STABILIZATION AND TRACKING OF THE VAN DER POL OSCILLATOR

by

XIN ZHAO
B.S. Xidian University, 1998

A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Science
in the Department of Electrical and Computer Engineering
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

Summer Term
2005

ABSTRACT

In this thesis, the stabilization and tracking problem of the Van der Pol oscillator is studied by using advanced control techniques. First, the linear state feedback and linear adaptive state feedback controllers for the stabilization problem are designed. Then, nonlinear state feedback and output feedback controllers are proposed for the tracking problem with known parameters. Finally, a dynamic output feedback controller based on adaptive backstepping technique is introduced for the tracking problem when all parameters of the Van der Pol system are unknown.

ACKNOWLEDGMENTS

I would like to thank Dr. Michael G. Haralambous, my advisor, for his suggestions, guidance, constructive criticism, compliments, incredible patience and financial assistance during this research. I will always remember him as the key factor that geared my career towards this path. I am thankful to Dr. Zhihua Qu for his guidance through the early years of chaos and confusion, and the financial support. I also wish to express my gratitude of my committee member Dr. Yi Guao.

I have the pleasure of having a nice husband, Guangyu Zhou. His support made the successful completion of this thesis possible.

Of course, I wish to thank the following: Postdoctor Jing Wang, Jian Yang, Vatana Ana and Honglian Yuan (for their friendship and expertise).

Finally, I am grateful to my parents and my sister in China for their support and *love*. Without them this work would never have come into existence.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The van der Pol system is one of the simplest forms of nonlinear oscillators and is used for modelling a variety of mechanical, electrical, and laser oscillators [2, 7, 17]. The van der Pol oscillator was first investigated by Van der Pol in 1927. In 1928, Van der Pol and Van der Mark presented it as the first dynamic model of oscillatory activity in the heart. Since then, the van der Pol oscillator has been extensively investigated.

Many techniques have been proposed in the control problem of the van der Pol system. For example, Basin and Pinsky used an impulse controller to stabilize the van der Pol system [1]; Su and Kermiche introduced a learning control method to improve the performance of a class of nonlinear systems, and applied it to the van der Pol system [18]; a dynamic neural network was proposed by Hovakimyan, Rysdyk and Calise for output feedback control of the van der Pol system [10]; Lefeber introduced nonlinear bounded controllers for stabilization and tracking problems of the periodic forced Van der Pol system [26].

In our work, linear state feedback and linear adaptive state feedback controllers are proposed for the stabilization problem of the van der Pol system. Nonlinear state feedback and output feedback controllers are proposed for the tracking problem. When the parameters are unknown, an adaptive output feedback controller is introduced, using the adaptive backstepping method [21], to force the van der Pol oscillator to follow any desired trajectory.

We review some important definitions and stability theorems, and give background for nonlinear control in Chapter 2. In Chapter 3 and 4 of this thesis, we consider the design of linear state feedback and linear adaptive state feedback controllers for the stabilization of the van der Pol oscillator. Then, nonlinear state feedback and output feedback controllers for the tracking problem with known parameters are presented in Chapter 5. An adaptive output feedback controller based on the adaptive backstepping method is introduced in Chapter 6. The proposed control strategies are illustrated with simulation examples in Chapter 7. Conclusions are presented in Chapter 8.

CHAPTER 2

PRELIMINARIES

2.1 Nonlinear Control

Nonlinear systems with either inherent nonlinear characteristics or nonlinearities deliberately introduced into the system to improve their dynamic characteristics have found wide application in diverse fields of engineering [16]. Unfortunately, the development of nonlinear methods faces real difficulties for various reasons [16]. There are no universal mathematical methods for the solution of nonlinear differential equations. The existing methods available deal with specific classes of nonlinear equations and therefore have only limited applicability to system analysis. The classification of a given system and the choice of an appropriate method of analysis are not easy tasks. Furthermore, even in simple nonlinear problems, there are numerous new phenomena qualitatively different from those expected in linear system behavior, and it is impossible to encompass all these phenomena in a single and unique method of analysis.

Linear control is a mature subject with a variety of powerful methods and a long history of successful industrial applications. However, all the physics systems are nonlinear. Not all of them can be simplified and described by linear models. Therefore, it is natural for many researchers and designers continue to show active interest in the development and applications of nonlinear control methodologies [17]. Moreover, nonlinear control techniques have several advantages over linear ones:

- Nonlinear controllers can improve the performance of the control systems.
- Nonlinear analysis techniques can process the system's hard nonlinearities.
- Nonlinear controllers can deal with model uncertainties.
- Good nonlinear control designs are simpler than their linear counterparts.

The stability analysis of nonlinear systems, heavily based on the work of Lyapunov, is a powerful approach to the qualitative study of system global behavior. By this approach, the global behavior of the system is investigated utilizing the given form of the nonlinear differential equations but without explicit knowledge of the solutions [16]. The Lyapunov stabilization theory will be introduced later.

2.2 van der Pol Oscillator

The mathematical model of the van der Pol oscillator is given by the following second-order nonlinear differential equation:

$$\ddot{x} + p_1(x^2 - 1)\dot{x} + p_2 x = 0 \tag{2.2.1}$$

where p_1, p_2 are constants and greater than zero.

The electrical circuit equivalent of the van der Pol oscillator is shown in Figure 2.1, where the inductor and capacitor are assumed to be constant. The resistive element is an active element circuit characterized by the voltage-controlled i-v characteristic $i = g(v)$. Applying Kirchhoff's current law, we have,

$$C\frac{dv}{dt} + \frac{1}{L}\int v(\tau)d\tau + g(v) = 0.$$

Differentiating with respect to t and rescaling the time variable, results in:

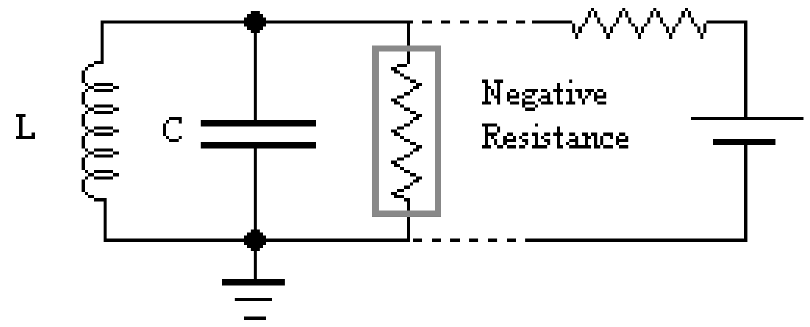

Figure 2.1: Electrical Circuit Equivalent of The van der Pol Oscillator

$$\ddot{v} + \frac{1}{C}g^{'}(v)\dot{v} + \frac{1}{LC}v = u^{'},$$

where $u^{'}$ is a source of control voltage that is added to the circuit. If

$$g(v) = -v + \frac{1}{3}v^3$$

the circuit equation takes the form

$$\ddot{v} + p_1(v^2 - 1)\dot{v} + p_2 v = u^{'} \tag{2.2.2}$$

where

$$p_1 = \frac{1}{C} > 0,\ p_2 = \frac{1}{LC} > 0,$$

Equation (2.2.2) is known as the forced van der Pol oscillator. This equation exhibits chaotic behavior for certain parameter values.

This study examines a controlled version of the forced van der Pol oscillator expressed as:

$$m\ddot{x} + 2c^{\star}(x^2 - 1)\dot{x} + kx = u \tag{2.2.3}$$

where m, $c^{\star}$ and k are positive constants and u is the physically realizable control input.

2.3 Lyapunov Stability

Lyapunov stability theory plays an important role in both system analysis and control design [14]. It provides an effective means of analyzing the stability of nonlinear differential equations where the solutions to these equations are difficult to obtain. Lyapunov theory is used to make conclusions about trajectories of a system without finding the trajectories. The fundamental approach of Lyapunov stability analysis consists of finding a generalized energy function, or so-called Lyapunov function $V(x)$, and examining its time derivative $\dot{V}(x)$. If the energy of the system is always positive and continually decreasing ($\dot{V}(x)$ negative definite), this means the energy of the system will eventually settle at some minimum energy state. Therefore, the system under investigation will become stable in that particular region.

Most of the Lyapunov stability results have been covered in existing literature. In the following subsections, we will summarize some of the main definitions and stability results [14, 17].

2.3.1 Definitions

Consider nonlinear time-invariant system

$$\dot{x} = f(x) \qquad\qquad f : D \longrightarrow R^n, \tag{2.3.1}$$

where D is an open and connected subset of R^n and f is a locally Lipschitz map from D into R^n. In what follows we will assume that $x = x_e$ is an equilibrium point of the system (2.3.1). In other words, x_e is such that

$$f(x_e) = 0.$$

We now introduce the following definition [14].

Definition 2.3.1. The equilibrium point $x = x_e$ of the system (2.3.1) is said to be stable if for each $\epsilon > 0$, $\exists \delta = \delta(\epsilon) > 0$

$$\| x(0) - x_e \| < \delta \implies \| x(t) - x_e \| < \epsilon \qquad \forall t \geq t_0,$$

where $\forall$ means "for all" and $\exists$ means "there exists"; otherwise, the equilibrium point is said to be unstable (see Figure 2.2).

Definition 2.3.2. The equilibrium point $x = x_e$ of the system (2.3.1) is said to be convergent if there exists $\delta_1 > 0$:

$$\| x(0) - x_e \| < \delta_1 \implies \lim x(t) = x_e.$$

Equivalently, x_e is convergent if for any given $\epsilon_1 > 0$, there exists a T such that

$$\| x(0) - x_e \| < \delta_1 \implies \| x(t) - x_e \| < \epsilon_1 \qquad \forall t \geq t_0 + T.$$

Definition 2.3.3. The equilibrium point $x = x_e$ of the system (2.3.1) is said to be asymptotically stable if it is both stable and convergent (see Figure 2.3).

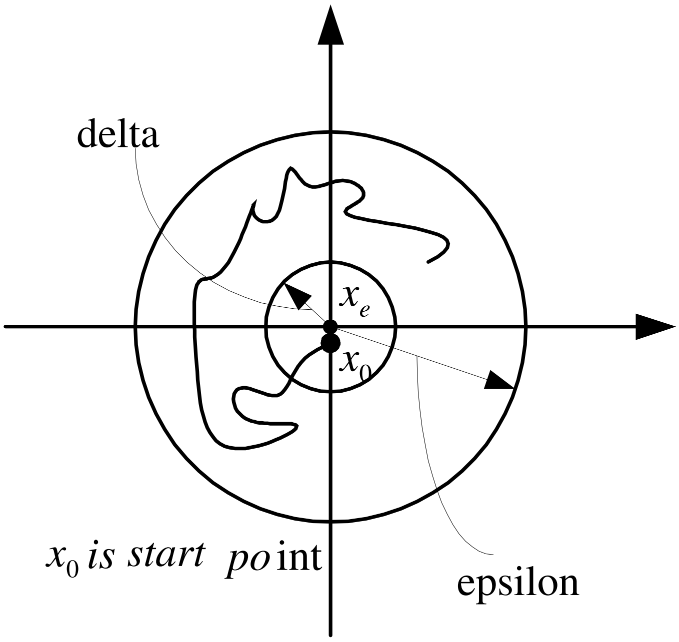

Figure 2.2: Stable Equilibrium Point

Definition 2.3.4. The equilibrium point $x = x_e$ of the system (2.3.1) is said to be (locally) exponentially stable of there exist two real constants $\alpha, \lambda > 0$ such that

$$\| x(t) - x_e \| \leqslant \alpha \| x(0) - x_e \| e^{-\lambda t} \quad \forall t > 0 \tag{2.3.2}$$

wherever $\| x(0) - x_e \| < \delta$. It is said to be globally exponentially stable if (2.3.2) holds for any $x \in R^n$.

Definition 2.3.5. A function $V : D \to R$ is said to be positive semidefinite in D if it satisfies the following conditions:

$$(i) \quad 0 \in D \; and \; V(0) = 0$$

$$(ii) \quad V(x) \geq 0, \quad \forall x \; in \; D$$

$V : D \to R$ is said to be positive definite in D if condition (ii) is replaced by $(ii^{'})$

$$(ii^{'}) \quad V(x) > 0 \; in \; D$$

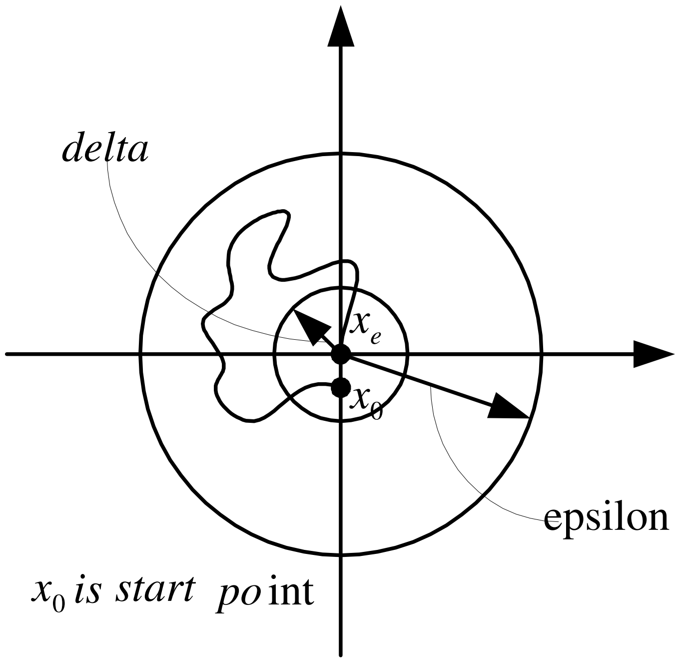

Figure 2.3: Asymptotically Stable Equilibrium Point

Finally, $V : D \to R$ is said to be negative definite (semidefinite) in D if $-V$ is positive definite (semidefinite).

Definition 2.3.6. Let $V : D \to R$ and $f : D \to R$. The Lie derivative of V along f, denoted by $L_f V$ is defined by

$$L_f V(x) = \frac{\partial V}{\partial x} f(x)$$

Thus, according to this definition, we have that

$$\dot{V}(x) = \frac{\partial V}{\partial x} f(x) = \nabla V \cdot f(x) = L_f V(x) \tag{2.3.3}$$

2.3.2 Lyapunov Stability Theory

Theorem 2.3.1. (Lyapunov Boundedness Theorem) Let $x = 0$ be an equilibrium point of

$$\dot{x} = f(x) \qquad f : D \longrightarrow R^n,$$

and let $V : D \to R$ be a continuously differentiable function such that

$$(i) \quad V(0) = 0,$$

$$(ii) \quad V(x) > 0 \quad in \;\; D - \{0\},$$

$$(ii') \quad \dot{V}(x) \leq 0 \quad in \;\; D - \{0\}.$$

We can conclude that $x = 0$ is stable. In other words, a sufficient condition for the stability of the equilibrium point $x = 0$ is that there exists a continuously differentiable positive definite function $V(x)$ such that $\dot{V}(x)$ is negative semidefinite in a neighborhood of $x = 0$.

Theorem 2.3.2. (Lyapunov Asymptotic Stability Theorem) Under the condition of Theorem 2.3.1, let $x = 0$ be an equilibrium point of

$$\dot{x} = f(x) \qquad f : D \longrightarrow R^n,$$

and let $V : D \rightarrow R$ be a continuously differentiable function such that

$$(i) \quad V(0) = 0,$$

$$(ii) \quad V(x) > 0 \quad in \;\; D - \{0\},$$

$$(ii^{'}) \quad \dot{V}(x) < 0 \quad in \;\; D - \{0\}.$$

Then $x = 0$ is asymptotically stable. In other words, the theorem states that asymptotic stability is achieved if the conditions of Theorem 2.3.1 are strenghtened by requiring $\dot{V}(x)$ to be negative definite, rather than semidefinite.

2.3.3 Barbalat's Lemma

Lemma 2.3.1. (Barbalat) If the differentiable function $f(t)$ has a finite limit as $t \longrightarrow \infty$, and if $\dot{f}$ is uniformly continuous, then $\dot{f}(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

Definition 2.3.7. A function $g(t)$ is continuous on $[0, \infty)$ if

$$\forall\, t_1 \geq 0, \forall\, R > 0, \exists\, \eta(R, t_1) > 0, \forall\, t \geq 0, \mid t - t_1 \mid < \eta \quad \Rightarrow \;\mid g(t) - g(t_1) \mid < R$$

Definition 2.3.8. A function $g(t)$ is said to be uniformly continuous on $[0, \infty)$ if

$$\forall\, R > 0, \exists\, \eta(R) > 0, \forall\, t_1 \geq 0, \forall\, t \geq 0, \mid t - t_1 \mid < \eta \quad \Rightarrow \;\mid g(t) - g(t_1) \mid < R$$

Lemma 2.3.2. (The Extended Version of Barbalat's Theorem)

If a differentiable function $f(t)$ converges to a limit value as $t \longrightarrow \infty$, and if its derivative $\dot{f}(t)$ is the sum of two terms, one being uniformly continuous and another tending to zero as t tends to infinity, then $\dot{f}(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

2.3.4 Finding Lyapunov Functions

There are many different types of Lyapunov theorems, but the key in all cases is to find a Lyapunov function and verify that it has the required properties. There is no universal method for constructing Lyapunov functions [26]. One common approach to finding Lyapunov function is to decide on a parametrized Lyapunov function candidate, and then try to find the values of the parameters so that the required hypotheses hold.

CHAPTER 3

LINEAR STATE FEEDBACK CONTROLLER FOR STABILIZATION

3.1 Linear State Feedback

The van der Pol equation with external control is given by [17]

$$m\ddot{x} + 2c^{\star}(x^2 - 1)\dot{x} + kx = u \tag{3.1.1}$$

where m, $c^{\star}$ and k are positive constants. For the convenience of controller design, the system (3.1.1) can be rewritten as

$$\ddot{x} + p_1(x^2 - 1)\dot{x} + p_2 x = p_3 u \tag{3.1.2}$$

where $p_1 = \frac{2c^{\star}}{m}$, $p_2 = \frac{k}{m}$ and $p_3 = \frac{1}{m}$ are positive constants.

This chapter presents the design of linear state feedback controller for setpoint regulation, where $[x, \dot{x}]^T$ is considered to be the state vector. All parameters of (3.1.2) are assumed to be known in this chapter.

Denote x_d as the setpoint and $y = x$ as the output. Our first control task is to design a linear controller that regulates y to x_d. Letting $e = y - x_d$, we have $\dot{x} = \dot{e}$ and $\ddot{x} = \ddot{e}$. Consider the linear state feedback controller

$$p_3 u = p_2 x_d + p_1 x_d^2 \dot{e} - k_p e - k_d \dot{e} \tag{3.1.3}$$

where $k_p = c\lambda + 1 - p_2$ and $k_d = c + \lambda + p_1$, $c > p_1 x_d^2$ and $\lambda > 0$ are design parameters.

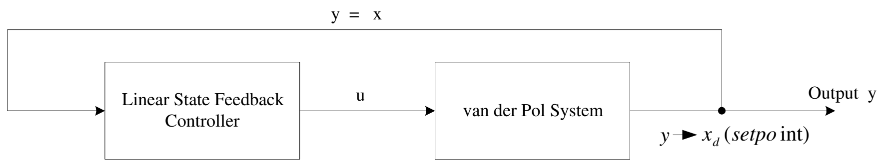

Figure 3.1: Linear State Feedback Control System

The closed-loop stability property of system (3.1.2) and (3.1.3) is summarized in the following theorem.

Theorem 3.1.1. For the van der Pol system described by (3.1.2), with the linear control law (3.1.3), the closed-loop signals $y(t) - x_d$ and $\dot{x}(t)$ tend to zero as $t \to \infty$.

Proof: Notice that

$$x^2\dot{x} = x_d^2\dot{e} + 2x_d e\dot{e} + e^2\dot{e} \tag{3.1.4}$$

Substituting (3.1.4) and (3.1.3) into (3.1.2), we obtain

$$\ddot{e} - p_1\dot{e} + p_2 e + 2p_1 x_d e\dot{e} + p_1 e^2\dot{e} + k_p e + k_d\dot{e} = 0 \tag{3.1.5}$$

Let $e_1 = e$ and $e_2 = \dot{e}$, (3.1.5) becomes

$$\begin{cases} \dot{e}_1 &= e_2 = -\lambda e_1 + (e_2 + \lambda e_1) \\ \dot{e}_2 &= -ae_1 - be_2 - 2p_1 x_d e_1 e_2 - p_1 e_1^2 e_2 \end{cases} \tag{3.1.6}$$

where $a = p_2 + k_p$ and $b = k_d - p_1$.

Consider the Lyapunov function candidate

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}(e_2 + \lambda e_1)^2 + \frac{\lambda p_1}{4}e_1^4 \tag{3.1.7}$$

The derivative of V along the solutions of (3.1.6) is

$$\begin{aligned}
\dot{V} &= e_1\dot{e}_1 + (e_2+\lambda e_1)(\dot{e}_2+\lambda\dot{e}_1) + \lambda p_1 e_1^3 e_2 \\
&= -\lambda e_1^2 + e_1(e_2+\lambda e_1) + (e_2+\lambda e_1)(-ae_1 \\
&\quad -be_2 - 2p_1 x_d e_1 e_2 - p_1 e_1^2 e_2 + \lambda e_2) + \lambda p_1 e_1^3 e_2 \\
&= -\lambda e_1^2 + (e_2+\lambda e_1)(-ae_1 - be_2 + e_1 + \lambda e_2) \\
&\quad -2p_1 x_d e_1 e_2 (e_2+\lambda e_1) - p_1 e_1^2 e_2^2 \\
&= -\lambda e_1^2 - c(e_2+\lambda e_1)^2 - 2p_1 x_d e_1 e_2 (e_2+\lambda e_1) \\
&\quad -p_1 e_1^2 e_2^2 \\
&= -\lambda e_1^2 - c[(e_2+\lambda e_1) + \frac{p_1}{c} e_1 e_2 x_d]^2 \\
&\quad -(p_1 - \frac{p_1^2}{c} x_d^2) e_1^2 e_2^2 \qquad (3.1.8)
\end{aligned}$$

When we select $c > p_1 x_d^2$, i.e., $p_1 > \frac{p_1^2 x_d^2}{c}$, $\dot{V}$ is negative definite. Therefore, it follows that $e_1(t)$ and $e_2(t)$ tend to zero, i.e., $y(t)$ tends to x_d and $\dot{x}(t)$ tends to zero.

3.2 Summary

In this chapter, the linear state feedback controller is designed to stabilize the van der Pol system to a setpoint. The proof is conducted based on the Lyapunov stability theorem. We give simulation results to exhibit the effectiveness of the proposed method in Chapter 7.

CHAPTER 4

LINEAR ADAPTIVE STATE FEEDBACK CONTROLLER FOR STABILIZATION

4.1 Linear Adaptive State Feedback

Consider the van der Pol system (3.1.2). In this chapter we assume that the parameter p_3 is known, and that p_1 and p_2 are unknown. The controller is selected as

$$p_3 u = \hat{p_2} x + \hat{p_1}(x_d^2 - 1)\dot{e} - k_p e - k_d \dot{e} \tag{4.1.1}$$

where $\hat{p_2}$ is the estimate of p_2 and $\hat{p_1}$ is the estimate of p_1. The parameter estimates used in (4.1.1) are obtained using the following adaptive law

$$\dot{\hat{p_1}} = -\Gamma_1(e_2 + \lambda e_1)(x_d^2 - 1)e_2 \tag{4.1.2}$$

and

$$\dot{\hat{p_2}} = -\Gamma_2(e_2 + \lambda e_1)x \tag{4.1.3}$$

where Γ_1 and Γ_2 are postive constants.

The closed-loop stability property of system (3.1.1), (4.1.1), (4.1.2) and (4.1.3) is summarized in Theorem 4.1.1.

Theorem 4.1.1. For the Van der Pol system described by (3.1.1), with the linear controller (4.1.1) and adaptive law (4.1.2) and (4.1.3), the closed-loop signals $y(t) - x_d$ and $\dot{x}(t)$ tend to zero as $t \to \infty$.

Proof: Substituting (3.1.4) and (4.1.1) into (3.1.2), we obtain

$$\ddot{x} + p_1(x^2-1)\dot{x} + p_2 x - \hat{p_2}x - \hat{p_1}(x_d^2-1)\dot{e} + k_p e + k_d\dot{e} = 0 \tag{4.1.4}$$

$$\begin{aligned} &\ddot{e} - p_1\dot{e} + p_1(x_d^2\dot{e} + 2x_d e\dot{e} + e^2\dot{e}) + \tilde{p_2} + (p_1 - \hat{p_1} - p_1) \\ &x_d^2\dot{e} + (p_1 - p_1 + \hat{p_1})\dot{e} + k_p e + k_d\dot{e} = 0 \end{aligned} \tag{4.1.5}$$

$$\ddot{e} + p_1(2x_d e\dot{e} + e^2\dot{e}) + k_p e + k_d\dot{e} + \tilde{p_1}(x_d^2-1)\dot{e} + \tilde{p_2}x = 0 \tag{4.1.6}$$

where $\tilde{p_1} = p_1 - \hat{p_1}$ and $\tilde{p_2} = p_2 - \hat{p_2}$. Then we have

$$\dot{\hat{p_1}} = -\dot{\tilde{p}}_1 \tag{4.1.7}$$

$$\dot{\hat{p_2}} = -\dot{\tilde{p}}_2 \tag{4.1.8}$$

Error equation (4.1.6) can be transformed to

$$\begin{cases} \dot{e}_1 &= e_2 = -\lambda e_1 + (e_2 + \lambda e_1) \\ \dot{e}_2 &= -k_p e_1 - k_d e_2 - 2p_1 x_d e_1 e_2 - p_1 e_1^2 e_2 \\ & \quad -\tilde{p}_1(x_d^2-1)e_2 - \tilde{p}_2 x \end{cases}$$

Consider Lyapunov function candidate

$$V_2 = V + \frac{1}{2\Gamma_1}\tilde{p}_1^2 + \frac{1}{2\Gamma_2}\tilde{p}_2^2 \tag{4.1.9}$$

where V is given by equation 3.1.7, and repeated below

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}(e_2 + \lambda e_1)^2 + \frac{\lambda p_1}{4}e_1^4$$

The derivative of V_2 along the solutions of (4.1.2), (4.1.3), (4.1.7), (4.1.8) and (4.1.9)is

$$\begin{aligned}
\dot{V}_2 &= -\lambda e_1^2 + (e_2 + \lambda e_1)(-k_p e_1 - k_d e_2 + e_1 + \lambda e_2) \\
&\quad -2p_1 x_d e_1 e_2 (e_2 + \lambda e_1) - p_1 e_1^2 e_2^2 \\
&\quad -\tilde{p}_1 (e_2 + \lambda e_1)(x_d^2 - 1) e_2 - \tilde{p}_2 (e_2 + \lambda e_1) x \\
&\quad -\frac{1}{\Gamma_1} \tilde{p}_1 \dot{\hat{p}}_1 - \frac{1}{\Gamma_2} \tilde{p}_2 \dot{\hat{p}}_2 \\
&= -\lambda e_1^2 + (e_2 + \lambda e_1)(-k_p e_1 - k_d e_2 + e_1 + \lambda e_2) \\
&\quad -2p_1 x_d e_1 e_2 (e_2 + \lambda e_1) - p_1 e_1^2 e_2^2 \\
&= -\lambda e_1^2 - c_1 (e_2 + \lambda e_1)^2 - c_2 (e_2 + \lambda e_1) e_1 \\
&\quad -2p_1 x_d e_1 e_2 (e_2 + \lambda e_1) - p_1 e_1^2 e_2^2 \\
&= -\lambda [e_1 + \frac{c_2}{2\lambda}(e_2 + \lambda e_1)]^2 - p_1 [e_1 e_2 + x_d (e_2 \\
&\quad +\lambda e_1)]^2 - (c_1 - \frac{c_2^2}{4\lambda} - p_1 x_d^2)(e_2 + \lambda e_1)^2 \qquad (4.1.10)
\end{aligned}$$

where $c_1 = k_d - \lambda$ and $c_2 = (k_p - 1) - (k_d - \lambda)\lambda$.

When we select k_p and k_d to make $c_1 - \frac{c_2^2}{4\lambda} - p_1 x_d^2$ larger than zero, $\dot{V}_2$ is negative semidefinite. Then e_1, e_2, $\tilde{p}_1$ and $\tilde{p}_2$ are all bounded. Furthermore, from (4.1.9), $\dot{e}_1$ and $\dot{e}_2$ are bounded. It follows that $\dot{V}_2$ is unifomly continuous. By applying Barbalat's Lemma[17], we have $\dot{V}_2$ tending to zero, i.e., $y(t)$ tends to x_d and $\dot{x}(t)$ tends to zero.

Remark 4.1.1. Note that

$$c_1 = k_d - \lambda \qquad (4.1.11)$$

$$\begin{aligned}
c_2 &= (k_p - 1) - (k_d - \lambda)\lambda \\
&= k_p - \lambda k_d - 1 + \lambda^2 \qquad (4.1.12)
\end{aligned}$$

When we select $k_p = \lambda k_d$, we have

$$c_2 = \lambda^2 - 1 \qquad (4.1.13)$$

For bounded λ and x_d, from (4.1.11) and (4.1.13), $\frac{c_2^2}{4\lambda} + p_1 x_d^2$ is bounded. We can select $k_p = \lambda k_d$ and $k_d > m(\lambda + \frac{(\lambda^2-1)^2}{4\lambda} + p_1 x_d^2)$ to make $c_1 - \frac{c_2^2}{4\lambda} - p_1 x_d^2 > 0$.

4.2 Summary

In this chapter, we assume that parameters p_1 and p_2 are unknown. The linear adaptive state feedback controller is proposed to force the van der pol oscillator to stable to a setpoint. The stability property of the system (3.1.1)is proofed by finding a suitable Lyapunov function. The corresponding simulation results for the proposed adaptive technique are showed in Chapter 7.

CHAPTER 5

NONLINEAR CONTROLLER FOR TRACKING: KNOWN PARAMETERS

Nonlinear state feedback and output feedback controllers for the tracking problem with known parameters are proposed in this chapter.

The tracking problem is stated as follows: consider $x_d(t)$ with bounded $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ as the desired time-varying trajectory. Our control task is to design a nonlinear controller to force y to follow x_d.

Let us begin with the state feedback controller design.

5.1 State Feedback

The state feedback controller is selected as

$$\begin{aligned} p_3 u &= \ddot{x}_d - p_1\dot{x}_d + p_2 x_d + p_1 x_d^2\dot{x}_d + 2p_1 x_d\dot{x}_d e \\ &\quad + p_1 x_d^2\dot{e} + p_1\dot{x}_d e^2 - k_p e - k_d\dot{e} \end{aligned} \tag{5.1.1}$$

where $k_p = c\lambda + 1 - p_2$ and $k_d = c + \lambda + p_1$, $c > p_1$ and $\lambda > 0$ are design parameters.

The closed-loop stability property of (3.1.2) and (5.1.1) is summarized in **Theorem 5.1.1**.

Theorem 5.1.1. For the Van der Pol system described by (3.1.2), with the nonlinear state feedback control law (5.1.1), the closed-loop signals $y(t) - x_d(t)$ and $\dot{x}(t) - \dot{x}_d(t)$ tend to zero as $t \to \infty$.

Proof: Notice that

$$x^2\dot{x} = x_d^2\dot{x}_d + 2x_d\dot{x}_d e + x_d^2\dot{e} + \dot{x}_d e^2 + 2x_d e\dot{e} + e^2\dot{e} \tag{5.1.2}$$

Substituting (5.1.2) and (5.1.1) into (3.1.2), we have

$$\ddot{e} - p_1\dot{e} + p_2 e + 2p_1 x_d e\dot{e} + p_1 e^2\dot{e} + k_p e + k_d\dot{e} = 0 \tag{5.1.3}$$

We find that error equation (5.1.3) is the same as equation (3.1.5). So, using the same argument as in Chapter 3, we can prove that, under controller (5.1.1), $y(t) - x_d(t)$ and $\dot{x}(t) - \dot{x}_d(t)$ converge to zero as $t \to \infty$.

5.2 Output Feedback

When $\dot{x}$ cannot be measured and only the output $y = x$ is available to the designer, an output feedback controller is needed to solve the tracking problem. In this case, an observer is first designed [20].

Let $h_1 = x$ and $h_2 = \dot{x}$, then the original model of the van der Pol equation (3.1.2) changes into

$$\left\{ \begin{array}{lcl} \dot{h}_1 & = & h_2 \\ \dot{h}_2 & = & -p_1(h_1^2 - 1)h_2 - p_2 h_1 + p_3 u \end{array} \right. \tag{5.2.1}$$

Define

$$\left\{ \begin{array}{lcl} x_1 & = & h_1 \\ x_2 & = & \frac{p_1}{3}h_1^3 - p_1 h_1 + h_2 \end{array} \right. \tag{5.2.2}$$

From (5.2.1) and (5.2.2), we have

$$\begin{array}{rcl} \dot{x}_2 & = & p_1 h_1^2 h_2 - p_1 h_2 - p_1 h_1^2 h_2 + p_1 h_2 \\ & & -p_2 h_1 + p_3 u \\ & = & -p_2 h_1 + p_3 u \end{array} \tag{5.2.3}$$

$$\dot{x}_1 = \dot{h}_1 = h_2 = x_2 - \frac{p_1}{3}h_1^3 + p_1 h_1 \tag{5.2.4}$$

Then, system (5.2.1) is transformed into

$$\begin{cases} \dot{x}_1 &= x_2 + p_1(x_1 - \frac{1}{3}x_1^3) \\ \dot{x}_2 &= -p_2 x_1 + p_3 u \end{cases} \tag{5.2.5}$$

From the transformation (5.2.2), it is apparent that

$$y = x_1 = h_1 = x \tag{5.2.6}$$

Define

$$g = x_2 - Lx_1 \tag{5.2.7}$$

where $L > 0$. Differentiating g along the solutions of (5.2.5) gives

$$\begin{aligned} \dot{g} &= \dot{x}_2 - L\dot{x}_1 \\ &= -p_2 x_1 + p_3 u - Lx_2 - Lp_1(x_1 - \frac{1}{3}x_1^3) \\ &= -L(x_2 - Lx_1) - (p_2 + L^2)x_1 - Lp_1(x_1 \\ &\quad -\frac{1}{3}x_1^3) + p_3 u \\ &= -Lg - (p_2 + L^2)x_1 - Lp_1(x_1 - \frac{1}{3}x_1^3) \\ &\quad +p_3 u \end{aligned} \tag{5.2.8}$$

The observer of g is selected as

$$\dot{\hat{g}} = -L\hat{g} - (p_2 + L^2)x_1 - Lp_1(x_1 - \frac{1}{3}x_1^3) + p_3 u \tag{5.2.9}$$

Define $\varepsilon_g = g - \hat{g}$. We have

$$\dot{\varepsilon}_g = -L\varepsilon_g \tag{5.2.10}$$

Since $L > 0$, ε_g tends to zero at an exponential rate of L. Let $\hat{x}_2$ be the estimate of x_2, $\hat{h}_2$ be the estimate of h_2, and $\hat{g}$, $\hat{x}_2$ and $\hat{h}_2$ have the following relation

$$\hat{g} = \hat{x}_2 - Lx_1 = \hat{h}_2 + \frac{p_1}{3}h_1^3 - p_1 h_1 - Lh_1 \tag{5.2.11}$$

Then we have

$$\begin{aligned}\varepsilon_g &= g - \hat{g} \\ &= x_2 - \hat{x}_2 \\ &= h_2 - \hat{h}_2\end{aligned} \tag{5.2.12}$$

Furthermore, defining $\eta = \hat{h}_2 - \dot{x}_d$, from (5.2.12), we obtain

$$\begin{aligned}\dot{e} - \eta &= h_2 - \dot{x}_d - \hat{h}_2 + \dot{x}_d \\ &= h_2 - \hat{h}_2 \\ &= \varepsilon_g\end{aligned} \tag{5.2.13}$$

We use η to substitute $\dot{e}$ into the state feedback controller (5.1.1) to obtain the output feedback controller

$$\begin{aligned}p_3 u &= \ddot{x}_d - p_1\dot{x}_d + p_2 x_d + p_1 x_d^2 \dot{x}_d + 2p_1 x_d \dot{x}_d e \\ &\quad + p_1 x_d^2 \eta + p_1 \dot{x}_d e^2 - k_p e - k_d \eta\end{aligned} \tag{5.2.14}$$

The closed-loop stability property of system (3.1.2), (5.2.9) and (5.2.14) is summarized in Theorem 5.2.1.

Theorem 5.2.1. For the van der Pol system described by (3.1.2), with the nonlinear output feedback control law (5.2.14) and reduced-order observer (5.2.9), the closed-loop signals $y(t)$, $\dot{x}(t)$ and $\hat{g}(t)$ are bounded. Moreover, $y(t) - x_d(t)$ and $\dot{x}(t) - \dot{x}_d(t)$ tend to zero as $t \to \infty$.

Proof: Substituting (5.1.2), (5.2.13) and (5.2.14) into (3.1.2), we have

$$\begin{aligned}&\ddot{e} - p_1\dot{e} + p_2 e + 2p_1 x_d e\dot{e} + p_1 e^2 \dot{e} + k_p e + k_d \dot{e} \\ &\qquad - (k_d - p_1 x_d^2)\varepsilon_g = 0\end{aligned} \tag{5.2.15}$$

i.e.,

$$\dot{e}_2 = -ae_1 - be_2 - 2p_1 x_d e_1 e_2 - p_1 e_1^2 e_2 + (k_d - p_1 x_d^2)\varepsilon_g \tag{5.2.16}$$

Consider the Lyapunov function candidate

$$V_a = V + \frac{1}{2}\varepsilon_g^2 \tag{5.2.17}$$

where V is given by equation 3.1.7, and repeated below

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}(e_2 + \lambda e_1)^2 + \frac{\lambda p_1}{4}e_1^4$$

The derivative of V_a along the solutions of (5.2.10) and (5.2.16) is

$$\begin{aligned}\dot{V}_a &= -\lambda e_1^2 - c(e_2 + \lambda e_1)^2 - 2p_1 x_d e_1 e_2 (e_2 + \lambda e_1) \\ &\quad -p_1 e_1^2 e_2^2 + (k_d - p_1 x_d^2)\varepsilon_g (e_2 + \lambda e_1) - L\varepsilon_g^2 \\ &= -\lambda e_1^2 - p_1[e_1 e_2 + x_d(e_2 + \lambda e_1)]^2 \\ &\quad -L[\varepsilon_g - \frac{k_d - p_1 x_d^2}{2L}(e_2 + \lambda e_1)]^2 \\ &\quad -(c - \frac{(k_d - p_1 x_d^2)^2}{4L} - p_1 x_d^2)(e_2 + \lambda e_1)^2 \end{aligned} \tag{5.2.18}$$

When we select $c > \frac{(k_d - p_1 x_d^2)^2}{4L} + p_1 x_d^2$, $\dot{V}_a$ is negative definite. Thus, it follows that $e_1(t)$, $e_2(t)$, and ε_g are bounded and tend to zero, i.e., $y(t) - x_d(t)$ and $\dot{x}(t) - \dot{x}_d(t)$ tend to zero. Since $e_1(t)$ and $e_2(t)$ are bounded, $y(t)$ and $\dot{x}(t)$ are bounded. Then, from (5.2.2), x_1 and x_2 are bounded. Furthermore, from (5.2.11), $\hat{g}$ is bounded.

Remark 5.2.1. Since we can always select L to make $\frac{(k_d - p_1 x_d^2)^2}{4L} < \varepsilon_c$ for any given $\varepsilon_c > 0$, we can find some c such that $c > \varepsilon_c + p_1 x_d^2$.

5.3 Summary

In this chapter, the nonlinear controllers are presented to force the van der Pol oscillator following the desired trajectory. First, a state feedback controller is proposed when the system states is measurable. Then an output feedback controller is designed, only if the system output is available. The proposed methods have been supported by simulation results, which are shown in Chapter 7.

CHAPTER 6

NONLINEAR OUTPUT FEEDBACK CONTROLLER FOR TRACKING: UNKNOWN PARAMETERS

In this chapter, the unknown parameters case is considered. That is, parameters p_1, p_2 and p_3 are not required to be known. We use the popular adaptive backstepping with tuning function [21] to design an adaptive output feedback controller for the tracking problem. First, a K-filter is designed. Then, an adaptive output feedback controller is proposed incorporating the K-filter.

Let $y_r(t)$ be the trajectory that the output $y(t)$ of system (3.1.2) is forced to follow, and suppose $\dot{y}_r(t)$ and $\ddot{y}_r(t)$ are known and piecewise continuous and bounded.

6.1 Introduction

During the last decade, backstepping-based designs have emerged as powerful tools for stabilizing nonlinear systems both for tracking and regulation purposes [21]. The main advantage of these designs is the systematic construction of a Lyapunov function for closed loop systems, allowing the analysis of their stability properties. The adaptive version of these designs, especially the tuning functions design, offers a systematic way to design controllers for a wide class of nonlinear systems whose structures are known but with unknown parameters [9].

In order to apply the adaptive backstepping techniques into our application. We can rewrite the van der Pol system(5.2.5)

$$\begin{cases} \dot{x}_1 &= x_2 + p_1(x_1 - \frac{1}{3}x_1^3) \\ \dot{x}_2 &= -p_2 x_1 + p_3 u \end{cases}$$

$$y = x_1$$

into

$$\dot{X} = AX + \phi(y) + \Phi(y)a + \begin{bmatrix} 0 \\ b \end{bmatrix} \sigma(y)u,$$

$$y = e_1^T X, \tag{6.1.1}$$

where $X = [x_1, x_2]^T$, $\phi(y) = 0$, $a = [p_1, p_2]^T$, $b = p_3$, $\sigma(y) = 1$, $e_1^T = [1, 0]$,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{6.1.2}$$

and

$$\Phi(y) = \Phi(X_1) = \begin{bmatrix} x_1 - \frac{x_1^3}{3} & 0 \\ 0 & -x_1 \end{bmatrix}.$$

Further, we transfer the van der Pol system (6.1.1) to

$$\dot{X} = AX + \phi(y) + F(y, u)^T \theta$$

$$y = e_1^T X, \tag{6.1.3}$$

where $\theta = [p_3, p_1, p_2]^T$ and

$$F(y, u)^T = \begin{bmatrix} 0 & x_1 - \frac{x_1^3}{3} & 0 \\ u & 0 & -x_1 \end{bmatrix}.$$

6.2 K-filter and Observer Design

We design an observer for system (6.1.3) based on the K-filter of [21]. Such an estimate $\hat{X}$ of unmeasured X is given as

$$\hat{X} = \xi + \Omega^T \theta \tag{6.2.1}$$

where

$$\dot{\xi} = A_0 \xi + KY \tag{6.2.2}$$

$$\dot{\Omega}^T = A_0 \Omega^T + F(y, u)^T \tag{6.2.3}$$

$K = [k_1, k_2]^T$ and

$$A_0 = A - K e_1^T = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \tag{6.2.4}$$

is Hurwitz, that is

$$P A_0 + A_0^T P = -I, P = P^T > 0 \tag{6.2.5}$$

$\Omega^T = [v_0, \Xi]$ and $\Xi = [\Xi_1, \Xi_2]$

$$\begin{bmatrix} \dot{v}_{0,1} \\ \dot{v}_{0,2} \end{bmatrix} = A_0 \begin{bmatrix} v_{0,1} \\ v_{0,2} \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix} \tag{6.2.6}$$

i.e.,

$$\dot{v}_{0,1} = v_{0,2} - k_1 v_{0,1} \tag{6.2.7}$$

$$\dot{v}_{0,2} = -k_2 v_{0,1} + u \tag{6.2.8}$$

$$\dot{\Xi} = A_0 \Xi + \Phi(y) \tag{6.2.9}$$

Define $\varepsilon = X - \hat{X}$, from K-filter design [21], we have

$$\dot{\varepsilon} = A_0 \varepsilon \tag{6.2.10}$$

6.3 Adaptive Controller Design

From (6.2.1), we have

$$\begin{aligned} x_2 &= \hat{x}_2 + \varepsilon_2 \\ &= \xi_2 + \Omega_{(2)}^T \theta + \varepsilon_2 \\ &= \xi_2 + [v_{0,2}, \Xi_{1,2}, \Xi_{2,2}][p_3, p_1, p_2]^T + \varepsilon_2 \end{aligned} \tag{6.3.1}$$

Then from (5.2.5) and (6.3.1), we have

$$\begin{aligned} \dot{y} &= \dot{x}_1 \\ &= x_2 + p_1(x_1 - \frac{x_1^3}{3}) \\ &= \xi_2 + \omega^T \theta + \varepsilon_2 = p_3 v_{0,2} + \xi_2 + \bar{\omega}^T \theta + \varepsilon_2 \end{aligned} \tag{6.3.2}$$

where

$$\omega^T = [v_{0,2}, \Xi_{1,2} + (x_1 - \frac{x_1^3}{3}), \Xi_{2,2}] \tag{6.3.3}$$

$$\bar{\omega}^T = [0, \Xi_{1,2} + (x_1 - \frac{x_1^3}{3}), \Xi_{2,2}] \tag{6.3.4}$$

We define the change of coordinates as

$$\begin{cases} z_1 = y - y_r \\ z_2 = v_{0,2} - \hat{\varrho}\dot{y}_r - \alpha_1 \end{cases} \tag{6.3.5}$$

where $\hat{\varrho}$ is an estimate of $\varrho = 1/p_3$ and define $\hat{\varrho} = \varrho - \tilde{\varrho}$, and α_1 is a virtual control.

Step-1: The equation for the tracking error z_1 obtained from (6.3.2) and (6.3.5) is

$$\begin{aligned} \dot{z}_1 &= \dot{y} - \dot{y}_r \\ &= p_3 v_{0,2} + \xi_2 + \bar{\omega}^T \theta + \varepsilon_2 - \dot{y}_r \end{aligned} \tag{6.3.6}$$

Let $\alpha_1 = \hat{\varrho}\bar{\alpha}_1$. From (6.3.5), we have

$$v_{0,2} = z_2 + \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1) \tag{6.3.7}$$

So equation (6.3.6) is rewritten as

$$
\begin{aligned}
\dot{z}_1 &= p_3 z_2 + p_3 \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1) + \xi_2 + \bar{\omega}^T \theta + \varepsilon_2 - \dot{y}_r \\
&= p_3 z_2 + p_3(\varrho - \tilde{\varrho})(\dot{y}_r + \bar{\alpha}_1) + \xi_2 + \bar{\omega}^T \theta + \varepsilon_2 - \dot{y}_r \\
&= \bar{\alpha}_1 + \xi_2 + \bar{\omega}^T \theta + \varepsilon_2 - p_3 \tilde{\varrho}(\dot{y}_r + \bar{\alpha}_1) + p_3 z_2 \qquad (6.3.8)
\end{aligned}
$$

Then the choice of the stabilizing function

$$\bar{\alpha}_1 = -c_1 z_1 - d_1 z_1 - \xi_2 - \bar{\omega}^T \hat{\theta} \qquad (6.3.9)$$

where c_1 and d_1 are positive scale. Then, we have

$$\dot{z}_1 = -c_1 z_1 - d_1 z_1 + \varepsilon_2 + \bar{\omega}^T \tilde{\theta} - p_3(\dot{y}_r + \bar{\alpha}_1)\tilde{\varrho} + p_3 z_2 \qquad (6.3.10)$$

From (6.3.3), (6.3.5) and (6.3.7), we obtain,

$$
\begin{aligned}
\bar{\omega}^T \tilde{\theta} + p_3 z_2 &= \bar{\omega}^T \tilde{\theta} + \tilde{p}_3 z_2 + \hat{p}_3 z_2 \\
&= \bar{\omega}^T \tilde{\theta} + (v_{0,2} - \hat{\varrho}\dot{y}_r - \alpha_1)\tilde{p}_3 + \hat{p}_3 z_2 \\
&= \omega^T \tilde{\theta} - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)\eta_1^T \tilde{\theta} + \hat{p}_3 z_2 \\
&= (\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)\eta_1)^T \tilde{\theta} + \hat{p}_3 z_2 \qquad (6.3.11)
\end{aligned}
$$

where $\eta_1 = [1, 0, 0]^T$. Substituting (6.3.11) into (6.3.10), we get

$$\dot{z}_1 = -c_1 z_1 - d_1 z_1 + \varepsilon_2 + (\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)\eta_1)^T \tilde{\theta} - p_3(\dot{y}_r + \bar{\alpha}_1)\tilde{\varrho} + \hat{p}_3 z_2 \qquad (6.3.12)$$

The first tuning function is selected as

$$\tau_1 = (\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1 \eta_1))z_1 \qquad (6.3.13)$$

and

$$\dot{\hat{\varrho}} = -\gamma sign(p_3)(\dot{y}_r + \bar{\alpha}_1)z_1 \qquad (6.3.14)$$

where $\gamma > 0$.

Step-2: Differentiating z_2, with the help of (6.2.2), (6.2.8), (6.2.9), (6.3.2) and (6.3.5), we have

$$\begin{aligned}
\dot{z}_2 &= \dot{v}_{0,2} - \hat{\varrho}\ddot{y}_r - \dot{\hat{\varrho}}\dot{y}_r - \dot{\alpha}_1 \\
&= u - k_2 v_{0,1} - \hat{\varrho}\ddot{y}_r - \frac{\partial\alpha_1}{\partial y}(\xi_2 + \omega^T\theta + \varepsilon_2) - \frac{\partial\alpha_1}{\partial\xi_2}(-k_2\xi_1 + k_2 y) \\
&\quad -\frac{\partial\alpha_1}{\partial\Xi}(A_0\Xi + \Phi) - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - (\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}})\dot{\hat{\varrho}} \\
&= u - k_2 v_{0,1} - \hat{\varrho}\ddot{y}_r - \frac{\partial\alpha_1}{\partial y}(\xi_2 + \omega^T\hat{\theta}) - \frac{\partial\alpha_1}{\partial\xi_2}(-k_2\xi_1 + k_2 y) \\
&\quad -\frac{\partial\alpha_1}{\partial\Xi}(A_0\Xi + \Phi) - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 - (\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}})\dot{\hat{\varrho}} \\
&\quad -\frac{\partial\alpha_1}{\partial y}\varepsilon_2 - \frac{\partial\alpha_1}{\partial y}\omega^T\tilde{\theta} - \frac{\partial\alpha_1}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \Gamma\tau_2)
\end{aligned} \tag{6.3.15}$$

where $\Gamma \in R^{3\times 3}$ and $\Gamma = \Gamma^T > 0$.

Select the controller

$$\begin{aligned}
u &= -\hat{p}_3 z_1 - c_2 z_2 - d_2(\frac{\partial\alpha_1}{\partial y})^2 z_2 + k_2 v_{0,1} + \hat{\varrho}\ddot{y}_r + \frac{\partial\alpha_1}{\partial y}(\xi_2 + \omega^T\hat{\theta}) \\
&\quad +\frac{\partial\alpha_1}{\partial\xi_2}(-k_2\xi_1 + k_2 y) + \frac{\partial\alpha_1}{\partial\Xi}(A_0\Xi + \Phi) + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 \\
&\quad +(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}})\dot{\hat{\varrho}}
\end{aligned} \tag{6.3.16}$$

and adaptive law

$$\tau_2 = \tau_1 - \frac{\partial\alpha_1}{\partial y}\omega z_2 \tag{6.3.17}$$

So equation (6.3.15) is rewritten as

$$\dot{z}_2 = -\hat{p}_3 z_1 - c_2 z_2 - d_2(\frac{\partial\alpha_1}{\partial y})^2 z_2 - \frac{\partial\alpha_1}{\partial y}\varepsilon_2 - \frac{\partial\alpha_1}{\partial y}\omega^T\tilde{\theta} - \frac{\partial\alpha_1}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \Gamma\tau_2) \tag{6.3.18}$$

The closed-loop system can be compactly written as

$$\dot{z} = A_z(z)z + W_\varepsilon(z)\varepsilon_2 + W_\theta^T(z)\tilde{\theta} - p_3(\dot{y}_r + \bar{\alpha}_1)\eta_1\tilde{\varrho} \tag{6.3.19}$$

where

$$A_z(z) = \begin{bmatrix} -c_1 - d_1 & \hat{p}_3 \\ -\hat{p}_3 & -c_2 - d_2(\frac{\partial\alpha_1}{\partial y})^2 \end{bmatrix} \tag{6.3.20}$$

$$W_\varepsilon(z) = \begin{bmatrix} 1 \\ -\frac{\partial \alpha_1}{\partial y} \end{bmatrix} \tag{6.3.21}$$

$$W_\theta^T(z) = W_\varepsilon(z)\omega^T - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)\eta_1\eta_1^T \tag{6.3.22}$$

The adaptive law can be written as.

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\Gamma W_\theta(z) z = -\Gamma \tau_2 \tag{6.3.23}$$

$$\dot{\tilde{\varrho}} = \dot{\hat{\varrho}} = \gamma sgn(p_3)(\dot{y}_r + \bar{\alpha}_1)\eta_1^T z \tag{6.3.24}$$

The closed-loop stability property of system (3.1.2), (6.2.2), (6.2.3), (6.2.10), (6.3.16), (6.3.23) and (6.3.24) is summarized in Theorem 6.3.1.

Theorem 6.3.1. For the van dero Pol system described by (3.1.2), with the output feedback control law (6.3.16), adaptive law (6.3.23) and (6.3.24), and K-filter (6.2.2) and (6.2.3), the closed-loop signals z, $\hat{\theta}$, $\hat{\varrho}$ and ε are bounded. Moreover $y(t) - x_d(t)$ tends to zero as $t \to \infty$.

Let us prove our work. Consider the Lyapunov function candidate.

$$V = \frac{1}{2} z^T z + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{\mid p_3 \mid}{2\gamma}\tilde{\varrho}^2 + \sum_{i=1}^{2} \frac{1}{4d_i}\varepsilon^T P \varepsilon \tag{6.3.25}$$

The derivative of V along (6.2.10), (6.3.19), (6.3.23) and (6.3.24) is

$$\begin{aligned}
\dot{V} &= z^T(A_z/2 + A_z^T/2)z + z^T W_\varepsilon \varepsilon_2 + z^T W_\theta^T \tilde{\theta} - z^T p_3(\dot{y}_r + \bar{\alpha}_1)\eta_1 \tilde{\varrho} \\
&\quad - \tilde{\theta} W_\theta z + \tilde{\varrho} p_3 (\dot{y}_r + \bar{\alpha}_1)\eta_1^T z - \sum_{i=1}^{2} \frac{1}{4d_i} \mid \varepsilon \mid^2 \\
&= -\sum_{i=1}^{2} c_i z_i^2 - \sum_{i=1}^{2} d_i (\frac{\partial \alpha_{i-1}}{\partial y})^2 z_i^2 - \sum_{i=1}^{2} z_i \frac{\partial \alpha_{i-1}}{\partial y}\varepsilon_2 - \sum_{i=1}^{2} \frac{1}{4d_i} \mid \varepsilon \mid^2 \\
&= -\sum_{i=1}^{2} c_i z_i^2 - \sum_{i=1}^{2} d_i (z_i \frac{\partial \alpha_{i-1}}{\partial y} + \frac{1}{2d_i}\varepsilon_2)^2 - \sum_{i=1}^{2} \frac{1}{4d_i}\varepsilon_1^2 \\
&\leq -\sum_{i=1}^{2} c_i z_i^2
\end{aligned} \tag{6.3.26}$$

where for notational convenience, we have introduced $\frac{\partial \alpha_0}{\partial y} = -1$.

From (6.3.26), we can conclude that z, $\hat{\theta}$, $\hat{\varrho}$ and ε are bounded. By applying the LaSalle-Yoshizawa Theorem 2.1 in [21] to (6.3.26), it further follows that $z(t) \to 0$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$.

6.4 Summary

In the previous chapter, we discussed the tracking problem with known parameters. In this chapter, an adaptive output feedback controller are designed for the tracking problem, when parameters of van der Pol system are unknown. The controller design is based on adaptive backstepping techniques. The proposed adaptive approach is proved theoretically and by simulation results given in Chapter 7.

CHAPTER 7

SIMULATIONS

To support our results, we simulated all the designs by Matlab with the controlled van der Pol system(3.1.2). In the simulations that follow, the parameters of system (3.1.2) are selected as $p_1 = 1.0$, $p_2 = 1.0$ and $p_3 = 1.0$.

7.1 Linear Output Feedback Controller for Stabilization

In the simulation of the linear out put feedback controller for stabilization,the reference signal is $x_d = 2.0$. The controller gains in (3.1.3) are $k_p = 2$ and $k_d = 4$. The simulation results are shown in Figure (7.1) and (7.2). From Figure (7.1), we can see that $x(t) \to x_d$ as $t \to \infty$, thus system is asymptotically stable. In Figure (7.2), the control signal $u(t)$ tends to zero when system is stable.

When we increase the controller gains $k_p = 14$ and $k_d = 30$, we can see in Figure(7.3,$x(t)$ goes to x_d when time is around 1 to 2 seconds. Note that for the high gain controller $x(t) - x_d(t)$ converges to zero more quickly than it does for the low gain case.But from Figure(7.4, we can see it need lots energy in the first second.

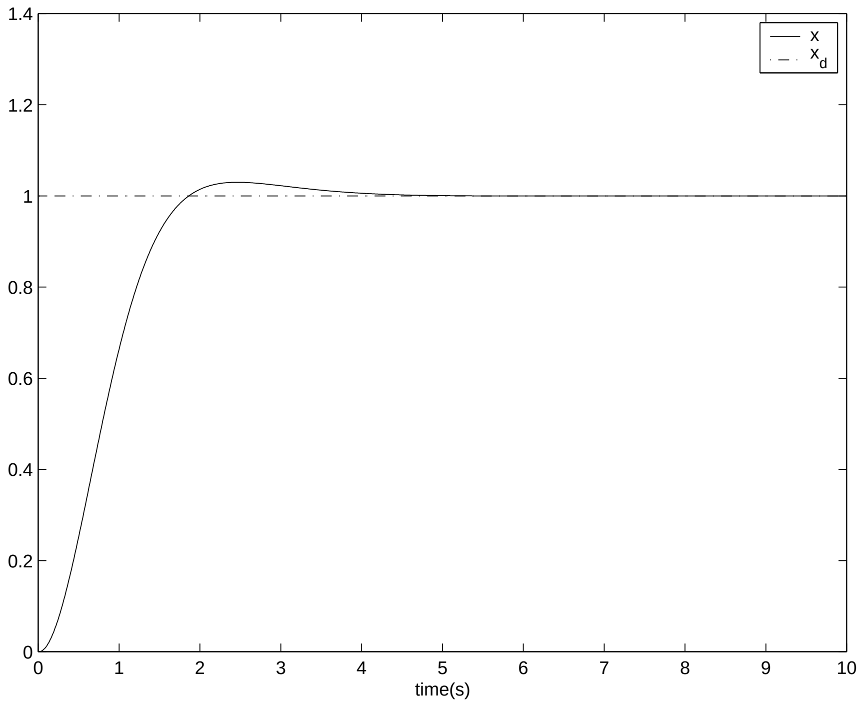

Figure 7.1: Linear Controller For Stabilization: x and x_d.

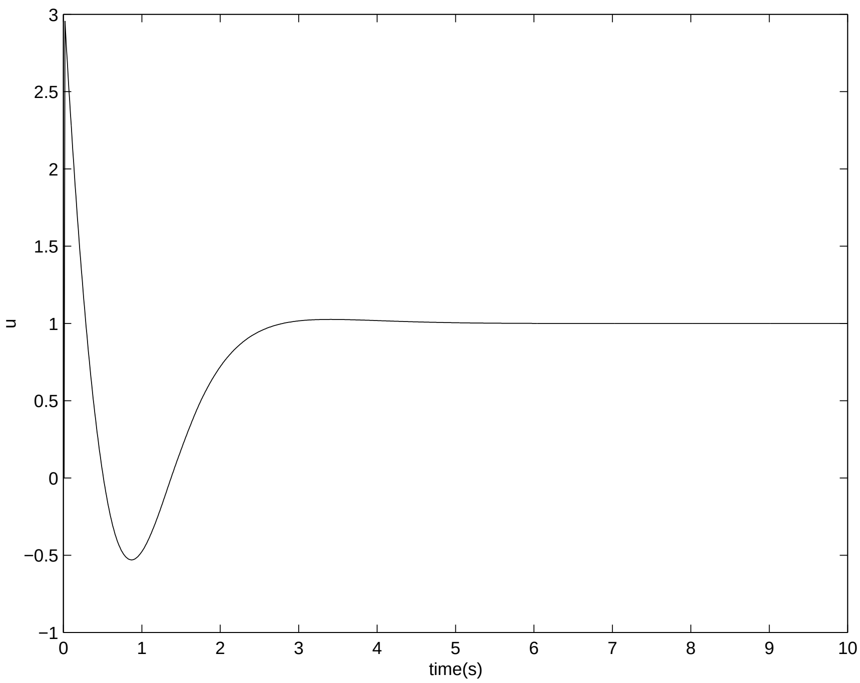

Figure 7.2: Linear Controller For Stabilization: u.

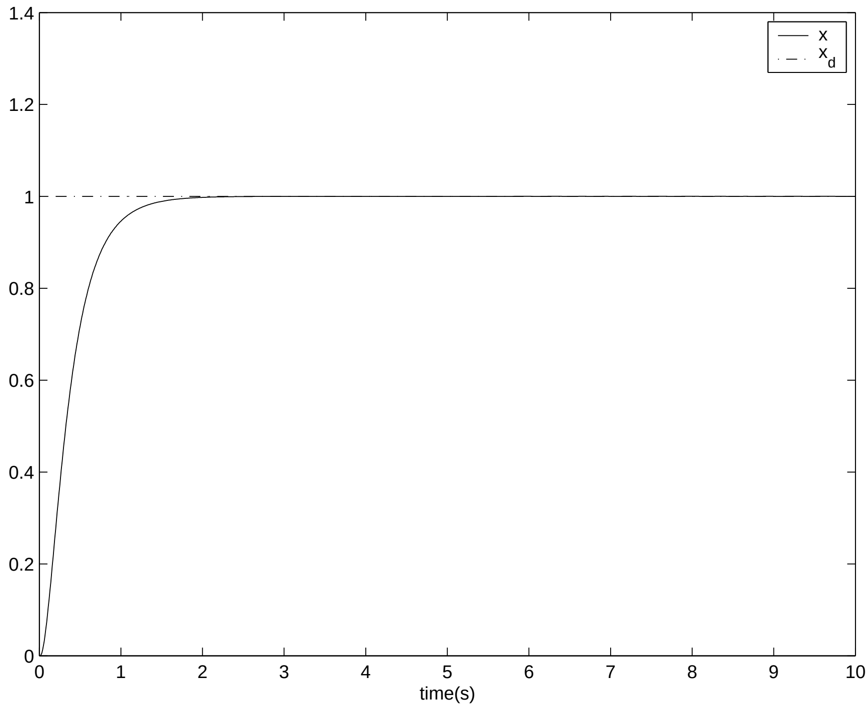

Figure 7.3: Linear Controller For Stabilization(with increased control gain): x and x_d.

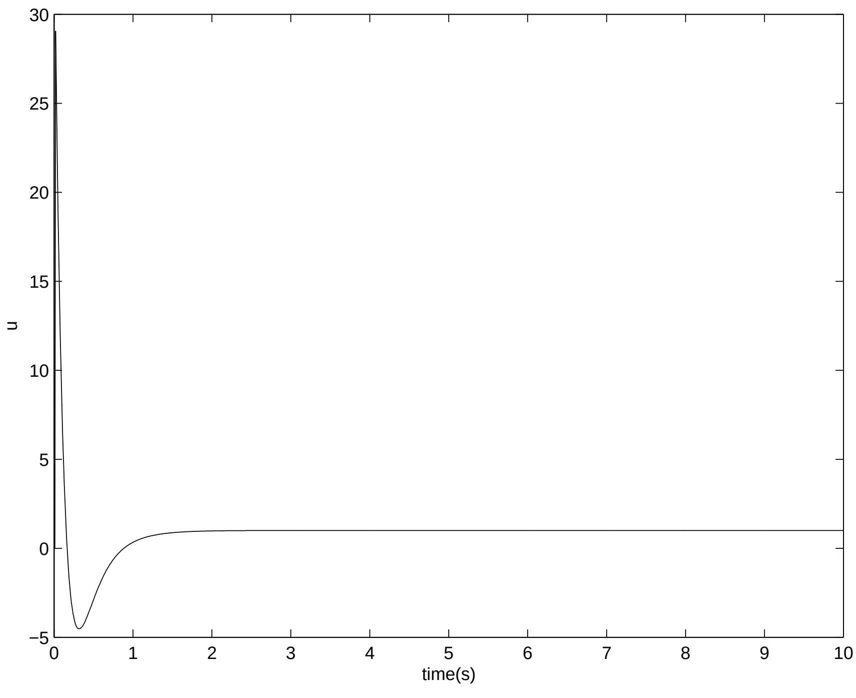

Figure 7.4: Linear Controller For Stabilization(with increased control gain): u.

7.2 Linear Adaptive State Feedback Controller for Stabilization

In this simulation, the reference signal is $x_d = 2.0$. The controller gains in (3.1.3) are $k_p = 6$ and $k_d = 6$. The simulation results are shown in Figures (7.5) and (7.6). From Figure (7.5), we can see that $x(t) \to x_d$ as $t \to \infty$, thus system is asymptotically stable. In Figure (7.6), the control signal $u(t)$ tends to zero when system is stable.

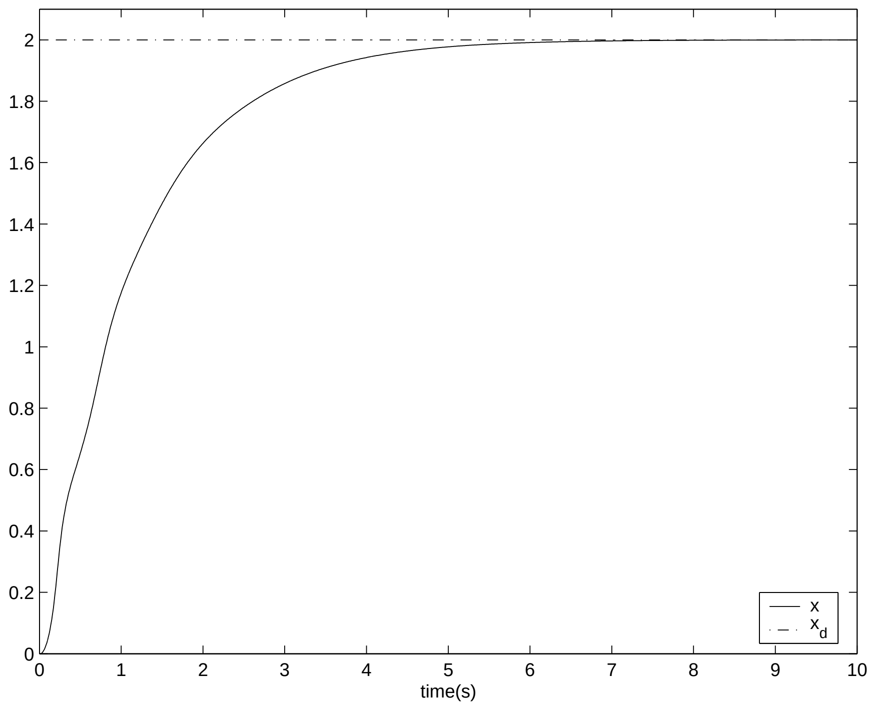

Figure 7.5: Linear Adaptive Controller For Stabilization: x and x_d.

7.3 Nonlinear Controller for Tracking

The reference signal is $x_d(t) = sin(2t)$ in this section. The controller gains in (5.1.1) and (5.2.14) are $k_p = 2$, $k_d = 4$, and $L = 2.0$ when observer (5.2.9) is used. The simulation results are shown in Figure (7.7) and (7.8) for state feedback controller and in Figure (7.9)

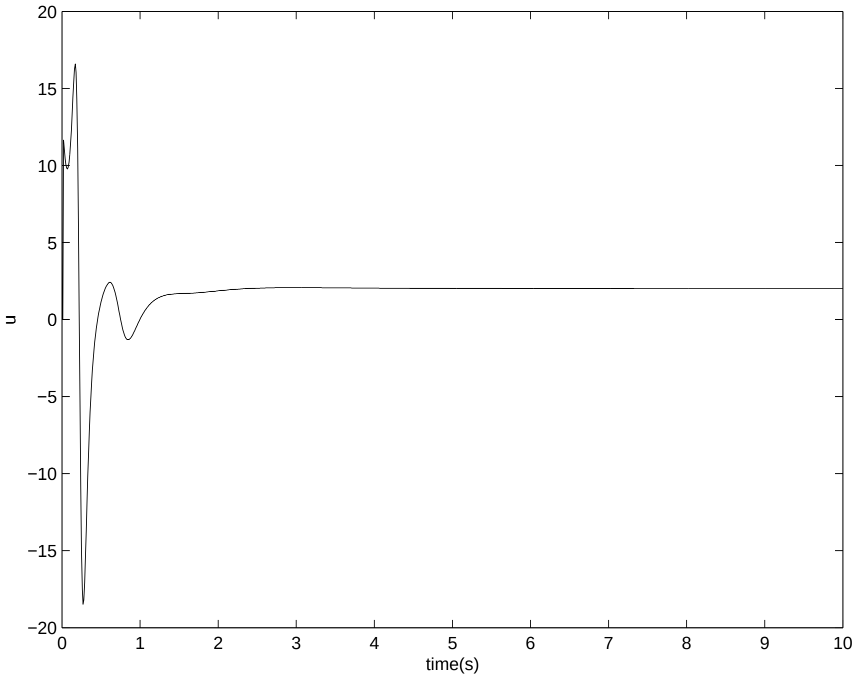

Figure 7.6: Linear Adaptive Controller For Stabilization: u.

and (7.10) for output feedback controller. From the simulation results in Figure (7.7) and Figure (7.9), we can see that $x(t)$ follows $x_d(t)$ within two and five seconds, respectively. Note that for the state feedback controller, $x(t) - x_d(t)$ converges to zero more quickly than it does for the output feedback case. The reason for this is that an observer is used in the later case.

7.4 Nonlinear Adaptive Controller for Tracking

The reference signal is $x_d(t) = sin(2t)$ in the case of parameters being unknown. The feedback gains for the K-filter (6.2.2) is selected as $K = [2, 1]$. The adaptive gain for (6.3.23) is $\Gamma = I$ and for (6.3.24) is $\gamma = 2$. The simulation results are shown in Figure (7.11) and (7.12). In the Figure (7.11), we can see that $y(t) - y_r(t)$ goes to zero at a relatively slower speed due to parameter adaption.

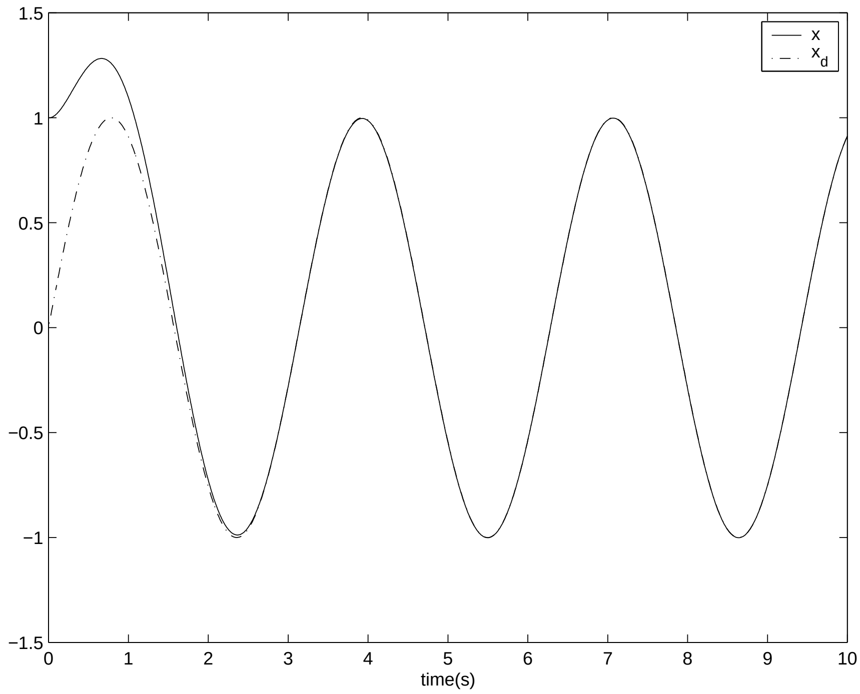

Figure 7.7: Nonlinear State Feedback Controller For Tracking: x and x_d.

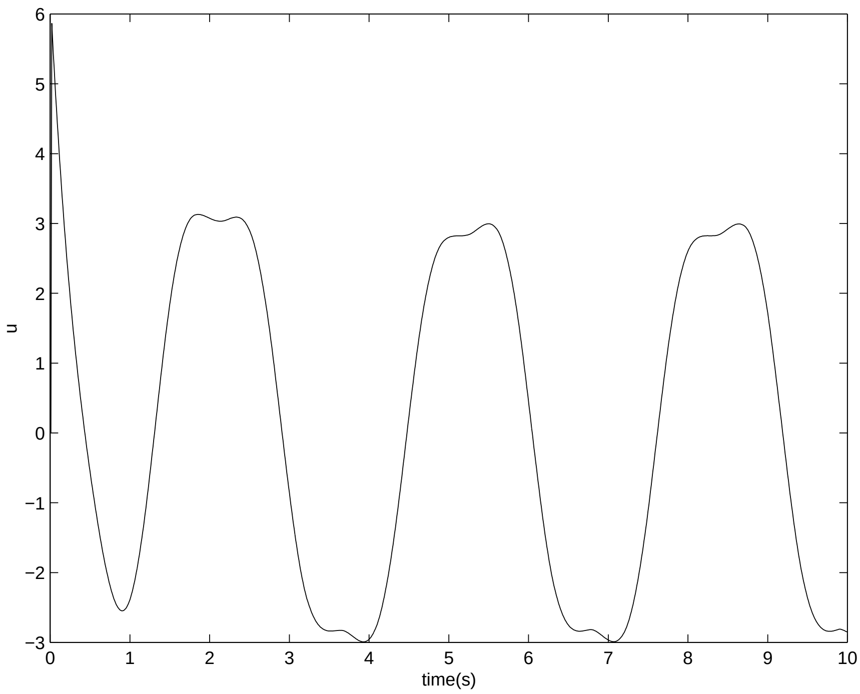

Figure 7.8: Nonlinear State Feedback Controller For Tracking: u.

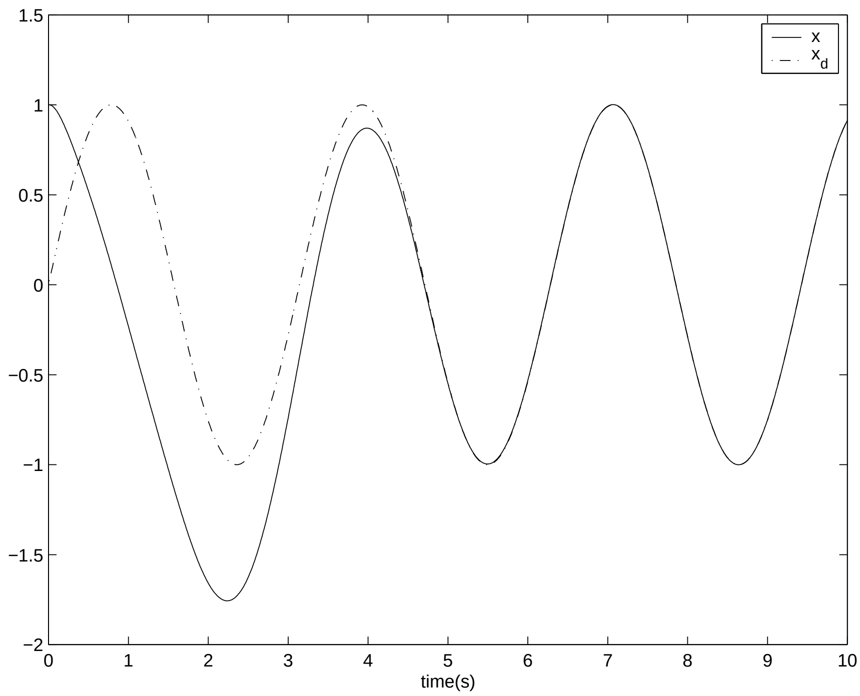

Figure 7.9: Nonlinear Output Feedback Controller For Tracking: x and x_d.

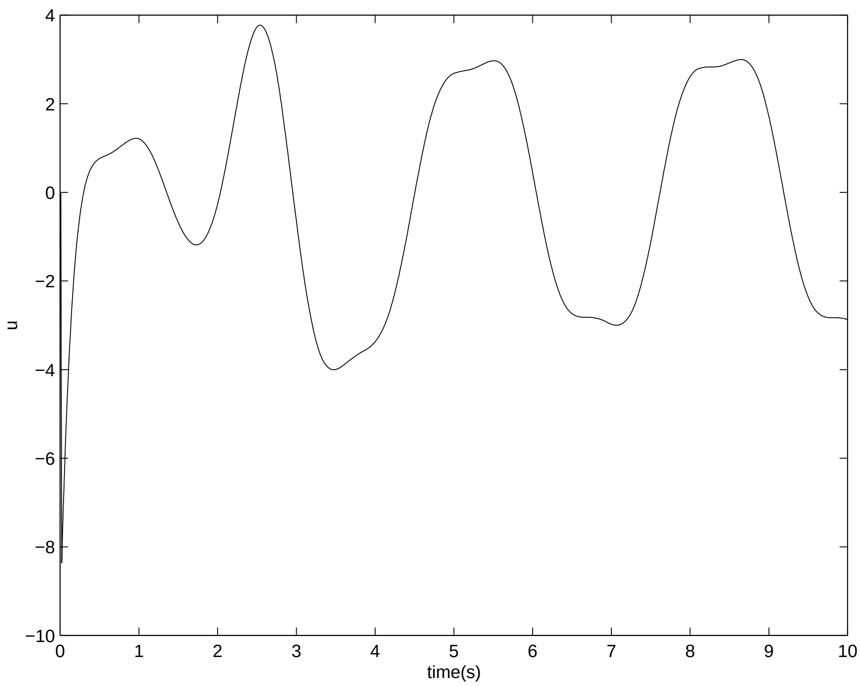

Figure 7.10: Nonlinear Output Feedback Controller For Tracking: u.

The simulation results validate our proposed algorithms.

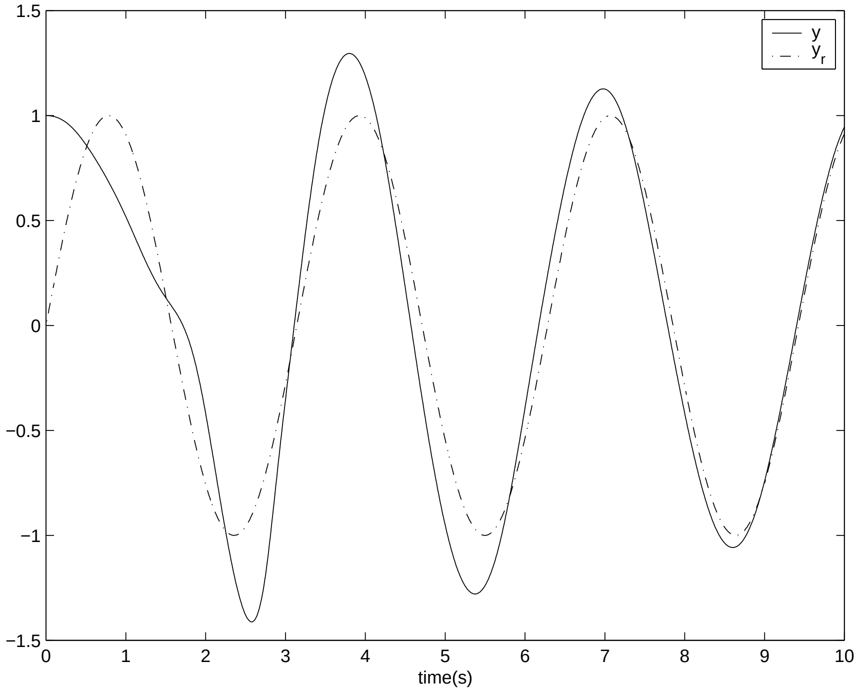

Figure 7.11: Nonlinear Adaptive Controller For Tracking: y and y_r.

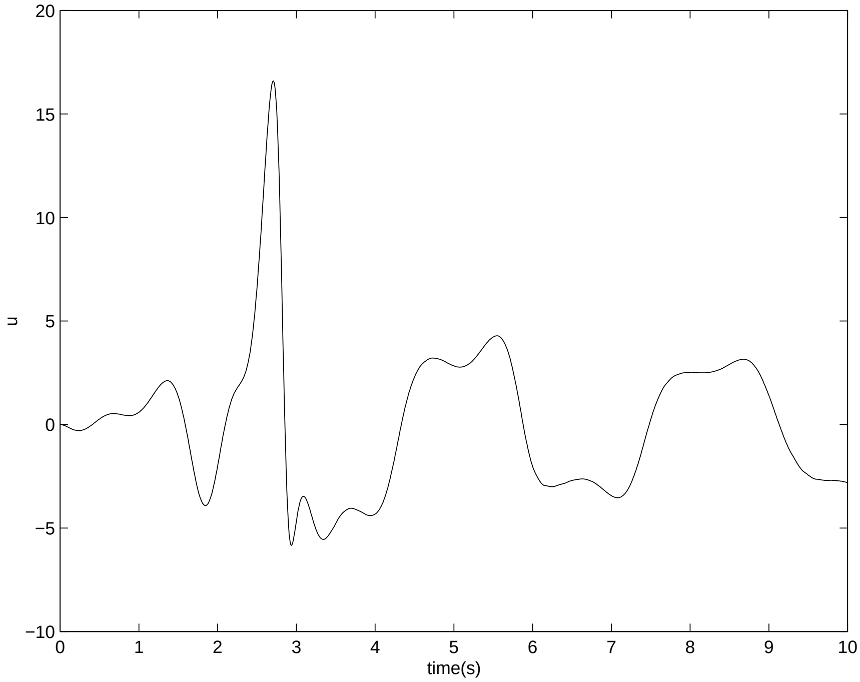

Figure 7.12: Nonlinear Adaptive Controller For Tracking: u.

CHAPTER 8

CONCLUSIONS

In the thesis, the stabilization and tracking problem of the Van der Pol oscillator has been investigated by using advanced control theory. First, a linear state feedback controller was proposed for the stabilization problem. Then, nonlinear state feedback and output feedback controllers were designed for the tracking problem with known parameters. Finally, a dynamic output feedback controller based on adaptive backstepping technique was introduced for the tracking problem when all parameters are unknown. The proposed schemes have been used in the simulations to demonstrate their effectiveness.

APPENDIX A
PROGRAMS CODE

PROGRAMS CODE

1. linear state feedback controller for Stabilization Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% This M file is used to simulate the linear state feedback            %
%% controller for Stabilization                                         %
%% Writed by Xin Zhao                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
clc;
%---setup the initial conditions-----
x0 = [0,0,0];
tf = 10;
step = 1000;
stepsize= tf/step;
for
i=1:step
tspan(i) = i*stepsize;
end
%--simulation---------------
[t,y]=ode45('Van1',tspan,x0);
%--plot the results--------
for
i = 1:step
y1(i) = y(i,1);
y2(i) = y(i,2);
end
%--calculate u-------------
```

```
u(1) = (y(1,3)-0)/stepsize;
for
i = 1:step-1
u(i+1) = (y(i+1,3)-y(i,3))/stepsize;
end
%--Linear controller for stablization:x,xd-------
for
i = 1:step
xd(i) = 2.0;
end
plot(t,y1,'k',t,xd,'k-.');
legend('x','x_{d}');
xlabel('time(s)');
print -deps Fig1.eps;
%--Linear controller for stablization:u----------
figure;
plot(t,u,'k');
xlabel('time(s)');
ylabel('u');
print -deps Fig2.eps;
%end of the M file
%--Function Van1
function xdot = Van1(t,x,flag);
p1 = 1.0;
p2 = 1.0;
p3 = 1.0;
xd=2.0;
%--------Initial Values----
```

```
h1 = x(1);
h2 = x(2);
e = h1-xd;
edot = h2;
%-------Parameters----------
c = 2.0;
lambda = 1.0;
b = c+lambda;
a = c*lambda+1; kp = a-p2;
kd= b+p1;
%-----Linear state feedback controller---
u = (p2*xd+p1*(xd^2)*edot-kp*e-kd*edot)/p3;
%-------Van der Pol System-----
h1dot = h2;
h2dot = -p1*(h1^2-1)*h2-p2*h1+p3*u;
uidot = u;
xdot = [h1dot;h2dot;uidot];
%end of function
```

2.linear adaptive state feedback controller for Stabilization Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% This M file is used to simulate the linear adaptive state            %
%% feedback controller for Stabilization                                %
%% Writed by Xin Zhao                                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
clc;
%---setup the initial conditions-----
```

```
x0 = [0,0,0,0,0];

tf = 10;

step = 1000;

stepsize= tf/step;

for i=1:step

   tspan(i) = i*stepsize;

end

%--simulation---------------

[t,y]=ode45('Van12',tspan,x0);

%--plot the results--------

for

i = 1:step

y1(i) = y(i,1);

y2(i) = y(i,2);

end

%--calculate u-------------

u(1) = (y(1,5)-0)/stepsize;

for i = 1:step-1

    u(i+1) = (y(i+1,5)-y(i,5))/stepsize;

end

%--Linear controller for stablization:x,xd-------

for

i = 1:step

xd(i) = 2.0;

end

plot(t,y1,'k',t,xd,'k-.');

axis([0,10,0,2.1]);

legend('x','x_{d}',0);
```

```
xlabel('time(s)');
print -deps Fig12.eps;
%--Linear controller for stablization:u---------
figure;
plot(t,u,'k');
xlabel('time(s)');
ylabel('u');
print -deps Fig22.eps
%end of the M file

%--Function Van12
function xdot = Van1(t,x,flag) p1 = 1.0; p2 = 1.0; p3 = 1.0;
xd = 2.0;
h1 = x(1);
h2 = x(2);
hb = x(3);
hk = x(4);
e = h1-xd;
edot = h2;
e1 = e;
e2 = edot;

%-------Parameters----------
lambda = 1.0;
kd = 6.0;
kp = kd*lambda;
Gamma1 = 5;
Gamma2 = 5;
```

```
%-----Linear adaptive state feedback controller---
u = hk*x(1)+2*hb*(xd^2-1)*edot-kp*e-kd*edot;

%-------Van der Pol System-----
h1dot = h2;
h2dot = -p1*(h1^2-1)*h2-p2*h1+p3*u;
hbdot = -Gamma1*(e2+lambda*e1)*(xd^2-1)*e2;
hkdot = -Gamma2*(e2+lambda*e1)*x(1);
uidot = u;
xdot = [h1dot;h2dot;hbdot;hkdot;uidot];
%end of function
```

3.The nonlinear state feedback controller for the tracking problem Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% This M file is used to simulate the nonlinear state feedback controller     %
%% for the tracking problem                                                    %
%% Writed by Xin Zhao                                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
clc;
%---setup the initial conditions-----
x0 = [1,0,0];
tf = 10;
step = 1000;
stepsize= tf/step;
```

```
for i=1:step
   tspan(i) = i*stepsize;
end
%--simulation---------------
[t,y]=ode45('Van21',tspan,x0);
%--plot the results--------
for
i = 1:step
y1(i) = y(i,1);
y2(i) = y(i,2);
end
%--calculate u-------------
u(1) = (y(1,3)-0)/stepsize;
for
i = 1:step-1
u(i+1) = (y(i+1,3)-y(i,3))/stepsize;
end
% Nonlinear state feedback for tracking:X and Xd
for i = 1:step
   yr(i) = sin(2*i*stepsize);
end
plot(t,y1,'k',t,yr,'k-.');
legend('x','x_{d}');
xlabel('time(s)');
print -deps Fig3.eps;
% Nonlinear state feedback for tracking:u
figure;
plot(t,u,'k');
```

```
xlabel('time(s)');
ylabel('u');
print-depsFig4.eps;
%end of the M file

%--Function Van21
function xdot = Van21(t,x,flag);
p1 = 1.0;
p2 = 1.0;
p3 = 1.0;
xd = sin(2*t);
xddot = 2*cos(2*t);
xdddot = -4*sin(2*t);
h1 = x(1);
h2 = x(2);
e = h1-xd;
edot = h2-xddot;
%-------Parameters----------
c = 2.0;
lambda = 1.0;
b = c+lambda;
a = c*lambda+1;
kp = a-p2;
kd = b+p1;
%-------Output feedback controller---
u=xdddot-p1*xddot+p2*xd+p1*(xd^2)*xddot+2*p1*xd*xddot*e
+p1*(xd^2)*edot+p1*xddot*(e^2);
u = (u-kp*e-kd*edot)/p3;
```

```
%-------Van der Pol System-----
h1dot = h2;
h2dot = -p1*(h1^2-1)*h2-p2*h1+p3*u;
uidot = u;
xdot = [h1dot;h2dot;uidot];
%end of function
```

4. The nonlinear output feedback controller for the tracking problem Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% This M file is used to simulate the nonlinear output feedback controller    %
%% for the tracking problem                                                    %
%% Writed by Xin Zhao                                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
clc;
%---setup the initial conditions-----
x0 = [1,0,1,0];
tf = 10;
step = 1000;
stepsize= tf/step;
for
i=1:step
tspan(i) = i*stepsize;
end
%--simulation---------------
[t,y]=ode45('Van22',tspan,x0);
```

```
%--plot the results--------
for i = 1:step
   y1(i) = y(i,1);
   y2(i) = y(i,2);
   y3(i) = y(i,3);
end
%--calculate u-------------
u(1) = (y(1,4)-0)/stepsize;
for
i = 1:step-1
u(i+1) = (y(i+1,4)-y(i,4))/stepsize;
end
% Nonlinear output feedback for tracking:X and Xd
for i = 1:step
   yr(i) = sin(2*i*stepsize);
end
plot(t,y1,'k',t,yr,'k-.');
legend('x','x_{d}');
xlabel('time(s)');
print -deps Fig5.eps;
% Nonlinear output feedback for tracking:u
figure; plot(t,u,'k');
xlabel('time(s)');
ylabel('u');
print -deps Fig6.eps
%end of the M file


%--Function Van22
```

```
function xdot = Van22(t,x,flag);
p1 = 1.0;
p2 = 1.0;
p3 = 1.0;
xd = sin(2*t);
xddot = 2*cos(2*t);
xdddot = -4*sin(2*t);
%--------Initial Values----
h1 = x(1);
h2 = x(2);
gh = x(3);
%--------State Transision---
x1 = h1;
x2 = (1/3)*p1*(h1^3)-p1*h1+h2;
%-------Parameters----------
L = 2.0;
c = 2;
lambda = 1.0;
b = c+lambda;
a = c*lambda+1;
kp = a-p2;
kd = b+p1;
%--------Output Error--------
e = h1-xd;
%-------h2h means h2hat------
%-------h2hat is the estimate of h2
h2h = gh-1/3*p1*h1^3+p1*h1+L*x1;
%-------ehdot means eta------
```

```
%It is the estimate error of xddot
ehdot = h2h-xddot;
%-------Output feedback controller---
%u = (xdddot-p1*xddot+p2*xd+p1*(xd^2)*xddot+2*p1*xd*xddot*e
%+p1*(xd^2)*ehdot+p1*xddot*(e^2)-kp*e-kd*ehdot)/p3;
u=xdddot-p1*xddot+p2*xd+p1*(xd^2)*xddot
+2*p1*xd*xddot*e+p1*(xd^2)*ehdot+p1*xddot*(e^2);
u=(u-kp*e-kd*ehdot)/p3;
%-------Van der Pol System-----
h1dot = h2;
h2dot = -p1*(h1^2-1)*h2-p2*h1+p3*u;
%------Observer of g---------
ghdot = -L*gh-(p2+L^2)*x1-L*p1*(x1-1/3*(x1^3))+p3*u;
uidot = u;
xdot = [h1dot;h2dot;ghdot;uidot];
%end of function
```

5. Adaptive controller for the tracking problem Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% This M file is used to simulate the adaptive controller  %
%% for the tracking problem                                 %
%% Writed by Xin Zhao                                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
clc;
```

```
%---setup the initial conditions-----
x0 = [0,0,0,0];
x0 = [1,0,0,0,x0,x0,x0,0];
tf = 10;
step = 1000;
stepsize= tf/step;
for
i=1:step
tspan(i) = i*stepsize;
end
%--simulation---------------
[t,y]=ode45('Van',tspan,x0);
%--plot the results--------
for
i = 1:step
y1(i) = y(i,1);
y2(i) = y(i,2);
end
u(1) = (y(1,17)-0)/stepsize;
for
i = 1:step-1
u(i+1) = (y(i+1,17)-y(i,17))/stepsize;
end
for i = 1:step
yr(i) = sin(2*i*stepsize);
end
plot(t,y1,'k',t,yr,'k-.');
legend('y','y_{r}');
```

```
xlabel('time(s)');
print -deps Fig7.eps;
figure;
plot(t,u,'k');
xlabel('time(s)');
ylabel('u');
print -deps Fig8.eps;
%end of the M file
function xdot = Van(t,x,flag)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This function is as a defile of Van der Pol Equation %
%h1dot = h2                                           %
%h2dot = -p1(h1^2-1)h2-p2h1+p3u                       %
%Writed by Xin Zhao                                   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
p1 = 1;
p2 = 1;
p3 = 1;
k = [2;1];
A0= [-k(1),1;-k(2),0];
gamma1 = 2;
Gamma2 = [1,0,0;0,1,0;0,0,1];
c1 = 1;
c2 = 1;
d1 = 1;
d2 = 1;
yr = sin(2*t);
yrdot = 2*cos(2*t);
```

```
yrddot = -4*sin(2*t);
h1 = x(1);
y = x(1);
h2 = x(2);
xi = [x(3);x(4)];
v0 = [x(5);x(6)];
Xi1 = [x(7);x(8)];
Xi2 = [x(9);x(10)];
hrho = x(11);
htheta = [x(12);x(13);x(14)];
epsilon = [x(15);x(16)];
hp3 = htheta(1);
hp1 = htheta(2);
hp2 = htheta(3);
e1 = [1;0;0];
omega = [v0(2);Xi1(2)+(y-y^3/3);Xi2(2)];
bomega = [0;Xi1(2)+(y-y^3/3);Xi2(2)];
z1 = y-yr;
balpha1 = -c1*z1-d1*z1-xi(2)-bomega'*htheta;
alpha1 = hrho*balpha1;
z2 = v0(2)-hrho*yrdot-alpha1;
pa1phrho = -(c1+d1)*y+(c1+d1)*yr-xi(2)-[Xi1(2)+(y-(y^3)/3)]*hp1-Xi2(2)*hp2;
pa1py = -hrho*(c1+d1)-hrho*hp1*(1-y^2); pa1pyr = hrho*(c1+d1);
pa1pxi2 = -hrho; pa1pXi12 = -hrho*hp1; pa1pXi22 = -hrho*hp2;
pa1php1 = -hrho*(Xi1(2)+(y-y^3/3)); pa1php2 = -hrho*Xi2(2);
pa1phtheta =[0,pa1php1,pa1php2];
tau1 = (omega- hrho*(yrdot+balpha1)*e1)*z1;
tau2 = tau1-pa1py*omega*z2;
```

```
u = -hp3*z1-c2*z2-d2*(pa1py^2)*z2+k(2)*v0(1)+hrho*yrddot+pa1py*(xi(2)
     +(omega')*htheta)+pa1pxi2*(-k(2)*xi(1)+k(2)*y);
u = u+pa1pXi12*(-k(2)*Xi1(1))+pa1pXi22*(-k(2)*Xi2(1)-y)
     +pa1pyr*yrdot+pa1phtheta*Gamma2*tau2;
u = u+(yrdot+pa1phrho)*[-gamma1*(yrdot+balpha1)*z1];
h1dot = h2;
h2dot = -p1*(h1^2-1)*h2-p2*h1+p3*u;
xidot = A0*xi+k*y;
v0dot = A0*v0+[0,u]';
Xi1dot = A0*Xi1+[y-(y^3)/3; 0];
Xi2dot = A0*Xi2+[0; -y];
hrhodot = -gamma1*(yrdot+balpha1)*z1;
hthetadot = +Gamma2*tau2;
epdot = A0*epsilon;
uidot = u;
xdot = [h1dot;h2dot;xidot;v0dot;Xi1dot;Xi2dot;hrhodot;hthetadot;epdot;uidot];
%end of the fuction
```

APPENDIX B
EXAMPLE

EXAMPLE

Lyapunov Stabilization Example:

Consider the nonlinear system:

$$\dot{x} = u + \theta\varphi(x) \tag{8.0.1}$$

If θ were known, the control

$$u = -\theta\varphi(x) - c_1 x, \quad c_1 > 0 \tag{8.0.2}$$

would render teh derivative of $V_0 = \frac{1}{2}x^2$ negative definite: $\dot{V}_0 = -c_1 x^2$. Of course the control law8.0.2 can not be implemented, since θ were unknown. Instead, one can employ its certainty-equivalence from in which θ is replaced by an estimate $\hat{\theta}$:

$$u = -\hat{\theta}\varphi(x) - c_1 x, \quad c_1 > 0. \tag{8.0.3}$$

Substituting 8.0.3 to 8.0.1, we obtain

$$\dot{x} = -c_1 x + \tilde{\theta}\varphi(x), \tag{8.0.4}$$

where $\tilde{\theta}$ is the parameter error:

$$\tilde{\theta} = \theta - \hat{\theta}. \tag{8.0.5}$$

The derivative of $V_0 = \frac{1}{2}x^2$ becomes

$$\dot{V}_0 = -c_1 x^2 + \tilde{\theta} x \varphi(x).. \tag{8.0.6}$$

Since the second term is indefinite and contains the unknown parameter error $\tilde{\theta}$, we can not conclude anything about the stability of 8.0.1. We make the controllor dynamic with an adaptive law for $\hat{\theta}$. To design this update law, we expend V_0 with a quadratic term in the parameter error $\tilde{\theta}$,

$$V_1(x, \tilde{\theta}) = \frac{1}{2}x^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \tag{8.0.7}$$

where $\gamma > 0$ is the adaptation gain. The derivative of this function is

$$\begin{aligned} \dot{V}_1 &= x\dot{x} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} && (8.0.8) \\ &= -c_1 x^2 + \tilde{\theta} x \varphi(x) + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} && (8.0.9) \\ &= -c_1 x^2 + \tilde{\theta}\left[\ x\varphi(x)\tfrac{1}{\gamma}\dot{\tilde{\theta}}\ \right]. && (8.0.10) \end{aligned}$$

The second term is still indefinite and contains $\tilde{\theta}$ as a factor. However, the situation is much better tan in 8.0.6, because we now have the dynamics of $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$ at our disposal. With teh appropriate choice of $\dot{\hat{\theta}}$ we can cancel the indefinite term. Thus, we choose the adaptive law

$$\dot{\hat{\theta}} = -\dot{\tilde{\theta}} = \gamma x \varphi(x), \tag{8.0.11}$$

which yields

$$\dot{V}_0 = -c_1 x^2 \leq 0. \tag{8.0.12}$$

So for the system described by 8.0.1, with the nonlinear controller (8.0.3) and adaptive law (8.0.11), the closed-loop signals $x(t)$ and $\tilde{\theta}$ tend to zero as $t \to \infty$.

REFERENCES

[1] M. V. Basin and M. A. Pinsky, "Stability impulse control of faulted nonlinear systems," *IEEE Transactions on Automatic Control,* Vol. 43, pp. 1604-1608, 1998.

[2] K. Buckholtz, D. W. Repperger and M. Daniels, " A study on the stabilization of the van der Pol limited cycle," *Submitted to International Journal of System Science.*

[3] G. Chen, *Chios, Bifurcation, and Their Control,* Wiley Encyclopedia of Electrical and Electronics Engineering, 1998.

[4] G. Chen, X. Dong, *From Chaos to Order-Perspectives, Methodologies, and Applications,* World Scientific Pub. Co.: Singapore,1998.

[5] Arun V. Holden, *Chaos* Princeton, N.J. : Princeton University Press, 1986.

[6] Ziauddin Sardar and Iwona Abrams , *Introducing Chaos* New York : Totem Books ; Lanham, Md. : Distributed to the trade in the U.S. by National Book Network, 1999.

[7] K.Y. Chen, P.D. Biernacki, A. Lahrichi and A. Mickelson, "Analysis of an experimental technique for determining van der Pol parameters of a transistor oscillator," *IEEE Transactions on Microwave Theory and Techniques,* Vol. 46, pp. 914-922, 1998.

[8] J. D. Parsons and E. M. Deeley "Some Properties of Two Series-Connected Synchronized van der Pol Oscillators," *IEEE Transactions on Circuit Theory,* Vol. 5, pp. 292-294, 1972.

[9] F. Ikhouane and M. Krstic, "Adaptive backstepping with parameter projection : Robustness and asymptotic performance," *Automatica,* Vol. 34, pp.429-435, 1998.

[10] N. Hovakimyan, R. Rysdyk and A. J. Calise, "Dynamic neural networks for output feedback control," *Proceedings of the 38th IEEE Conference on Decision and Control,* Vol. 2, pp. 1685-1690, 1999.

[11] H.Sirtra Ramirez, "Harmonic Response of Variable-Structure-Controlled Van der Pol Oscillators," *IEEE Transactions on Circuits and Systems,*Vol. CAS-34, No.1, pp. 103-106, 1987

[12] R. Olfati-Saber,"Exponential ϵ-tracking and ϵ-stabilization of second-order nonholonomic SE(2) vehicles using dynamic state feedback," *American Control Conference,*Vol. 5, No.1, pp. 3961 - 3967, 2002.

[13] A. Sinha and H. Yurtseven, "Nonlinear adaptive tracking and stabilization of a robot manipulator," *Industrial Electronics, Control, and Instrumentation,*Vol. 3, pp. 1814 - 1818, 1993.

[14] Horacio J. Marquez, *Nonlinear Control Systems Analysis and Design,* New York: John Wiley & Sons, Inc., 2003.

[15] Khoo,Micheal C.K., *Physiological control systems,* IEEE Press,2000.

[16] Dragoslav D. Siljak, *Nonlinear System,* New York: John Wiley & Sons, Inc., 1969.

[17] J. J. Slotine and W. Li, *Applied Nonlinear Control,* London: Prentice-Hall, 1991.

[18] R. Su and N. Kermiche, "Learning control for a class of nonlinear systems," *IEEE International Symposium on Intelligent Control,* pp. 582-585, 1989.

[19] Karl. J. Åstrom and Bjiörn.Wittenmark, *Adaptive Control,* Reading,Massachusetts Menlo Park,Addison-Wesley Longman,1995.

[20] John S. Bay, *Fundamentals of Linear State Space Systems,* McGraw-Hill Companies, 1999.

[21] M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design,* New York: John Wiley & Sons, Inc., 1995.

[22] M. Krstic and H. Deng, *Stabilization of Nonlinear Uncertain Systems,* Springer-Verlag London, 1998.

[23] G. Chan and X. Yu, *Chaos Control,* Springer-Verlag London, 2003.

[24] D. Schröder (Ed.),*Intelligent Observer and Control Design for Nonlinear Systems,* Springer-Verlag London, 2000.

[25] J. Hogan (Ed.), *Nonlinear dynamics and chaos : where do we go from here?,* Bristol ; Philadelphia : Institute of Physics, 2003.

[26] A.A.J.Lefeber, *(Adaptive) Control of chaotic and robot systems via bounded feedback control,* Master thesis, 1996.