication) properties, and these are quite visible when one examines students' strategies. Although standard algorithms are also built on number concepts, they are not quite visible for children to understand their conceptual underpinnings (Kilpatrick et al., 2001). When students learn standard algorithms without understanding, the reasoning behind them like why the "ones" are being "carried," is often unclear which consequently causes students to develop some flawed procedures (Carroll & Porter, 1998), which result in systematic errors (Kilpatrick, Martin, & Schifter, 2003). Romberg and Collis (1985) concluded that children who have the capacity to reason about quantitative problems often do not use algorithmic procedures even though they know how to use them. On the other hand children, whose capacity to reason about quantitative problems is suspicious, and who have not acquired other skills like direct modeling and counting, may use the standard algorithm, but often make errors.

Murray and Olivier (1989) suggested that level four (seeing numbers as groups of tens and some ones) understanding is a prerequisite to execute the standard algorithm meaningfully. In general, when level one (count all by ones strategy) and level two (count on by ones strategy) students have difficulty in computation with larger numbers, teachers seem to "help" them by introducing the standard algorithm. However, researchers argued that even if the teachers try to build a conceptual basis for the algorithms (level four), such efforts would be ill fated if level two and level three (seeing numbers as composite units of decade and ones) are bypassed. They concluded that superficial facility in executing the algorithm might hide serious deficiencies.

The results of this study support the results of Murray and Olivier (1989), because the students in the *invented algorithms* group had a significantly higher mean score than the students in any other strategy groups, and the students in the *higher standard algorithm* group (at least one *invented algorithm* or *concrete modeling with tens*) had a significantly higher mean score on the ITBS math problem solving section than students in the *lower standard algorithm* and *unitary strategies* groups. The results of this study suggest that teachers should refrain from introducing the procedures of standard algorithms to their students unless they acquire a level four (seeing numbers as groups of tens and some ones) understanding, which will give them more opportunities to use *invented algorithms*, and which will consecutively increase their mathematics achievement.

Implications of the Study

This study has concluded that teachers' attending the CGI professional developments had a positive effect on students' use of single-digit strategies at the second grade level. The students in the classes of treatment teachers showed more progression towards using *derived facts/recall* strategies, which is the most advanced progression level in the literature to solve single-digit problems. Additionally, the second grade students that were in the most advanced strategy groups (*derived facts/recall* for single-digit problems, and *invented algorithms* for multi-digit problems) scored significantly higher on a standardized mathematics achievement test than the students who were in less advanced strategy groups.

The results of this study suggest that all first and second grade teachers should have the knowledge of students' thinking and the progression that they show in dealing with numbers. One way to accomplish this is to provide teachers with the CGI professional development.

Therefore, CGI professional development may be recommended for all first and second grade teachers. Additionally, students in the most advanced strategy groups had a significantly higher mathematics achievement. If we would like our students to have higher mathematics achievement, all first and second grade teachers should have a goal for their students to have a progression from using the simplest strategies to most advanced strategies to add and subtract single-digit and multi-digit numbers. First and second grade teachers should not introduce the procedures of standard algorithms before their students are provided with sufficient opportunities to make sense of more advanced student invented strategies and actually are able to use them.

Limitations

This study had several limitations that must be noted when interpreting the study's results and conclusions. First of all, the number of single-digit and multi-digit problems that were used in the classification of students into strategy groups was relatively low. Secondly, due to the low number of single-digit and multi-digit problems used to classify students into strategy groups, the cut off point for classification of students into strategy groups was not as high as it should be, which consecutively may affect the differences between strategy groups.

The third limitation was that gender and socioeconomic status were not included in the analysis of this study. The research indicates that gender might have an influence on students' academic achievement. Although some studies showed that gender differences in mathematics achievement are minimal or nonexistent during the primary school years (Lachance & Mazzocco, 2005), it has been reported that gender differences increases with age in favor of males (Braswell et al., 2001; Grigg et al., 2007). In addition, the research about gender differences in upper grades reported conflicting results. While some studies reported that males outperform females significantly (Mau & Lynn, 2000; Mullis et al. 1998), others reported no significant differences between males and females. (Haciomeroglu, Chicken, & Dixon, 2013; Fennema & Sherman, 1977). Likewise, socioeconomic status might also have an influence on students' mathematics achievement. Studies examining the relation between socioeconomic status and academic achievement reported inconsistent results since their results range from a strong relation (e.g., Sutton & Soderstrom, 1999) to no significant correlation at all (e.g., Ripple & Luthar, 2000). Therefore, future studies should take into account the effect of gender and socioeconomic status on students' academic achievement.

Lastly, there was no control on participants' prior experiences. Blote et al. (2001) suggested that the effect of instruction might depend, in part, on the kind of knowledge that students previously acquired. Therefore the results of this study should be interpreted cautiously.

Recommendations for Future Research

This study investigated the impact of teachers' attending the CGI professional development on their students' use of strategies, and the impact of students' use of strategies on their mathematics achievement. The study was conducted at the end of the first year of a two-year CGI professional development for teachers. Therefore, it is recommended for future research to examine the impact of this intervention on students' use of strategies at the end of the CGI study, and after teachers having more experience with the use of CGI principles in their instruction, because research indicates that teachers' use of CGI principles in their instruction related to their numbers of years of experience with CGI (Jacobs, Lamb, & Philipp, 2010).

In this study, the researcher classified students into the most advanced strategy groups that they used for at least three problems. It is recommended for future researchers to use an instrument that includes a greater number of single-digit, and multi-digit problems when classifying students into strategy groups. Using a greater number of problems will enable the researcher to classify students into strategy groups in a way that will make the differences between strategy groups much more explicit.

APPENDIX A: INSTITUTIONAL REVIEW BOARD APPROVAL

FLORIDA STATE UNIVERSITY

Florida State
UNIVERSITY

Office of the Vice President For Research
Human Subjects Committee
Tallahassee, Florida 32306-2742
(850) 644-8673 · FAX (850) 644-4392

APPROVAL MEMORANDUM

Date: 03/14/2013

To: Robert Schoen <rschoen@lsi.fsu.edu>

Address: 2540

Dept.: LEARNING SYSTEMS INSTITUTE

From: Thomas L. Jacobson, Chair

Re: Use of Human Subjects in Research
Primary Grades Math Study

The application that you submitted to this office in regard to the use of human subjects in the research proposal referenced above has been reviewed by the Human Subjects Committee at its meeting on 06/13/2012 Your project was approved by the Committee.

The Human Subjects Committee has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval does not replace any departmental or other approvals which may be required.

If you submitted a proposed consent form with your application, the approved stamped consent form is attached to this approval notice. Only the stamped version of the consent form may be used in recruiting research subjects.

If the project has not been completed by 06/12/2013 you must request a renewal of approval for continuation of the project. As a courtesy, a renewal notice will be sent to you prior to your expiration date; however, it is your responsibility as the Principal Investigator to timely request renewal of your approval from the Committee.

You are advised that any change in protocol for this project must be reviewed and approved by the Committee prior to implementation of the proposed change in the protocol. A protocol change/amendment form is required to be submitted for approval by the Committee. In addition, federal regulations require that the Principal Investigator promptly report, in writing, any unanticipated problems or adverse events involving risks to research subjects or others.

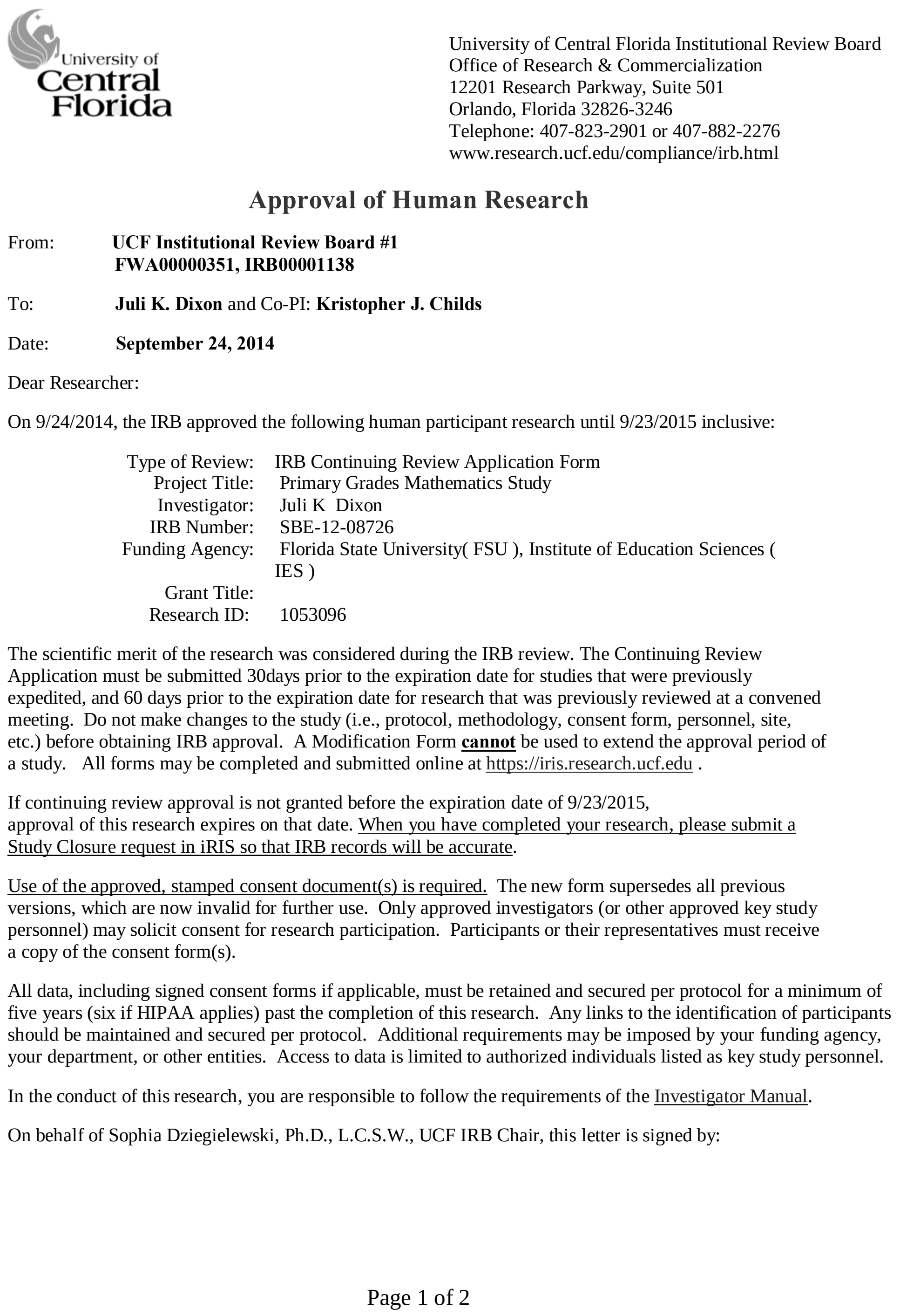
By copy of this memorandum, the chairman of your department and/or your major professor is reminded that he/she is responsible for being informed concerning research projects involving human subjects in the department, and should review protocols as often as needed to insure that the project is being conducted in compliance with our institution and with DHHS regulations.

This institution has an Assurance on file with the Office for Human Research Protection. The Assurance Number is IRB00000446.

Cc: Laura Lang <llang@lsi.fsu.edu>, Chair
HSC No. 2012.8326

APPENDIX B: INSTITUTIONAL REVIEW BOARD APPROVAL

UNIVERSITY OF CENTRAL FLORIDA



University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Juli K. Dixon** and Co-PI: **Kristopher J. Childs**

Date: **September 24, 2014**

Dear Researcher:

On 9/24/2014, the IRB approved the following human participant research until 9/23/2015 inclusive:

Type of Review:	IRB Continuing Review Application Form
Project Title:	Primary Grades Mathematics Study
Investigator:	Juli K Dixon
IRB Number:	SBE-12-08726
Funding Agency:	Florida State University(FSU), Institute of Education Sciences (IES)
Grant Title:	
Research ID:	1053096

The scientific merit of the research was considered during the IRB review. The Continuing Review Application must be submitted 30days prior to the expiration date for studies that were previously expedited, and 60 days prior to the expiration date for research that was previously reviewed at a convened meeting. Do not make changes to the study (i.e., protocol, methodology, consent form, personnel, site, etc.) before obtaining IRB approval. A Modification Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at https://iris.research.ucf.edu .

If continuing review approval is not granted before the expiration date of 9/23/2015, approval of this research expires on that date. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

Use of the approved, stamped consent document(s) is required. The new form supersedes all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Participants or their representatives must receive a copy of the consent form(s).

All data, including signed consent forms if applicable, must be retained and secured per protocol for a minimum of five years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained and secured per protocol. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Joanne Muratori
IRB Coordinator

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