

DECISION-MAKER TRADE-OFFS IN  
MULTIPLE RESPONSE SURFACE OPTIMIZATION

by

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## ABSTRACT

The focus of this dissertation is on improving decision-maker trade-offs and the development of a new constrained methodology for multiple response surface optimization. There are three key components of the research: development of the necessary conditions and assumptions associated with constrained multiple response surface optimization methodologies; development of a new constrained multiple response surface methodology; and demonstration of the new method.

The necessary conditions for and assumptions associated with constrained multiple response surface optimization methods were identified and found to be less restrictive than requirements previously described in the literature. The conditions and assumptions required for a constrained method to find the most preferred non-dominated solution are to generate non-dominated solutions and to generate solutions consistent with decision-maker preferences among the response objectives. Additionally, if a Lagrangian constrained method is used, the preservation of convexity is required in order to be able to generate all non-dominated solutions. The conditions required for constrained methods are significantly fewer than those required for combined methods.

Most of the existing constrained methodologies do not incorporate any provision for a decision-maker to explicitly determine the relative importance of the multiple objectives. Research into the larger area of multi-criteria decision-making identified the interactive surrogate worth trade-off algorithm as a potential methodology that would provide that capability in multiple response surface optimization problems. The ISWT algorithm uses an  $\epsilon$ -constraint formulation to guarantee a non-dominated solution, and then interacts with the decision-maker

after each iteration to determine the preference of the decision-maker in trading-off the value of the primary response for an increase in value of a secondary response. The current research modified the ISWT algorithm to develop a new constrained multiple response surface methodology that explicitly accounts for decision-maker preferences. The new Modified ISWT (MISWT) method maintains the essence of the original method while taking advantage of the specific properties of multiple response surface problems to simplify the application of the method. The MISWT is an accessible computer-based implementation of the ISWT.

Five test problems from the multiple response surface optimization literature were used to demonstrate the new methodology. It was shown that this methodology can handle a variety of types and numbers of responses and independent variables. Furthermore, it was demonstrated that the methodology can be successful using a priori information from the decision-maker about bounds or targets or can use the extreme values obtained from the region of operability. In all cases, the methodology explicitly considered decision-maker preferences and provided non-dominated solutions. The contribution of this method is the removal of implicit assumptions and includes the decision-maker in explicit trade-offs among multiple objectives or responses.

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## LIST OF ACRONYMS/ABBREVIATIONS

BSRD	Bias-Specified Robust Design
DEA	Data Envelopment Analysis
DMUs	Decision Making Units
EVM	Extreme Values Method
GLM	Generalized Linear Models
GRG	Generalized Reduced Gradient
ISWT	Interactive Surrogate Worth Trade-Off
KKT	Karush-Kuhn-Tucker
LM	Lagrange Multiplier(s)
MBE	Minimum Bias Estimator
MCDM	Multi-Criteria Decision-Making
MDO	Multidisciplinary Design Optimization
MISWT	Modified Interactive Surrogate Worth Trade-Off
MRSM	Multiple Response Surface Methodology
MRSO	Multiple Response Surface Optimization
MSE	Mean Square Error
RPD	Robust Parameter Design
RSM	Response Surface Methodology
SWT	Surrogate Worth Trade-Off
VOP	Vector Optimization Problem

# CHAPTER ONE: INTRODUCTION

## Response Surface Methodology

Response Surface Methodology (RSM) was first introduced by Box and Wilson (1951) to address problems where the goal is to determine optimum settings in an experimental environment. RSM is often used in the design of new products and processes. RSM utilizes experimental design, search methods, model fitting, and optimization to find the optimal levels of design factors that minimize or maximize one or more response variables with the minimum number of experiments.

In general, response surface methodology can be outlined as follows:

- Phase One:
  1. An appropriate experimental design is selected to conduct an initial screening experiment.
  2. A first-order model is fit from the results of the screening experiment, usually by the method of least squares.
    - A first-order response function is of the form:
$$\hat{y} = b_0 + b_1x_1 + \dots + b_nx_n$$
, where  $x_i$  are the factors and  $b_i$  are the coefficients of the factors.
  3. A search algorithm such as steepest ascent is used to find the direction for improvement.
  4. The first three steps are repeated until no further improvement is sought.

- Phase Two:
  5. If needed, additional observations are made through experimental design in order to fit a higher order model.
  6. The response(s) is fitted using a model building technique such as regression.
    - A second-order response function is of the form:
 
$$\hat{y} = b_0 + b_1x_1 + \dots + b_nx_n + b_{11}x_1^2 + b_{12}x_1x_2 + \dots + b_{mm}x_n^2$$
, where  $x_i$  are the factors and  $b_i$  are the coefficients of the factors for  $n$  factors. This can also be in matrix form,  $\hat{\mathbf{y}} = \mathbf{X}\mathbf{b}$ , where  $\hat{\mathbf{y}} = [\hat{y}_1, \hat{y}_2, \dots, \hat{y}_m]'$  the vector of  $m$  responses;  $\mathbf{X}$  is a  $m \times p$  matrix of the factor settings where  $p = (n + 1)(n + 2) / 2$ ; and  $\mathbf{b}$  is a vector of the estimates of the coefficients.
  7. The response(s) is optimized to determine the optimal level of the design factors.

### **Multiple Response Surface Methodology**

When there are two or more responses, the procedure is often referred to as Multiple Response Surface Methodology (MRSM). MRSM is a subset of the larger field of multi-criteria decision-making. The focus of this research is on response surface methodologies that are able to solve problems with two or more responses.

The optimization of problems with multiple responses can be classified into one of two categories. The first, which will be defined as the “combined” approach, is an optimization of a single response which contains all of the response objectives. The second, which will be defined

as the “constrained” approach, optimizes a primary response while treating the other responses as constraints. Osborne (1997) called these two approaches MRSM1 and MRSM2, respectively.

The formulation of a typical combined model is as follows:

$$\min \sum_{i=1}^m w_i \hat{y}_i(x_1, \dots, x_n), \text{ where } w_i \geq 0 \text{ for all } i = 1, \dots, m \text{ responses and } \sum_{i=1}^m w_i = 1.$$

Subject to  $x \in R$ , where  $R$  is the experimental region.

The formulation of a typical constrained model is as follows:

$$\min \hat{y}_j(x_1, \dots, x_n)$$

Subject to  $l_i \leq \hat{y}_i \leq u_i$ , for all  $i = 1, \dots, m, i \neq j$

$x \in R$ , where  $j \in \{1, \dots, m\}$ ,  $u_i$  are upper bounds and  $l_i$  are lower bounds for the responses  $i \neq j$  and  $R$  is the experimental region.

An example of a combined response surface method is used to provide an illustrative overview. Derringer and Suich (1980) described the development of a tire tread compound. There are four response variables ( $y_1$ =PICO abrasion index;  $y_2$ =200% modulus;  $y_3$ =elongation at break;  $y_4$ =hardness) and three design factors ( $x_1$ =hydrated silica level;  $x_2$ =silane coupling agent level;  $x_3$ =sulfur level). The example begins at the beginning of Phase Two which assumes that the experimenter had already conducted screening experiments and utilized search methods to find the region of interest.

A rotatable, central composite design with six center points was used. Quadratic functions were fitted for each of the four response variables as follows:

$$\hat{y}_1 = 139.2 + 16.3x_1 + 17.7x_2 + 10.8x_3 - 3.9x_1^2 - 3.4x_2^2 - 1.6x_3^2 + 5.1x_1x_2 + 7.1x_1x_3 + 7.9x_2x_3$$

$$\hat{y}_2 = 1250.6 + 265.1x_1 + 243.7x_2 + 134.8x_3 - 73.7x_1^2 - 112.6x_2^2 + 192.8x_3^2 + 69.4x_1x_2 + 94.1x_1x_3 + 104.4x_2x_3$$

$$\hat{y}_3 = 400.2 - 98.5x_1 - 31.2x_2 - 73.0x_3 + 7.9x_1^2 + 16.7x_2^2 + 0.8x_3^2 + 8.9x_1x_2 + 6.3x_1x_3 + 1.3x_2x_3$$

$$\hat{y}_4 = 68.9 - 1.4x_1 + 4.3x_2 + 1.6x_3 + 1.5x_1^2 + 0.1x_2^2 - 0.3x_3^2 - 1.6x_1x_2 + 0.1x_1x_3 - 0.3x_2x_3$$

For illustration, the desirability function approach by Derringer and Suich (1980) was used to simultaneously optimize the responses given the following goals:

$$\max \hat{y}_1 \text{ with the constraint } 120 < \hat{y}_1 < 170$$

$$\max \hat{y}_2 \text{ with the constraint } 1000 < \hat{y}_2 < 1300$$

$$\hat{y}_3 = 500 \text{ with the constraint } 400 < \hat{y}_3 < 600$$

$$\hat{y}_4 = 67.5 \text{ with the constraint } 60 < \hat{y}_4 < 75$$

The desirability values for each response were calculated by the following:

$$d_i = \begin{cases} 0 & \hat{y}_i \leq l_i \\ \left[ \frac{\hat{y}_i - l_i}{u_i - l_i} \right]^r & l_i < \hat{y}_i < u_i \\ 1 & \hat{y}_i \geq u_i \end{cases}$$

for  $\hat{y}_1$  and  $\hat{y}_2$  given the upper ( $u_i$ ) and lower ( $l_i$ ) bounds given above and

$$d_i = \begin{cases} \left[ \frac{\hat{y}_i - l_i}{t_i - l_i} \right]^s & l_i \leq \hat{y}_i \leq t_i \\ \left[ \frac{\hat{y}_i - u_i}{t_i - u_i} \right]^t & t_i < \hat{y}_i < u_i \\ 0 & \hat{y}_i < l_i, \hat{y}_i > u_i \end{cases}$$



for  $\hat{y}_3$  and  $\hat{y}_4$  given the upper ( $u_i$ ) and lower ( $l_i$ ) bounds and target ( $t_i$ ) given above. The desirabilities were combined using the geometric mean and equal weights of importance to the responses were assumed. The resulting combined objective function was as follows: Maximize  $D$  where  $D = (d_1 * d_2 * d_3 * d_4)^{\frac{1}{m}}$ , where  $m = 4$  for the four response variables. The objective function was optimized using the Design Expert software. The nonlinear algorithm used by Design Expert is a Nelder-Mead search method. The solution was found to be:  $x_1=-0.04$ ;  $x_2=0.16$ ;  $x_3=-0.90$ ;  $y_1=129.363$ ;  $y_2=1300$ ;  $y_3=465.605$ ;  $y_4=68.0133$  for a predicted desirability  $D=0.582$ . A sample contour plot and response surface plot are shown below.

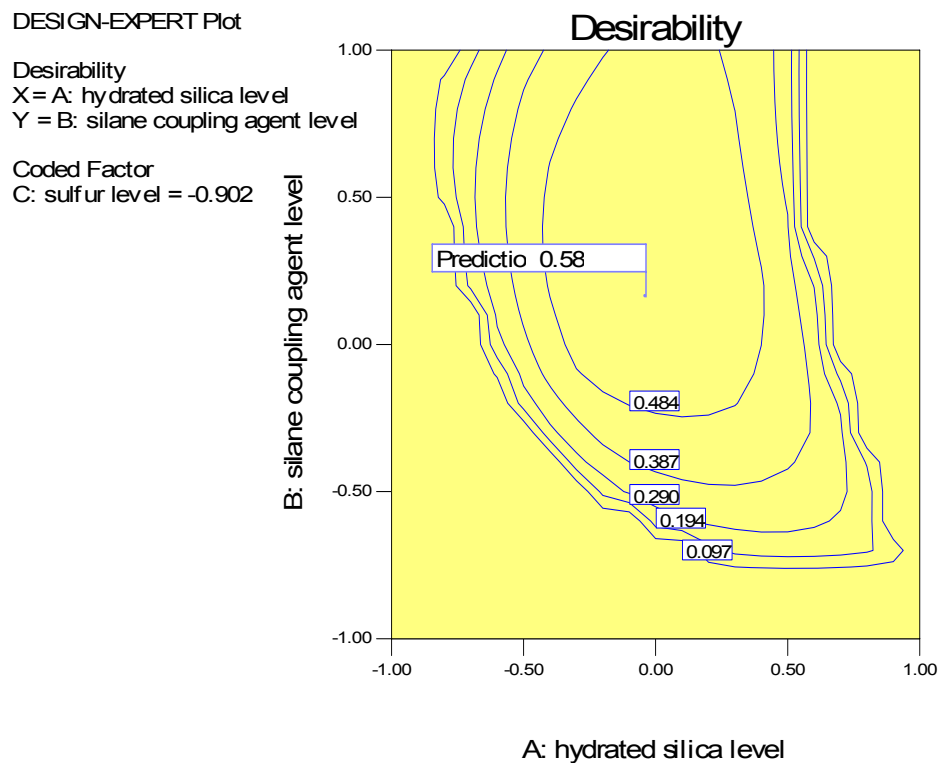


Figure 1-1 Sample Contour Plot

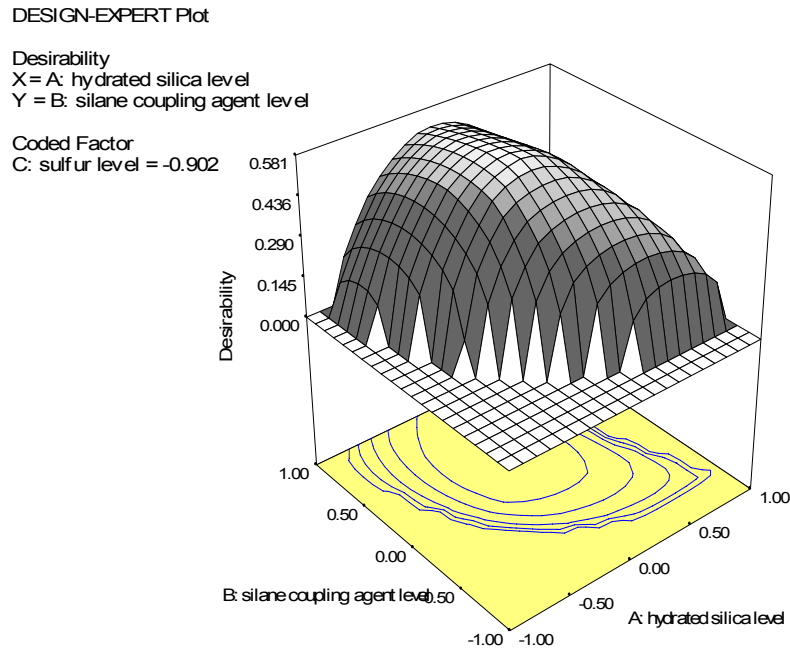


Figure 1-2 Sample Response Surface Plot

### Notations and Definitions

The section below provides general definitions found in response surface methodology and multi-criteria decision-making literature and the notations that will be utilized throughout this document.

*Definition:* A response variable is the performance measure or quality characteristic of interest to the decision-maker or, in other words, the dependent variable of the response function.

Notation:  $y_j$ , where there are  $m$  responses of interest.

*Definition:* Design factors are the other variables that are believed to impact the response variable. These are the independent variables.

Notation:  $x_i$ , where there are  $n$  factors that can be controlled in the experimental design.

*Definition: Region of operability* is the space defined by the independent variables.

*Definition: Steepest ascent* is a first-order search method used to find the direction of improvement.

*Definition: Region of interest*, also known as the *region of experimentation*, is a subregion of the region of operability usually cuboidal (square) or spherical near the optimum.

Notation:  $R$

*Definition: Pareto-optimal solutions* are those in which further improvement in one objective cannot be made without detriment to at least one other objective. A decision vector  $\mathbf{x}^* \in S$  is Pareto optimal if there does not exist another  $\mathbf{x} \in S$  such that  $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$  for all  $i = 1, \dots, m$  and  $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$  for at least one index  $j$ . An objective vector is Pareto optimal if the corresponding decision vector is Pareto optimal (Miettinen 2001). Pareto-optimal solutions may also be referred to as “noninferior, efficient, and non-dominated” interchangeably (Chankong and Haimes 1983).

*Pareto-optimal or non-dominated solutions* may also be classified as those alternatives that are not dominated by another alternative where *dominated* is defined as:  $\mathbf{x}_1$  dominates  $\mathbf{x}_2$  if and only if  $v(f(\mathbf{x}_1)) \geq v(f(\mathbf{x}_2))$ , where  $v$  is the value function (Chankong and Haimes 1983).

*Definition: A weakly Pareto optimal* vector is one where there does not exist any other vector for which all the components are better. The Pareto optimal set is a subset of the weakly Pareto optimal set (Miettinen 1999).

*Definition: Non-differentiable functions* are functions where there are points for which a first derivative cannot be calculated.

*Definition: Robustness* is a measure of the solution's ability to remain unaffected by small, but deliberate, variations in parameters.

*Definition: Bias* is the error associated with parameter estimates.

*Definition: Implicit preferences* are preferences among the response functions that are embedded in the optimization procedure.

*Definition: Explicit preferences* are preferences among the response functions that are expressed by the decision-maker.

*Definition: No-preference* methods do not take into consideration preferences or opinions of the decision-maker.

*Definition: A posteriori* methods generate the set of Pareto optimal solutions from which a decision-maker chooses the most preferred.

*Definition: A priori* methods require the decision-maker to provide preferences prior to conducting the analysis.

*Definition: Interactive* methods are a repetitive process of finding a solution, soliciting decision-maker input, and revising the process to obtain a new solution or set of solutions until an acceptable solution is found.

*Definition: A trade-off* is the ratio of the change in the values of two objective functions as one increases and the other decreases.

*Definition: Indifference curves* indicate alternative solutions which the decision-maker finds equally desirable.

*Definition: The marginal rate of substitution* is the change in the value of one objective function that the decision-maker feels offsets a one-unit change in the value of another function, with all other objectives remaining constant.

*Definition: Satisficing decision-making* means that the decision-maker is satisfied reaching certain aspiration levels for each objective rather than minimizing or maximizing any general value function.

*Definition: A value function* is an order-preserving real-valued function that serves to compare various levels of different attributes or responses indirectly.

*Definition: Reference point* is a vector consisting of aspiration levels desirable to the decision-maker.

*Definition: Local optima* are optimal solutions within a subregion of the region of operability.

*Definition: Global optima* are non-dominated solutions in the region of operability.

*Definition: Ideal objective vectors ( $z^*$ )* represent the values of the minimization of each objective function individually.

*Definition: Scalarization* converts problems into a single or multiple single objective optimization problems that have a real-valued objective function (i.e., scalarizing function).

## **Applications**

The scope of areas utilizing response surface methodologies has expanded in recent years beyond its initial use of finding optimum settings for a set of parameters for new product development. Multiple response surface methods have been used in applications such as quality

control (Simpson and Keats 1995; Carlyle, Montgomery, and Runger 2000), ergonomic designs (Ben-Gal and Bukchin 2002), simulation models (Rees, Clayton, and Taylor 1985; Meidt and Bauer 1992; Ankenman and McDaniel 1996; Gearhart and Wang 2001; Safizadeh 2002; Yang, Kuo, and Chou 2005), structural reliability (Kaymaz and McMahon 2005), machinery performance (Suresh, Venkateswara Rao, and Deshmukh 2002), health services (Facer and Muller 2003), and multidisciplinary design optimization problems (Rodriguez, Perez, Padmanabhan, and Renaud 2001; van Keulen and Vervenne 2004; Wang 2004; Youn and Choi 2004; Jianjiang, Renbin, and Yifang 2005).

### **Research Objectives**

With the expanding use of response surface methodology as just discussed, there continues to be a need to improve the formulation and understanding of multiple response surface methods. Chapter Two provides an overview of the available multiple response surface methods as well as recent topics in the literature. The largest gap identified in the multiple response surface literature was the lack of methods that explicitly take into consideration decision-maker preferences and allow for trade-offs among the responses.

Most MRSM approaches use implicit weighting schemes rather than explicit trade-offs between responses. A few utilize a posteriori approaches where multiple alternative optima are generated and then the decision-maker chooses among the solutions. However, the solutions provided to the decision-maker may or may not be representative of the set of non-dominated solutions. For example, convexity is required for all non-dominated solutions to be found using combined methods. More recent methods have looked at the ability to generate alternatives by

changing the values of the constraints, the values of the weights, or the allowable over or under achievement of the target. As indicated by Osborne (1997), in order to determine the most preferred solution, decision-maker preferences must be considered. However, no current methods explicitly allow the decision-maker to make trade-offs among the responses.

Chapter Three explores the broader area of multi-criteria decision-making. Interactive methods provide a platform for decision-makers to provide input throughout the optimization process. The interactive surrogate worth trade-off (ISWT) method is identified for potential application to MRSM problems. The method begins with the  $\epsilon$ -constraint method by having the analyst select one objective to optimize and provide upper bounds on the other objectives. The problem is solved and a Pareto optimal solution is presented to the decision-maker. The decision-maker then conducts a “worth assessment” to determine how (much) the decision-maker would like to make a trade-off between the primary response and each secondary response where the value of the primary response decreases by the value of the Lagrange multiplier for a one unit increase in value of the secondary response. The worth values are used to update the right-hand-side of the secondary responses and then the problem is re-optimized. The process continues until the decision-maker is satisfied with the solution.

The advantages to interactive methods are that they require less information from the decision-maker up front, the decision-maker is involved throughout the process, and the information can be obtained incrementally. These methods require fewer assumptions than a priori methods but many of these procedures are more complicated and can be time consuming for the decision-maker.

The identified area for further research is incorporating decision-maker trade-offs and the development of a new constrained multiple response surface optimization methodology.

Specifically, the research develops an interactive method for multiple response surface problems to explicitly allow the decision-maker to conduct trade-offs in order to guarantee a preferred solution.

Chapter Four develops the conditions and assumptions required for constrained methods and compares the results to those identified by Osborne (1997) for combined methods. The current constrained methods are evaluated against these conditions. Recommended characteristics for multi-criteria methodologies are also identified.

Chapter Five develops a new multiple response surface method based on the advantages of interactive methods and  $\epsilon$ -constraint methodology. Using the particular characteristics of multiple response surface problems, a methodology is developed that maintains the essence of ISWT while simplifying the process. Furthermore, the  $\epsilon$ -constraint foundation of the methodology guarantees that all non-dominated solutions can be found and convexity is not required.

Chapter Six demonstrates the use of the new Modified ISWT (MISWT) methodology on five problems from the literature and Chapter Seven provides conclusions and recommended areas for future research.



## **CHAPTER TWO: MULTIPLE RESPONSE SURFACE OPTIMIZATION METHODS**

This dissertation is a follow-up to the work completed by Osborne (1997). Osborne developed a structure to categorize and evaluate the different multiple response surface methodologies. The work focused on combined MRSM methodologies where necessary conditions and assumptions were developed as well as a new combined weighted methodology. The current work focuses on constrained methods.

Osborne's dissertation provided a good summary of the multiple response surface methodologies conducted prior to 1996. Myers (1999) provided a perspective on the current status and future directions of response surface methodologies. Myers identified the need to move to robust designs rather than optimal solutions including the possible use of sequential and Bayesian designs. Additionally, identified future areas of research included generalized linear models, multiple response models, nonparametric and semiparametric models, and models that can respond to problems when there are restrictions in randomization. The articles referenced in this chapter reflect the recent progress seen in these areas.

In 2004, a literature survey was published of response surface methodology work conducted since 1989, when the last comprehensive review was conducted (Myers, Montgomery, Vining, Borror, and Kowalski 2004). The article provided an extensive bibliography. Primary areas included in the review are robust parameter design (RPD), experimental designs including methods of evaluation, multiple responses, and generalized linear models (GLM).

The review of the state of the art in MRSM in this chapter will focus on response surface methodology activities since 1996 as well as include relevant seminal work conducted previous to this time period. In order to provide a broad view of the recent direction and questions being

addressed in response surface, recent work related to the early steps of the process will be discussed. However, the majority of the review will focus on the optimization step. Finally, issues in trade-offs and weighting methodologies will be examined in more detail.

### **Experimental Design Techniques**

Many authors have focused on developing experimental designs that are robust to environmental conditions or noise factors. Other factors include understanding interaction effects and minimizing experimental runs. Possible designs included running experiments at only the extreme environmental conditions, split plot designs, mixed resolution designs, and optimal non-regular experimental designs (Box and Jones 1992; Lucas 1994; Loepky, Bingham, and Sitter 2006). Furthermore, Myers (1999) recommended the use of Bayesian and multi-stage experimental designs to address robustness.

In the multidisciplinary design optimization (MDO) research, one main concern was the expense for large scale problems, so response surface experimental design and sampling techniques are being used to approximate the surface under study (Rodriguez, et al. 2001). Furthermore, Youn and Choi (2004) introduced the selective interaction sampling method in order to provide better information concerning the main effects and interactions needed for reliability analyses.

Some authors have focused on designing criteria to aid in the choice of an appropriate experimental design including using Bayesian criteria or progressive lattice sampling so that prior knowledge from earlier experiments can be used to determine future designs (Box and Jones 1992; Gilmour and Mead 2003; Romero, Swiler, and Giunta 2004; Loepky, Bingham,

and Sitter 2006). Another criterion recommended was the use of word-length patterns to rank designs (Loeppky, Bingham, and Sitter 2006). Huang and Fan (2004) reviewed minimizing the mean squared error (MSE) bias versus a minimum bias estimator (MBE) criterion where the MSE criterion considers the prediction error due to process variability and the bias due to model misspecification whereas the MBE criterion minimizes bias by reducing the number of terms included in the model. They showed that MBE depends only on the number of experimental runs and center replicates. Goupy (2005) developed a robustness quality ratio to compare the quality of various experimental designs.

### **Search Methods**

Response surface methodology has historically advocated the use of the path of steepest ascent as the search technique. An adapted steepest ascent method was recommended to address some of the issues with the steepest ascent methodology: scale dependency and step size selection (Kleijnen, den Hertog, and Angun 2004).

Dvorak (2000) introduced two new methods “to move from an initial region of experimentation to a new point or region where all of the response values are improved.” The hyperplane method transforms a multiple response non-linear problem into a linear solution set and searches the set to find solution points that represent the union for the solution sets for all responses. The sensitivity analysis method was applied to two different optimization formulations: 1) combined response with a distant constraint and 2) a minimized distance function with response constraints. The results of the sensitivity analysis help the analyst to

identify changes to make to the constraints that find a direction of improvement for all or some of the responses.

In situations involving simulations, a scatter-search method was recommended as part of the iterative process of defining the response surface and a smaller search region was recommended to reduce bias and variance (Safizadeh 2002; Yang, Kuo, and Chou 2005).

### **Model Fitting**

Box, Hunter, MacGregor, and Erjavec (1973) identified potential problems with fitting multiple response models. Specifically, they were concerned about three types of dependencies: correlation among the errors, linear dependence among the responses, and linear dependencies in the data often caused by normalizing the data. They recommended an eigenvalue-eigenvector analysis prior to fitting the model to identify these dependency issues. Furthermore, they stated that dependencies in the responses provide meaningless parameter estimates.

Recent work has focused on using methods other than regression to fit the response surface model. Particularly, as the use of response surface methodologies has expanded to other problems and disciplines beyond the traditional design problem, the types of methods used to fit response surface models has also expanded. Draper and Pukelsheim (2003) recommended the use of canonical reduction to determine the form and shape of the surface and to potentially simplify the model. For fitting structural reliability problems, Kaymaz and McMahon (2005) used a weighted regression methodology. Wang (2004) discussed the use of multiquadric approximation which allows for interpolation of the available data to fit multidisciplinary design optimization problems. For use with progressive or iterative experimental designs, three possible

fitting schemes were suggested: piecewise finite-element interpolation, polynomial regression, and kriging (Romero, Swiler, and Giunta 2004). Facer and Muller (2003) discussed the use of nonparametric regression with kernel smoothers to fit response surfaces.

Other recent work has included the development of new criteria to determine the best fit response model. Gearhart and Wang (2001) developed two Bayesian measures, under known and unknown uncertainty, of the fit of the response surface model to the original simulation model that take into consideration the number and robustness of the fitting parameters.

### **Optimization Methodologies**

Many overviews of optimization methodologies have been written in recent years with focuses on multiple response surface optimization (Carlyle, Montgomery, and Runger 2000; Murphy, Tsui, and Allen 2005), alternatives for robust parameter design (Myers, Khuri, and Vining 1992), nonlinear multiobjective optimization (Miettinen 2001), and a graphical overview of multiple objective programming (Steuer 2001).

The subsections below focus on a variety of methods that have been proposed to solve multiple response surface problems. The formulations for the constrained approaches, translated to a consistent notation, are included. A table comparing the various optimization methodologies is included in the summary section.

## **Contour Plots**

Hoerl (1959) introduced a graphical method, called Ridge Analysis, whereby a single response is plotted and the peaks are examined to determine the optimum levels of the variables. Furthermore, this methodology can be used for both types of multiple response problems. The combined method is analyzed like a single response problem. The constrained method requires the additional step of superimposing the contour plots of each of the responses to find the optimal values of the variables. The number of responses that can be accommodated by the method in the combined methodology is theoretically limitless although in practicality it becomes very difficult to interpret as the number of responses increases. Furthermore, no more than two variables can easily be analyzed simultaneously. One advantage to this methodology is that it allows the decision-maker to view multiple peaks and determine the trade-offs that reflect their needs.

## **Robust Parameter Design**

RPD is a method introduced by Taguchi in the 1980s to design a process by selecting the proper levels of controllable factors such that the system is robust or insensitive to changes in levels of the noise factors. Taguchi's methodology to the RPD problem involved two orthogonal arrays: an inner array for the control variables and an outer array for the noise variables. Taguchi developed a performance criterion called signal-to-noise ratio that provides information about the mean and variance dependent on the goal. For response surface, Taguchi had three goals: the

smaller the better, the larger the better, and target is best (Myers, Vining, Giovannitti-Jensen, and Myers 1992)

Myers, Khuri, and Vining (1992) indicated in a review of response surface alternatives to robust parameter design that, “Taguchi’s most significant contribution is that he formalized a notion...that product variability must be a performance response.” They also provided five criticisms of robust parameter design: “(1) inefficiency of the signal-to-noise ratio, (2) lack of flexibility in modeling design variables, (3) lack of economy in experimental design plan, (4) preoccupation with optimization, and (5) no formal allowance for sequential experimentation.”

### **Dual Response**

Myers and Carter first introduced the theory of dual response systems in 1973. Dual response is a method by which a set of optimal parameter settings are found by optimizing a primary response subject to the constraint of a secondary response. A secondary constraint may be applied to keep the solution within a region of interest. There is no limit to the number of independent variables; however, the method is limited to only quadratic response functions. Lagrange multipliers are used to find the optimum values of the parameters (Myers and Carter 1973).

Vining and Myers (1990) utilized the dual response methodology developed by Myers and Carter to solve the three goals identified by Taguchi by achieving the mean response objective and minimizing variance. For the target is best goal, the primary response is to minimize the variance and the mean target is the secondary response. For the other two goals, the mean is the primary response while the variance is the secondary response. In this case,

several possible targets for the variance may be considered and then a compromise solution is selected. This technique addresses four of the five criticisms discussed above under Robust Parameter Design.

Box and Jones (1992) addressed the dual response problem by minimizing the weighted sum of the mean response, as a function of the target mean, and variance response.

### **Nonlinear Programming – Dual Response**

Del Castillo and Montgomery (1993) developed a nonlinear programming approach to solve the same Taguchi goals that Vining and Myers addressed using dual response methods as well as having the flexibility to solve other types of problems including mixture experiments. Given that the fitted responses are usually quadratic functions, the response surface problems are nonlinear. Del Castillo and Montgomery suggested using the generalized reduced gradient (GRG) algorithm to solve the problems. GRG was recommended because it is “one of the most robust” nonlinear methods, is known to work well in practice, and the termination point is feasible.

As an extension of this work and to address the problem of sampling error, Del Castillo (1996) presented the idea of finding confidence regions for quadratic responses or confidence cones for linear responses. For quadratic responses, the Lagrangian function is maximized for each response subject to the Karush-Kuhn-Tucker conditions utilizing the GRG algorithm. For linear responses, Box and Draper’s methodology of finding confidence cones is utilized. Then a primary response is optimized subject to the confidence regions and confidence cones found in the previous step. The primary function can be optimized at various alpha levels to develop a



sensitivity table. In cases where there are conflicting linear responses, a combined weighted function is developed for the primary responses that take into account the best directions for each response.

Further, Miro-Quesada and Del Castillo (2004a) and Miro-Quesada and Del Castillo (2004b) provided a revised form of the variance response for the dual response problem that takes into account both the variability created by the randomness of the noise factors and the uncertainty in the parameter estimates.

### **Target Variance Methods (Process-Bias Methods)**

Lin and Tu (1995) expanded on the dual response work of Vining and Myers by minimizing a MSE criterion. Although this criterion allows for some bias, it provides a trade-off of some disparity from the target with a smaller variance. Unlike the previous method, the MSE method does not restrict models to be polynomials nor the constraints to be equality constraints. Copeland and Nelson (1996) proposed a variation to the MSE criterion, a direct function minimization, that places a restriction as to how far the mean can deviate from the target and recommended using the simplex search procedure of Nelder and Mead (1965). Myers, Brenneman, and Myers (2005) minimized a MSE criterion for a generalized linear model where a quality measure is a nonnormal response. Shin and Cho (2005) addressed the “target-is-best” goal by developing a bias-specified robust design (BSRD) model that, similar to Lin and Tu, allows for some process bias. Specifically, the model uses a nonlinear programming, Lagrangian method that allows the decision-maker to specify an upper bound on the amount that the mean can vary from the target. The authors reference the ability of the method to generate non-

dominated partial trade-off rates using the dual associated with the Lagrangian function, however, they do not demonstrate or discuss the use of this functionality.

### **Desirability Functions**

The desirability function was first introduced by Harrington in 1965. The decision-maker or user specifies the minimum, maximum, and/or preferred values acceptable for each of the original fitted response functions as well as the weights that determine the rate at which the desirability of a function increases or decreases over the range of acceptable values. This method transforms each of the fitted functions into a desirability value and then the desirability values for each response are combined using the geometric mean into a single objective to be optimized. Derringer and Suich (1980) utilized a more generalized method to achieve the individual desirability values than Harrington to allow greater flexibility in the creation of the desirability values.

However, this method creates non-differentiable points that do not allow the problem to be optimized using more efficient nonlinear programming techniques. Del Castillo, Montgomery, and McCarville (1996) overcame this by using a local polynomial approximation at each of the non-differentiable points and modified the desirability value functions accordingly. They suggested optimizing the revised desirability function using the GRG algorithm as they did with the dual response problem above. Although the formulation of the desirability values differ in the two approaches, and both allow the decision-maker to provide input into the relative importance of the different responses, neither approach allows the decision-maker to make any explicit trade-offs among the response functions.

Similarly to the desirability index, Plante (2001) proposed using the geometric mean of the individual process capability measures (Cpk or Cpm) as a criterion.

### **Generalized Distance Measure**

Khuri and Conlon (1981) developed a method to simultaneously optimize multiple responses. The method calls for the elimination of dependencies among the responses and then uses least squares to fit the models. Each response is individually optimized and if all responses produce the same set of solutions then the problem is solved. Otherwise, a distance measure is developed that takes into account the optimal values of each of the responses found in the previous step as well as their variability and optimized to minimize the distance.

### **Loss Functions**

The purpose behind loss functions is to capture the economic loss due to deviations of one or more quality characteristics from target where minimizing the loss function will simultaneously optimize the mean and variances. Pignatiello, Ames, and Vining all proposed approaches that utilize quadratic loss functions where Pignatiello and Vining incorporated the correlation structure while Ames, et al. did not (Pignatiello 1993; Ames, Mattucci, MacDonald, Szonyi, and Hawkins 1997; Vining 1998). Vining's (1998) work expanded on that of Pignatiello and included not only the correlation structure but also the quality of the predictions. Khuri and Conlon's approach was found to be a special case of this mean squared error loss function approach. Romano, Varetto, and Vicario (2004) expanded on Vining's work by adding an

additional term that takes into consideration producer loss which is the cost to fix the standard deviation of the noise factor. Kuhnt and Erdbrugge (2004) proposed the use of a sequence of possible weights assigned to each response rather than defining a cost matrix. Joint optimization plots are then used to visualize the optimal design settings for a sequence of weights.

### **Minimal Satisfaction**

Kim and Lin (2006) proposed an alternative to the generalized distance measure for simultaneously optimizing a set of responses including both the location and dispersion for all responses. The degree of satisfaction with any response decreases the further away the solution is from the target or the larger the standard deviation becomes. This approach is a maximin approach where the objective is to maximize the overall minimal degree of satisfaction with respect to all of the responses. Four variations of the approach were also discussed: 1) the responses are alternatives and the maximum satisfied response is selected, 2) consideration of predictive capability where the responses are weighted by the goodness of fit of the models, 3) assigning different weights to the responses, and 4) maximize the sum of the degrees of satisfaction.

### **Goal Programming**

Rees, Clayton, and Taylor (1985) expanded on the framework established by Biles and Swain in the late 70s of combining the techniques of response surface methodology and goal programming by introducing the use of the satisficing algorithm. This methodology requires that

the decision-maker rank the priority of the responses and provide a target or minimum level of attainment required for each response or goal. The satisficing algorithm, utilizing the path of steepest ascent, first optimizes the highest ranking response. Then the algorithm systematically attempts to find the optimal solution for each of the other responses within the region of interest that satisfies all higher priority goals without violating a higher goal.

Tang and Xu (2002) utilized a goal programming approach to simultaneously optimize the mean and standard deviation of a response, similar to the approach by Khuri and Conlon (1981); however the approach by Tang and Xu is more flexible. This method requires the decision-maker to provide a target for the mean and standard deviation as well as weights to indicate the importance of each response. A combined objective is developed that minimizes the weighted under or over-achievement of the targets subject to the constraints. Weights and targets can be defined that enable earlier methods to be special cases of this goal programming approach (Vining and Myers 1990; Del Castillo and Montgomery 1993; Lin and Tu 1995; Copeland and Nelson 1996; Ames, et al. 1997). Xu, Lin, Tang, and Xie (2004) discussed the use of this methodology in a more general sense for multiple responses using a minimax objective where the approach is to minimize the maximum weighted deviation from target.

### **Pareto Front Methods**

Pareto Front methods are those which aim to find multiple alternatives rather than one optimal solution.

Koksoy and Doganaksoy (2003) utilized a standard nonlinear multiobjective program, NIMBUS, to solve the “smaller-the-better” and the “larger-the-better” Taguchi goals by

simultaneously optimizing the mean and variance. This method provides many Pareto optimal solutions rather than the one solution that would be garnered using a combined weighted approach. Tang and Xu (2002) also provided for alternative Pareto solutions if the user varies the weights.

Tsui, Goh, Xie, and Loy (2001) introduced the idea of finding non-dominated fronts for each response individually and then looking at the intersection of those fronts to find optimal solutions or to find a compromise solution by looking at the solutions identified in the non-dominated fronts.

Liao (2004) used neural networks to find the signal-to-noise ratios for all combinations of factors and levels, also called decision making units (DMUs), including estimating incomplete data. Data envelopment analysis (DEA) is then used to find the optimal combination of factors/levels by ranking the DMUs by their relative efficiencies. DEA uses the ratio of inputs to outputs to create an efficient frontier that identifies the efficient and inefficient alternatives. Thus, a non-dominated solution is guaranteed (Mollaghasemi and Pet-Edwards 1997).

### **Weighted Multiple-Response Optimization Methodology**

Osborne (1997) provided an approach that utilized the necessary conditions for optimizing a combined methodology which include: generation of non-dominated solutions, convexity requirement, mutual preferential independence, identification of a preference structure, satisfaction of monotonicity of preference, use of value functions, requirement of similar magnitudes of the response objectives, and elicitation of decision-maker specified weights. The proposed approach is defined as a linear additive value function that can be maximized using

nonlinear programming techniques and ensures a non-dominated solution. Gheware (2003) recommended a change to the value function limit calculation such that the value function cannot be negative.

### **Randomly Valued – Independent Variable Method**

Dvorak (2000) proposed a methodology that considers the probability distribution of an independent noise variable. The method replaces the independent variable in the response surface model with the expected value of the probability distribution of that independent variable. Thus, the objective is to optimize the response to the mean of the randomly valued independent variable.

### **Genetic Algorithms**

Khoo and Chen (2001) developed a model that combines response surface methodology with a genetic algorithm to optimize the responses. Genetic algorithms have been around since the 1960s and are based on the idea of survival of the fittest. Each chromosome has a set of possible parameter values and the algorithm begins with a randomly generated set of chromosomes that are then evaluated against the objective function. The user provides the minimum and maximum values for each of the variables along with the genetic algorithm inputs of population size, probabilities of crossovers and mutations, and the number of iterations. The user can also specify weights to indicate the relative importance of the responses as well as to normalize the variables. Penalties are used to keep the responses as close to the targets as

possible. Thus the objective function simultaneously optimizes all responses by minimizing the sum of the weighted minimums minus the sum of the weighted maximums plus the sum of the weighted penalties.

An application of the combined RSM and genetic algorithm approach was used to find the optimal parameter values for the minimum and maximum values generated by the algorithm for a single objective function (Suresh, Venkateswara Rao, and Deshmukh 2002).

### **Special Cases**

Optimization methods for two special cases are discussed below: stochastic problems and multidisciplinary design optimization problems.

#### **Stochastic Models**

Six methods were described for solving the multiple response stochastic problem under the basic premise that the stochastic problem is transformed into a deterministic problem that can be solved by familiar optimization techniques. The first is the E-model which is essentially a minimization of the weighted average of the mean and variance responses. The second, the V-model, minimizes the variance of the objective function. The third, the P-model, maximizes the probability that the objective function is smaller than some fixed value. The fourth is a minimax objective. The fifth uses a lexicographic goal programming approach. The last one is based on minimizing the distances. Many of these methods align with response surface methods already in use for deterministic problems (Diaz-Garcia, Ramos-Quiroga, and Cabrera-Vicencio 2005).



## **Multidisciplinary Design Optimization Methods**

MDO “is a systematic methodology that synergistically exploits the interaction among disparate disciplines (also known as subsystems) to improve performance, low cost, and shorten product design cycle” (Jianjiang, Renbin, and Yifang 2005). Some of the issues concerning current MDO methods include the accumulation of uncertainties, the inability to use discrete design variables at the system level, difficulties with convergence, and being computationally expensive (Jianjiang, Renbin, and Yifang 2005). In order to address these issues, researchers were integrating response surface methods into their MDO methods. Artificial neural networks, gradient-enhanced response surfaces, fidelity response surface approximations, and hybrid mean value methods are some of the methods being considered (Rodriguez, et al. 2001; van Keulen and Vervenne 2004; Youn and Choi 2004; Jianjiang, Renbin, and Yifang 2005). Furthermore, Jones (2001) developed a taxonomy of optimization methods in multidisciplinary problems based on whether a one-stage or two-stage search method was used and whether or not interpolation was used.

### **Summary of Optimization Methodologies**

Figure 2-1 below provides a diagram that illustrates the evolution of the multiple response surface methods discussed in this section. Further, Table 2-1 provides a comparison of the methods utilizing the categories outlined by Osborne (1997). The table contains information about the number of responses, types of responses (mean and/or variance), type of approach (overlay, combined, constrained), articulation of preferences (implicit or explicit indication of

goals and weights), and assumptions (e.g. independence). Furthermore, the comments field indicates any explicit consideration of bias and robustness, weighting schemes, or issues that have been identified with the approach.

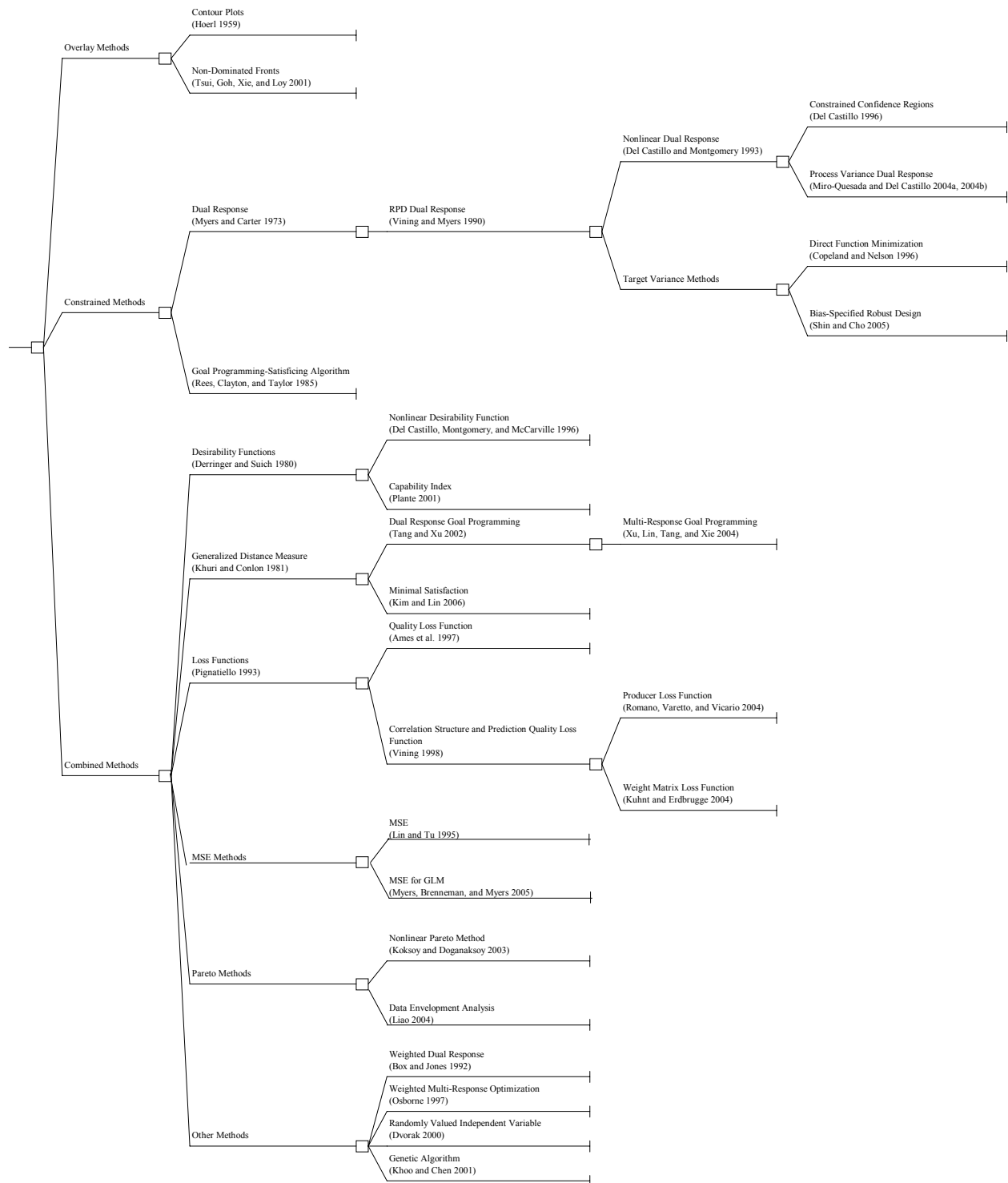


Figure 2-1 Multiple Response Surface Optimization Methods Tree

Table 2-1 Multiple Response Surface Optimization Methods Categorization

#	Method	Author	Year	# of Responses	Type of Responses	Type of Approach	Articulation of Preference	Assumptions	Comments
1	Contour Plots	Hoerl	1959	2 or more	$\mu$	Overlay	Explicit goals; Implicit weights		Not feasible for large number of variables
2	Non-Dominated Fronts	Tsui, Goh, Xie, Loy	2001	2	$\mu, \sigma^2$	Overlay	Explicit goals; Explicit weights		Does not require re-optimization when weights change
3	Dual Response	Myers and Carter	1973	2	$\mu$	Constrained	Explicit goals; Implicit weights	All response functions are quadratic	
4	RPD Dual Response	Vining and Myers	1990	2	$\mu, \sigma^2$	Constrained	Explicit goals; Implicit weights (choose compromise combination)	All response functions are quadratic	May lead to stationary point that is not a local optima
5	Nonlinear Dual Response	Del Castillo and Montgomery	1993	2 or 2 or more	$\mu, \sigma^2$ $\mu$	Constrained	Explicit goals; Implicit weights	All response functions are nonlinear	All solutions are feasible but may not be a global optimum
6	Constrained Confidence Regions	Del Castillo	1996	2 or 2 or more	$\mu, \sigma^2$ $\mu$	Constrained	Implicit goals; Implicit weights (more weight given to responses that exclude more directions)	Independence among responses	Addresses sampling error issue; use sensitivity analysis to find multiple optimal solutions for different levels of $\alpha$ and for some values of $\alpha$ there may be no feasible solutions

#	Method	Author	Year	# of Responses	Type of Responses	Type of Approach	Articulation of Preference	Assumptions	Comments
7	Process Variance Dual Response	Miro-Quesada and Del Castillo	2004	2	$\mu, \sigma^2$	Constrained	Explicit goals; Implicit weights	Independence between the noise factors and parameter estimates	Considers multiple sources of variability; robust to both noise factor and parameter estimation uncertainty; utilizes an unbiased estimator of the variance; calculates 95% prediction intervals
8	Direct Function Minimization	Copeland and Nelson	1996	2	$\mu, \sigma^2$	Constrained	Explicit goals; Implicit weights		Considers both bias and variance
9	Bias-Specified Robust Design	Shin and Cho	2005	2	$\mu, \sigma$	Constrained	Explicit goals; Implicit weights		Potential to provide tradeoffs between process bias and variability
10	Goal Programming Satisficing Algorithm	Rees, Clayton, and Taylor	1985	2 or more	$\mu$	Constrained	Explicit goals; Implicit weights (prioritized responses)		
11	Desirability Functions	Derringer and Suich	1980	2 or more	$\mu$	Combined	Explicit goals; Implicit weights (r, s, t)	Responses are continuous functions	Use contour plots to assess sensitivity; contains non-differentiable points

#	Method	Author	Year	# of Responses	Type of Responses	Type of Approach	Articulation of Preference	Assumptions	Comments
12	Nonlinear Desirability Function	Del Castillo, Montgomery, and McCarville	1996	2 or more	$\mu$	Combined	Explicit goals; Implicit weights (use breakpoints)		Use regression estimates to account for correlation between the responses
13	Capability Index	Plante	2001	2 or more	$\mu, \sigma^2$ as a Cp measure	Combined	Explicit goals; Implicit weights		Uses nonconformance costs to weight the measures
14	Generalized Distance Measure	Khuri and Conlon	1981	2 or more	$\mu$	Combined	Implicit goals; Implicit weights	Independence among responses; All responses have same set of controllable factors	Accounts for covariances and prediction uncertainty
15	Dual Response Goal Programming	Tang and Xu	2002	2	$\mu, \sigma^2$	Combined	Explicit goals; Explicit weights		Sensitivity analysis through experimenting with different convex combinations of weights
16	Multi-Response Goal Programming	Xu, Lin, Tang, and Xie	2004	2 or more	$\mu$	Combined	Explicit goals; Explicit weights	Responses must be twice differentiable	Considers predictive capability in the weighting scheme. Allows different sets of input variables for the r responses. Only considers the response with the maximum weighted deviation from target.

#	Method	Author	Year	# of Responses	Type of Responses	Type of Approach	Articulation of Preference	Assumptions	Comments
17	Minimal Satisfaction	Kim and Lin	2006	2 or more	$\mu$ and $\sigma^2$	Combined	Explicit goals; Implicit weights (a variation allows for explicit weights)		Uses best fit models; robust to dependencies among the responses
18	Loss Functions	Pignatiello	1993	2 or more	$\mu$	Combined	Explicit goals; Explicit weights (cost matrix)		Considers covariance matrix
19	Quality Loss Function	Ames, Mattucci, MacDonald, Szonyi, and Hawkins	1997	2 or more	$\mu$	Combined	Explicit goals; Explicit weights		Robust to variability in the process variables
20	Correlation Structure and Prediction Quality Loss Function	Vining	1998	2 or more	$\mu$	Combined	Explicit goals; Explicit weights (cost matrix)		Considers the covariance structure and prediction quality
21	Producer Loss Function	Romano, Varetto, and Vicario	2004	2 or more	$\mu, \sigma^2$	Combined	Explicit goals; Explicit weights (how limits are chosen or the cost coefficients)		Considers correlation structure and prediction uncertainty
22	Weight Matrix Loss Function	Kuhnt and Erdbrugge	2004	2 or more	$\mu$ and $\sigma^2$	Combined	Explicit goals; Explicit weights (Sets of weight matrices)		Utilizes a set of weight matrices to find alternative optima

#	Method	Author	Year	# of Responses	Type of Responses	Type of Approach	Articulation of Preference	Assumptions	Comments
23	MSE	Lin and Tu	1995	2	$\mu, \sigma^2$	Combined	Explicit goals; Explicit weights	Independence among responses	Considers both bias and variance; fit of the models. Does not limit how far the secondary response can vary from target
24	MSE for GLM	Myers, Brenneman, and Myers	2005	2	$\mu, \sigma^2$	Combined	Explicit goals; Implicit weights		
25	Nonlinear Pareto Method	Koksoy and Doganaksoy	2003	2	$\mu, \sigma$	Combined/ Simultaneous	Explicit goals; Implicit weights		Generates a set of Pareto optimal solutions using NIMBUS; ignores parameter estimation uncertainty
26	Data Envelopment Analysis	Liao	2004	2 or more	SN ratio	Combined	Explicit goals; Implicit weights		Neural network used to calculate SN ratios
27	Weighted Dual Response	Box and Jones	1992	2	$\mu, \sigma^2$	Combined	Explicit goals; Explicit weights		Dependent upon the magnitude and choice of the target mean
28	Weighted Multi-Response Optimization	Osborne	1997	2 or more	$\mu$	Combined	Explicit goals; Explicit weights		
29	Randomly Valued Independent Variable	Dvorak	2000	2	$\mu, \sigma^2$	Combined	Explicit goals; Implicit weights	Probability distributions are independent	Considers the distribution of the random variables
30	Genetic Algorithm	Khoo and Chen	2001	2 or more	$\mu$	Combined	Explicit goals; Explicit weights		



## Summary of Formulations for Constrained Optimization Methodologies

The formulations for the constrained approaches, translated to a consistent notation, for the above methodologies are shown in Table 2-2. Additionally, each method below has an additional constraint to restrict the region of interest.

Table 2-2 Constrained Optimization Method Formulations

Method	General case:	Target is best:	Larger is better:	Smaller is better:
Dual Response (Myers and Carter 1973):	optimize $\hat{y}_1$ subject to $y_2 = T$			
RPD Dual Response (Vining and Myers 1990)		minimize $\hat{\sigma}$ subject to $\hat{\mu} = T$	maximize $\hat{\mu}$ subject to $\hat{\sigma} = T$	minimize $\hat{\mu}$ subject to $\hat{\sigma} = T$
Nonlinear Dual Response (Del Castillo and Montgomery 1993)	optimize $\hat{y}_k$ subject to $l_j \leq \hat{y}_j \leq u_j$ for all $j = 1, \dots, m, j \neq k$	minimize $\hat{\sigma}$ subject to $\hat{\mu} = T$	maximize $\hat{\mu}$ subject to $\hat{\sigma} = T$	minimize $\hat{\mu}$ subject to $\hat{\sigma} = T$
Direct Function Minimization (Copeland and Nelson 1996)		minimize $\hat{\sigma}$ subject to $(\hat{\mu} - T)^2 \leq \Delta^2$	minimize $-\hat{\mu}$ subject to $\hat{\sigma} \leq T$	minimize $\hat{\mu}$ subject to $\hat{\sigma} \leq T$
Constrained Confidence Regions (Del Castillo 1996)	optimize $\hat{y}_k$ subject to $0 \leq \delta_j' V_j^{-1} \delta_j \leq k s_j^2 F_{\alpha_j, k, n-p}$ for all $j = 1, \dots, m$			
Process Variance Dual Response (Miro-Quesada and Del Castillo 2004b)		minimize $\text{var}_{z,\beta}(\hat{y}(x, z))$ subject to $\hat{\mu} = T$		
Bias-Specified Robust Design (Shin and Cho 2005)		minimize $\hat{\sigma}$ subject to $ \hat{\mu} - T  \leq \varepsilon$		

The final constrained method formulation is the goal programming satisficing algorithm (Rees, Clayton, and Taylor 1985) described below.

Step 1: The decision-maker prioritizes the objectives and sets goals for each response.

Step 2: Achievement of the goal for the highest ranking response is attempted. If achieved, then continue to Step 3.

Step 3: The goal for the next highest response is attempted, but not at the expense of achieving the goal for higher priority responses (i.e., search for response two is conducted in the region for which the optimum for response 1 was found). This continues for each response.

### **Trade-Offs and Weighting**

Some combined approaches, such as the desirability function, use implicit weighting schemes where a level of desirability for a response is established but no explicit trade-off between responses is determined. For the desirability function method, users provide values ( $r$ ,  $s$ , and  $t$ ) used in the transformation of the responses into desirabilities where large values indicate that values of the response closer to the extremes or target are most desirable whereas small values imply less importance for that response. A value of 1 indicates a linear transformation (Derringer and Suich 1980). A modified approach utilizes the breakpoints of the desirability functions set at different heights or targets for each of the responses to indicate priorities of the responses (Del Castillo, Montgomery, and McCarville 1996). The approaches do not permit the decision-maker to identify the relative importance of the response functions.

A few approaches identify alternative optima from which a decision-maker then looks at the trade-offs, for example between the mean and standard deviation of a response, and chooses an alternative. Some methods generate the alternatives by changing the values of the constraints (Koksoy and Doganaksoy 2003; Shin and Cho 2005) while others change the values of the

weights of the objectives or allowable over or under achievement to target (Jin, Okabe, and Sendhoff 2001; Tang and Xu 2002; Kuhnt and Erdbrugge 2004; Xu, et al. 2004).

Gheware (2003) explored the impact of concavity on solutions derived from multiple response surface optimization methods for two objective problems by varying the weights of each objective function. It was found that regardless of convexity that weighting methods generate only non-dominated solutions points. However, whether or not a complete set of Pareto optimal solutions were found was dependent on the sign of values in the Hessian matrix for second partial derivatives (test for convexity).

Jeong, Kim, and Chang (2005) proposed a systematic method for generating the weighting factors for a weighted MSE criteria. The method utilizes the decision-maker's expressed preferences utilizing pairwise comparisons of the bias and variance terms at each condition (set of factor levels). The first step consists of calculating the weights followed by resolving any inconsistencies. Jin, Okabe, and Sendhoff (2001) used an evolutionary methodology that systematically changes the weights in a multiobjective problem so that a Pareto front can be found.

Although, as discussed above, some work with respect to weighting and trade-offs has been conducted with respect to multiple response surface methods, much of the work that has been conducted is in the more general area of multiattribute or multi-criteria decision-making. Value trade-offs are two consequences with different measures for which the decision-maker is indifferent. Keeney (2002) described 12 common mistakes that when made result in a solution that does not represent the true intentions of the decision-maker. Miettinen (2001) provided an overview of nonlinear multiobjective optimization methods and discusses the role of the decision-maker in the various approaches. Chankong and Haimes (1983) provided an overview

of decision making theory including utility theory, assessment methods, noninteractive and interactive multiobjective programming methods, and the surrogate worth trade-off method.

Anderson and Hobbs (2002) introduced a procedure to eliminate scale compatibility bias in the use of trade-off weights for multiattribute analysis. Choo and Wedley (2004) compared 18 different methods for eliciting pairwise preference values when there is a finite number of alternatives. Jessop (2004) examined three methods for justifying a chosen alternative of a finite set of alternatives by showing insensitivity to the weighting of the attributes: 1) maximize the number of feasible scenarios for which the solution is best, 2) maximize the difference between the best and second best solutions, and 3) minimize the different between the chosen alternative and the best alternatives under different weighting scenarios.

An overview of the larger area of multi-criteria decision-making as it relates to preference methodologies and optimization techniques is provided in the next chapter.

### **Summary of Future MRSM Research**

Below are areas identified by the authors discussed in the literature review as topics for future research:

1. Impact of uncertainty on results:
  - a. How sensitive are the results to small changes in the input parameters (Kim and Lin 2006).
  - b. Impact of errors in the estimation of the covariance matrix (Miro-Quesada and Del Castillo 2004b).

- c. Problems with multiple correlated properties and incorporating estimation uncertainty (Koksoy and Doganaksoy 2003).
  - d. Multiple performance measure problems with correlated errors (Plante 2001).
  - e. Additional work on robustness (Myers, et al. 2004).
2. Determination of cost coefficients:
- a. How to select cost coefficients (Romano, Varetto, and Vicario 2004).
  - b. Investigate other ways to account for cost trade-offs (Plante 2001).
3. Extensions of current methods:
- a. Extension of fuzziness to the neural network and data envelopment analysis approach (Liao 2004).
  - b. Extend aggregation based method to more than two objectives (Jin, Okabe, and Sendhoff 2001).
  - c. Extend Peterson's approach to include noise variables and residual error distributions with heavier tails (Peterson 2004).
  - d. Computational improvements in algorithms used for parameter estimation and minimizing loss function and the replacement of loss function with other criteria (Kuhnt and Erdbrugge 2004).
  - e. New mutation scheme for multiple response problems (Khoo and Chen 2001).
4. Weighting and trade-offs:
- a. Consider how error and biases impact value function analyses (Anderson and Hobbs 2002).
  - b. More investigation of weighting methods (Anderson and Hobbs 2002).

- c. Optimization of multiple means and standard deviations, interpretability of results, and assessing weights (Osborne 1997).
5. New methodologies:
- a. Development of user-friendly nonlinear multiobjective programming methods and visual illustrations (Miettinen 2001).
  - b. Cases where responses are categorical (Kuhnt and Erdbrugge 2004).
  - c. Methods that can handle non-continuous variables (Yang, Kuo, and Chou 2005).
  - d. Methods for nonnormal and GLM models, semiparametric and nonparametric RSM (Myers 1999).
  - e. Efficient designs for larger problems (Myers, et al. 2004).
  - f. Methods for complex response objectives (Osborne 1997).
  - g. Methodologies when monotonicity of preference and mutual preferential independence do not hold (Osborne 1997).
6. Further theoretical development:
- a. Constrained methods conditions and assumptions including whether non-dominated solutions are obtained (Osborne 1997).
  - b. What types of problems should be solved with what methods (Osborne 1997).
  - c. Identifying the region of interest and what happens when regions for different responses are remote (Osborne 1997).

As demonstrated by the long list of topics, there exists numerous opportunities to contribute to the area of multiple response surface methods. The main topic of development in this research is the issue of trade-offs and incorporating decision-maker preferences. A new constrained method is developed and thus, Osborne's topic of determining the conditions and

assumptions necessary for constrained methods are also addressed. In order to have a better understanding of the issues with trade-offs among multiple responses the larger area of multi-criteria decision-making (MCDM) is explored in the next chapter. Current methods in MCDM are explored for potential application to multiple response surface problems.

## **CHAPTER THREE: MULTI-CRITERIA DECISION-MAKING**

Multiple response surface methods belong to a larger area of study called multi-criteria decision-making (MCDM). As indicated in Chapter Two, little work has been done within MRSM to explicitly address the role of the decision-maker and the impact that decision-maker preferences have on the solution to the problem. The first section below reviews the role of decision-makers in multiobjective problems and the second provides a high-level review of multiobjective optimization procedures within the area of MCDM.

### **Decision-Maker Preferences**

An integral part of any problem with multiple attributes, criteria, or responses is the preferences of the decision-maker(s). Kenney (2002) indicated that “making value trade-offs is one of the most difficult elements faced in important decisions.”

### **Role of the Decision-Maker**

The role of the decision-maker can be classified into four categories: no-preference, a posteriori, a priori, and interactive (Miettinen 1999). No-preference methods do not take into consideration preferences or opinions of the decision-maker. A posteriori methods generate a set of Pareto optimal solutions from which a decision-maker chooses the most preferred. A priori methods require the decision-maker to provide preferences prior to conducting the analysis. Interactive methods are a repetitive process of finding a solution, soliciting decision-maker input,



and revising the problem to obtain a new solution or set of solutions until an acceptable solution is found.

No-preference methods are often used when the decision-maker is satisfied with an optimal solution without any additional specifications. These types of methods often use simplifying assumptions such as equally weighted objectives or some global criterion which results in a single solution for the decision-maker to reject or accept. The disadvantage of this method is that the additional information to determine the most preferred solution for a particular decision-maker is not available. With a posteriori methods multiple optimal solutions are generated. Some methods have the capability to generate all potential alternative solutions while others are only able to generate a subset of all Pareto optimal solutions. The decision-maker then chooses from the set of alternative solutions. Disadvantages to this approach are that it can be computationally expensive to generate the multiple solutions and decision-makers may have difficulty making the trade-offs to select the most preferred solution as the size of the solution set increases. With a priori methods, the decision-maker provides input prior to the analysis. The disadvantage with this method is that the decision-maker may not know what the set of realistic expectations may be. The advantages of the interactive methods over the previous methods are that the decision-maker does not have to identify the preference structure up front and the entire set of Pareto optimal solutions does not need to be generated. Rather the decision-maker preferences can evolve throughout the process and provide direction to the generation of the next subset of Pareto optimal solutions (Miettinen 1999).

## **Methods of Soliciting Preference Structures**

The purpose of soliciting preferences is to be able to assess the relative importance of attributes or in the case of multiobjective problems the relative importance of the objectives in order to find the best compromise solution. Trading off an increased value in one objective for a lower value in another is subjective and requires the preference of the decision-maker (Clemen and Reilly 2001). Assessment methods can be classified into three categories: direct, sequential elimination, and spatial proximity (Chankong and Haimes 1983).

### **Direct Assessment**

The direct approach quantifies the decision-maker's preference up-front and reduces the problem to an a priori evaluation approach discussed below. This approach, including the multiattribute utility function method, develops a global representation of decision-maker preferences. There are five steps in the use of the multiattribute utility function method: 1) verify the existence of a value or utility function which in practice is assumed to be true, 2) find a suitable form of the function, 3) construct the component functions, 4) develop scaling constants, and 5) check for consistency and perform the final analysis (Chankong and Haimes 1983).

Suitable forms of the function include additive, quasi-additive, and multiplicative. Conditions of independence are tested to determine the appropriate form (Chankong and Haimes 1983). An additive utility function involves developing a utility score for each objective and then utilizing weights to demonstrate the relative importance of the objectives and then summing

the weighted utility scores (Clemen and Reilly 2001). A multiplicative utility function is the product of utility functions and their scaling constant (Chankong and Haimes 1983).

There are multiple methods for developing the weights used in individual utility functions. Proportional scoring has the decision-maker rate each attribute on a scale from 0 to 100; normalization can take place if needed. The midpoint method requires the decision-maker to identify the value at which one would have to give up the same amount of both attributes if you were to move in one direction or the other (Chankong and Haimes 1983). Another method is to price out the value of one additional unit of the attribute and then convert it to a weight. Ratios are used often when attributes are not quantitative. In this method, the decision-maker determines that one attribute/objective is twice as important as another attribute. Using swing weights involves the development of a table that compares scenarios for the worst outcomes for each objective (benchmark), and then swings the best outcome for one objective at a time. The scenarios are then ranked and rated on a scale of 1 to 100. The rates are then converted into weights. Swing weights have the advantage that the range of allowable values is considered. The lottery weight method is the development of the probability that makes the decision-maker indifferent between the lottery (best on all attributes) and the sure thing (best on one, worst on all others). By changing the item included as best in the “sure thing,” probabilities are developed for each attribute (Clemen and Reilly 2001). Once the individual utility functions are developed and the correct form is identified, scaling constants are determined. Many of the same approaches listed above to elicit weights can be used to determine the scaling constants.

## **Sequential Elimination**

Two examples of sequential elimination are included: lexicographical ordering and the ELECTRE method. Lexicographical ordering has the decision-maker rank order the importance of the objectives. The most important criteria is used in the first screening step and alternatives yielding the most preferred solution is kept and the other discarded. Next the second most important criteria is used to find the most preferred solution among this subset. This continues until only one solution is left or until all criteria have been used.

ELECTRE method can be used to make the final decision, classify alternatives as “rejected” or “nonrejected”, or classify alternatives into indifference classes and rank these classes given a small set of alternatives or solutions. The decision-maker indicates the relative importance of the objectives. Then for each pair of alternatives, outranking relations are developed. Alternatives that are outranked by another are eliminated. A new outranking relation can then be constructed to further narrow down this subset of alternatives.

## **Spatial Proximity**

An indifference curve depicts a set of alternatives (solutions) for which the decision-maker is indifferent. Multiple indifference curves can be created which demonstrate varying levels of utilities. The slope of the curve is the trade-off rate to be discussed in the following section (Clemen and Reilly 2001). This method provides graphical representation of two attributes/objectives at a time.

## Trade-Offs and Marginal Rates of Substitution

A trade-off is the ratio of the change in the values of two objective functions as one increases and the other decreases. The marginal rate of substitution is the change in the value of one objective function that the decision-maker feels offsets a one-unit change in the value of another objective, with all other objectives remaining constant. Weighting coefficients, used in many combined optimization methods, are the same as marginal rates of substitution when provided by the decision-maker. Miettinen (1999) provides a distinction between trade-offs and marginal rates of substitutions where trade-offs are mathematically defined compared to marginal rates of substitution that reflect the preferences of the decision-maker. Keeney (2002) uses the terminology “value trade-offs” to “define how much must be gained in the achievement of one objective to compensate for a lesser achievement on a different objective.”

Marginal rates of substitution can be used to create indifference curves. Indifference curves indicate alternative solutions for which the decision-maker finds equally desirable. Assume that given two objectives, X and Y, the decision-maker is “indifferent to the solutions  $(x_1, y_2)$  and  $(x_2, y_1)$ ”. The interpretation of this indifference is as follows:

- An increase in X from  $x_1$  to  $x_2$  is compensated by a decrease in Y from  $y_2$  to  $y_1$
- An increase in Y from  $y_1$  to  $y_2$  is compensated by a decrease in X from  $x_2$  to  $x_1$
- An increase in X from  $x_1$  to  $x_2$  is equal to an increase in Y from  $y_1$  to  $y_2$
- A decrease in X from  $x_2$  to  $x_1$  is equal to a decrease in Y from  $y_2$  to  $y_1$  (Keeney 2002).

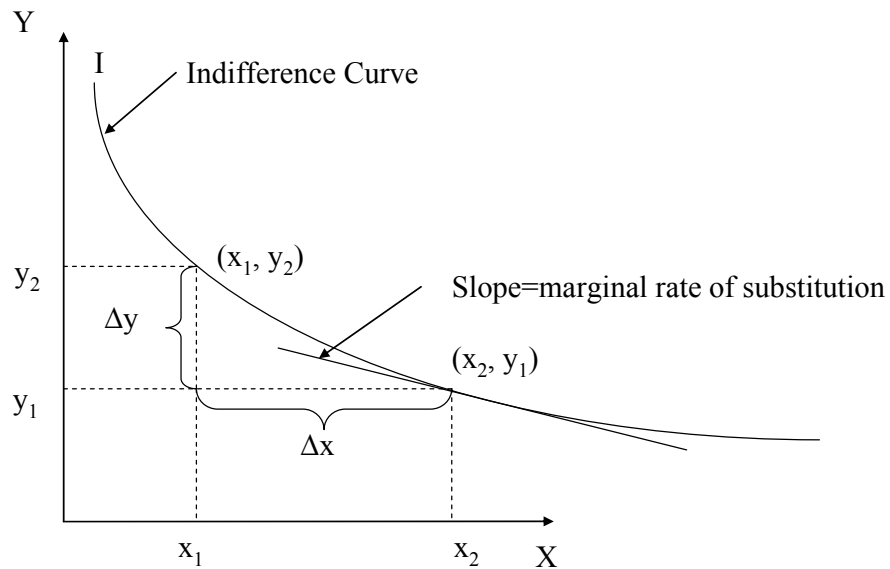


Figure 3-1 Marginal Rate of Substitution

### **Issues in Value Trade-Offs**

Keeney (2002) described twelve common mistakes in determining value trade-offs that when made result in a solution that does not represent the true intentions of the decision-maker. The first set of mistakes relate to not understanding the decision context including purpose and whose perspective the trade-offs should represent and using proxy objectives that do not adequately represent the decision problem. The second set of mistakes relate to measures and consequences including not having measures for consequences, using inadequate measures, and not knowing what the measures represent. The third set of mistakes relates to using inappropriate questions or assumptions to determine trade-offs including willingness to swap solutions, using screening criteria as absolute value judgments (i.e., hard limits), and using a “safety” factor to provide conservative trade-offs that bias the values. The fourth set relates to

mistakes made when calculating trade-offs including trying to calculate “correct” trade-offs without decision-maker input, determining trade-off rates without considering the range of possibilities, and extrapolating a trade-off over the entire range of consequences which assumes linearity. The final mistake discussed the failure to use consistency checks to identify bias and random errors and assess reasonableness.

Keeney (2002) provides a four step process for making good trade-offs. Step 1 is to appropriately understand the decision problem. Step 2 is to clearly define the objectives with clear measures, consequences, and allowable ranges. Step 3 is to find pairs of consequences (or solutions) for which the decision-maker is indifferent. Step 4 is to check the value trade-offs for reasonableness. The method developed in this research accomplishes Keeney’s four steps.

### **Decision Rules**

Chankong and Haimes (1983) defined two categories of decision rules. The first, the optimizing category, is a set of rules that provide a “best” alternative based on some criterion implied by the rule. The second, the satisficing category, is a set of rules that seeks a satisfactory alternative. The rules in this category may sacrifice optimality for simplicity, cost, and time. Generally, the alternatives are categorized as acceptable or unacceptable where the alternatives within the category are not differentiable by the decision-maker. In cases with a large number of alternatives, the satisficing rule may only look for an alternative where the objectives reach a certain aspiration level.

In problems with multiple responses, it is generally not possible to find one solution that would optimize all responses simultaneously. Therefore, solutions are optimal if one objective

cannot be improved without detriment to at least one other objective. This is what is meant by Pareto optimality. Most problems will have many (possibly infinite) Pareto optimal solutions. Somewhere in the process, the decision-maker's preferences must be taken into consideration to select one of the solutions in the Pareto optimal solution set as "best."

Satisficing decision making means that the decision-maker is satisfied reaching certain aspiration levels for each objective rather than minimizing or maximizing any general value function. Miettinen (1999) indicates that although in extreme cases the satisficing would not have to be Pareto optimal, he assumes throughout his book that the satisficing solution will always be within the Pareto optimal solution set (or at least weakly Pareto optimal).

### **Multiobjective Optimization Methods**

Miettinen (1999) provides a broad sample of multiobjective optimization methods. The sections below present MCDM optimization methods following Miettinen's structure of classifying the methods based on the role of the decision-maker. This section should not be seen as an exhaustive list of multi-criteria decision-making optimization methods and the reader is referred to the references included in Miettinen's text.

#### **No-Preference Methods**

Miettinen (1999) presented two no-preference methods of multiobjective optimization. The method of the global criterion minimizes the distance between a reference point and the feasible objective region where the analyst determines the reference point and metric to be used



to measure distance. For example, the  $L_p$  metric minimizes the sum of the objectives and the Tchebycheff metric returns the minimum of the maximum distances for all objectives. This method assumes that all functions are equally important using the scales of the individual functions. The second method, the multiobjective proximal bundle method, improves the values of all objectives simultaneously where the analyst must select a starting point from which all objectives can be improved.

### **A Posteriori Methods**

A posteriori methods generate a set of Pareto optimal solutions from which the decision-maker chooses a preferred solution. However, in many cases only one solution may be generated. Many multiple response surface methods fall into this category of MCDM methods.

The weighting method minimizes the weighted sum of the objectives where the weights are positive and sum to one. As such, this method combines the multiple objectives into a single objective to be optimized. A set of Pareto optimal solutions may be found by varying the values of the weights.

The  $\epsilon$ -constraint method optimizes one objective subject to treating the other objectives as constraints where the analyst selects an upper bound for each constraint. All Pareto optimal solutions can be found by varying the upper bounds of the constraint objective functions and by changing out the objective function to be treated as the primary response with each of the constraint responses.

The hybrid method combines elements of the weighting method and  $\epsilon$ -constraint method. The method minimizes the weighted sum of the objectives but subject to constraints on the objectives.

The method of weighted metrics is a variation of the method of global criterion where the metrics are weighted for each objective. Thus, this method minimizes the weighted distance between a reference point and the feasible objective region. The analyst must select the weights for each of the objectives, as well as the reference point and the metric (e.g.,  $L_p$ , Tchebycheff). Alternative solutions can be found by varying the weights. Achievement scalarizing functions can be used in place of the metrics used in the global criterion or method of weighted metrics. This option ensures weakly Pareto optimal solutions and allows alternative optima to be found by changing the reference point rather than the weighting coefficients.

### **A Priori Methods**

A priori methods capture the preferences of the decision-maker before beginning the optimization method. The value function method maximizes the value of the function defined by the mathematical formulation of the decision-maker's preferences. Once defined, the value function can be solved using any single objective optimization method. For example, the weighting method can be seen as a special case of a value function when utilizing decision-maker preferences.

Lexicographic ordering requires the decision-maker to rank order the objectives. The most important objective function is minimized and if a unique solution is not found then

subsequent objectives are optimized adding the previous objective as a constraint at its optimum value until a unique solution is found or all functions have been minimized.

Goal programming requires the decision-maker to define aspiration levels for each objective and the method minimizes the deviations from the aspiration levels. This method can be used with weighted methods or lexicographic ordering.

### **Interactive Methods**

Interactive methods provide a platform for decision-makers to provide input throughout the optimization process. The methods below differ in the optimization function and the amount of information required of the decision-maker.

The interactive surrogate worth trade-off (ISWT) method begins with the  $\epsilon$ -constraint method by having the analyst select one objective to optimize and provide bounds on the other objectives. The problem is solved and a Pareto optimal solution is presented to the decision-maker. The decision-maker then conducts a “worth assessment” to determine how (much) the decision-maker would like to make a trade-off between the primary response and each secondary response where the value of the primary response decreases by the value of the Lagrange multiplier for a one unit increase in value of the secondary response. An integer worth scale between -10 and 10 is used where 10 indicates a strong preference to make the trade-off, 0 indicates satisfaction with the current solution and the desire to make no trade-off and -10 indicates a strong preference to make the opposite trade-off. The worth values are used to update the right-hand-side of the secondary responses and then the problem is re-optimized. The process continues until the decision-maker is satisfied with the solution.

The Geoffrion-Dyer-Feinberg method maximizes a value function and requires the decision-maker to identify the reference function and then specify marginal rates of substitution between this function and the other objectives at the current solution point. The decision-maker also helps to determine the step size. The optimization is conducted iteratively with the decision-maker choosing the preferred solution among each set of solutions until the decision-maker wants to stop.

The sequential proxy optimization technique begins with the  $\epsilon$ -constraint method like the ISWT method. Then, like the Geoffrion-Dyer-Feinberg method, the decision-maker must specify the marginal rates of substitution. A proxy function and step size are determined. The optimization is conducted iteratively until the decision-maker is satisfied.

The Tchebycheff method minimizes the maximum weighted distance of the function from a “utopian objective” defined by the decision-maker where a utopian objective is a vector that represents values that are strictly better than every Pareto optimal solution.

The bounds on the weights are tightened to reduce the number of Pareto optimal solutions generated. The decision-maker chooses a most preferred objective vector among a subset of the generated ones at each iteration until a final solution is chosen.

The step method requires the decision-maker to be able to indicate functions that have acceptable values and those that have values that are too high. Then the weighted Tchebycheff problem is used to generate solutions. Then the decision-maker is asked to determine which of the objectives are satisfactory and relax the upper bounds on those objectives. The method repeats until the decision-maker is satisfied for all objectives.

The reference point method begins by providing the decision-maker with some information that provides a range of the Pareto optimal set. The decision-maker specifies a

reference point from which the achievement function is minimized. The solutions are provided to the decision-maker. If one of the solutions is satisfactory then the process stops, otherwise a new reference point is gathered and the process continues. An extension of this, the reference direction approach also projects the vector from the current iteration point to the reference point. This provides the decision-maker with more information to determine the next direction and also provides a wider part of the weakly Pareto optimal set to review. The reference direction method minimizes the computational effort by having the decision-maker determine the number of steps to be taken in the reference direction, thereby minimizing the number of alternatives that the decision-maker will review.

In the GUESS method the decision-maker specifies a reference point as well as upper or lower bounds to the objective functions. Then a function representing the minimum weighted deviation from the nadir objective vector is minimized, where the nadir objective vector contains the upper bounds of the Pareto optimal set, and the solution is presented to the decision-maker. If the solution is satisfactory the process stops, otherwise it is repeated with the decision-maker specifying new bounds and reference points.

The satisficing trade-off method begins by optimizing a scalarizing function and providing the solution to the decision-maker. The decision-maker reviews each objective and labels them as unacceptable, acceptable with the ability to relax, or acceptable as is. The decision-maker provides aspiration levels for the objectives to be improved. Then the modified scalarizing function is minimized and the process is repeated.

The light beam search method has the decision-maker identify bounds for each objective as well as to specify “indifference” thresholds. The achievement function is minimized and the solution as well as Pareto optimal neighbors are presented to the decision-maker. If one of the

alternatives are satisfactory then the process is stopped, otherwise the decision-maker can revise reference points or thresholds and the function is re-optimized. This method allows the decision-maker to “save” preferred solutions, explore other directions, and then select among the preferred solutions.

The NIMBUS method requires that at each solution point the decision-maker determines whether each objective should be decreased freely, decreased to a certain bound, satisfactory, increased to a certain bound, or changed freely. The function is minimized and the process repeats with input from the decision-maker until the decision-maker chooses a preferred solution.

### **Summary of MCDM Optimization Methods**

The following section summarizes the MCDM optimization methods discussed above and identifies similar methodologies that have been used for multiple response surface problems. The table indicates the decision-maker’s role, the type of method, and whether it can be considered a combined or constrained approach. From the table it can be seen that the interactive surrogate worth trade-off method and sequential proxy optimization technique are two constrained approaches that have not been used for multiple response surface problems.

Miettinen (1999) provides a decision tree diagram comparing the interactive methods and the reader is referred to this resource for further details.

Table 3-1 Multi-Criteria Decision-Making Optimization Methods Categorization

#	MCDM Method	DM Role	Type	Functional Types	Comparable Multiple Response Surface Methods
1	Global Criterion	No-Preference	Reference Point	Combined	Generalized Distance Measure (Khuri and Conlon 1981), MSE methods, Loss Function methods
2	Multiobjective Proximal Bundle	No-Preference	Simultaneous Improvement	Combined	
3	Weighting Method	A Posteriori	Weighted	Combined	Weighted Multi-Response Optimization (Osborne 1997); Weighted Dual Response (Box and Jones 1992)
4	$\epsilon$ -Constraint Method	A Posteriori	Constrained	Constrained	Dual Response (Myers and Carter 1973); RPD Dual Response (Vining and Myers 1990); Nonlinear Dual Response (Del Castillo and Montgomery 1993)
5	Hybrid Method	A Posteriori	Hybrid	Combined	
6	Method of Weighted Metrics	A Posteriori	Reference Point	Combined	
7	Achievement Scalarizing Function	A Posteriori	Reference Point	Combined	
8	Value Functions	A Priori	Weighted	Combined	Weighted Multi-Response Optimization (Osborne 1997); Desirability Functions (Derringer and Suich 1980)
9	Lexicographic Ordering	A Priori	Objective Ranking	Constrained	Goal Programming Satisficing Algorithm (Rees, Clayton, and Taylor 1985)
10	Goal Programming	A Priori	Aspiration Levels	Combined or Constrained	Dual Response Goal Programming (Tang and Xu 2002); Multi-Response Goal Programming (Xu, et al. 2004)
11	Interactive Surrogate Worth Trade-Off Method	Interactive	Objective Ranking	Constrained	
12	Geoffrion-Dyer-Feinberg Method	Interactive	Value Function	Combined	
13	Sequential Proxy Optimization Technique	Interactive	Value Function	Constrained	
14	Tchebycheff Method	Interactive	Weighted, Reference Point	Combined	
15	STEM Method	Interactive	Weighted, Reference Point	Combined	
16	Reference Point Method	Interactive	Reference Point	Combined	

#	MCDM Method	DM Role	Type	Functional Types	Comparable Multiple Response Surface Methods
17	Reference Direction Approach	Interactive	Achievement Function, Reference Point	Combined	
18	Reference Direction Method	Interactive	Reference Point	Combined	
19	GUESS Method	Interactive	Reference Point	Combined	
20	Satisficing Trade-Off Method	Interactive	Weighted, Reference Point	Combined	
21	Light Beam Search	Interactive	Achievement Function, Reference Point	Combined	
22	NIMBUS	Interactive	Simultaneous Improvement	Combined	Nonlinear Pareto Method (Koksoy and Doganaksoy 2003)



### **Area of Research**

The identified area of research for further development is constrained multiple response surface optimization methodologies and improving decision-maker trade-offs.

Osborne (1997) investigated the conditions and assumptions associated with combined MRSM methods and identified the need for parallel investigation into constrained methodologies. The majority of current multiple response surface methods utilize combined methodologies. The current constrained methods include the dual response method developed by Myers and Carter in 1973 and the evolution of additional dual response algorithms and target variance methods, and the goal programming satisficing algorithm developed by Rees, Clayton, and Taylor in 1985.

Only recently in the response surface literature has there been methods discussed for generating alternative optima for constrained methods from which a decision-maker then looks a posteriori and chooses an alternative. Some methods generate the alternatives by changing the values of the constraints (Koksoy and Doganaksoy 2003; Shin and Cho 2005) while others change the allowable over or under achievement to target (Jin, Okabe, and Sendhoff 2001; Tang and Xu 2002; Kuhnt and Erdbrugge 2004; Xu, et al. 2004). However, no methods have demonstrated the ability to explicitly allow the decision-maker to trade-off among the responses. It can be very difficult to correctly assess a decision-maker's preferences a priori. Most current methods require the ability to accurately develop the decision-makers value function or implicitly make assumptions about preferences, most often using evenly weighted responses.

Reviewing constrained methods found in the larger area of multi-criteria decision-making identified the interactive surrogate worth trade-off algorithm as one potential untapped MDCM methodology that has not been implemented in multiple response surface problems. ISWT uses an  $\epsilon$ -constraint methodology which guarantees non-dominated solutions and allows for the decision-maker to interactively make trade-offs among the responses.

Opportunity exists to contribute to the body of knowledge of constrained methods for multiple response surface optimization and improving the method and understanding of decision-maker trade-offs. The following three chapters develop and demonstrate a new approach to MRSM that provides explicit opportunities for a decision-maker to incorporate relative preferences for alternative solutions. In Chapter Four, conditions and assumptions associated with constrained multiple response optimization methodologies are developed and evaluated. In Chapter Five, a new constrained multiple response surface methodology that is based on the ISWT method is developed. In Chapter Six, the new constrained methodology is demonstrated using several diverse MRSM problems from the literature. A pictorial of the research methodology is provided in Figure 3-2.

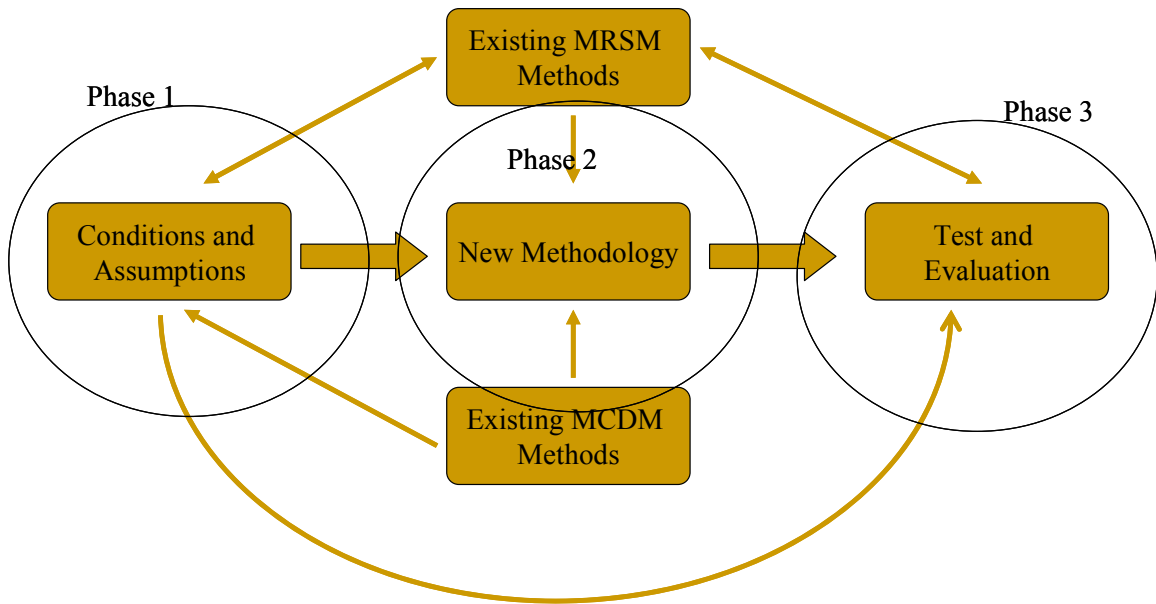


Figure 3-2 Research Methodology

## **CHAPTER FOUR: CONDITIONS AND ASSUMPTIONS FOR CONSTRAINED MULTIPLE RESPONSE SURFACE OPTIMIZATION METHODS**

Osborne, Armacost, and Malone (1996) described a partial set of necessary conditions for combined multiple response methods. Osborne (1997) found that independence between the factors, independence of the regression errors, independence between response functions, and uncorrelated response functions were not required for combined MRSM methodologies. It was found that monotonicity of preference, where for each response objective, an alternative having a smaller outcome is always preferred to a larger outcome, and mutual preferential independence, where the preference between outcomes is not dependent on the levels of the responses, are required. Furthermore, in order to find the most preferred solution using combined methods, the methodology needed to generate non-dominated solutions, preserve convexity, generate solutions consistent with the underlying preference structure of the response objective, account for different magnitudes in the response objectives, and generate solutions consistent with decision-maker preferences. Preserving convexity is a substantial requirement. When that does not occur, there is a “duality gap” in the objective space and combined methods will not generate all non-dominated solutions. A major challenge in using weighting methods is identifying and modeling the decision-maker’s preference structure.

After developing necessary conditions and assumptions for combined methods, Osborne (1997) identified the need for parallel investigation into constrained methodologies. This chapter develops the conditions and assumptions necessary for constrained methods, compares them to those required for combined methods, and evaluates the current constrained methods.

## **Development of Conditions and Assumptions for Constrained Methods**

Within the constrained multiple response surface methodologies literature, assumptions mentioned include all responses must be quadratic (Myers and Carter 1973; Vining and Myers 1990), all response functions are nonlinear (Del Castillo and Montgomery 1993), independence among the responses (Del Castillo 1996), and independence between the noise factors and parameter estimates (Miro-Quesada and Del Castillo 2004b). Concerns with some of the constrained methods are that the method may lead to a stationary point that is not a local optima (Vining and Myers 1990), that although solutions are feasible they may not be the global optimum (Del Castillo and Montgomery 1993), or that in the case of confidence regions that there may be no feasible solutions for some values of  $\alpha$  (Del Castillo 1996). This section will explore many of these issues along with those identified by Osborne for combined methods to determine their relevance for constrained methods.

Given the basic foundation of response surface methodologies the following assumptions apply to all MRSM methods and will carry throughout the rest of this research:

- Experimental designs are developed and conducted that contain the controllable factors and the response variables to be studied. As such, the same factors and factor settings are used in the development of the response functions for each response variable. However, this does not mean that all factors will be significant and included in all response functions.
- A region of interest is identified. In many methods an additional constraint is added to the problem formulation to identify this region.

- A model building technique, such as regression, is used to create the response functions. It is assumed that second-order response functions are sufficient for describing the functions.
- It is assumed that at least one response function is to be optimized.

### **Regression Assumptions**

Given that most MRSM techniques utilize regression to fit the second-order response model, Osborne (1997) evaluated two of the underlying assumptions of regression: uncorrelated errors and independence among the factors. Uncorrelated errors are important in conducting inferential tests due to the potential of creating bias in standard error estimation and inaccuracy in the values of the parameters. However, Osborne indicated that since response surface methodology only requires accurate estimates of the response variables and not the parameter estimates, RSM can be applied even when errors may be correlated. Independence among the factors, also known as multicollinearity, is also important in inferential testing and can cause regression coefficients to have a high standard error. Osborne indicated that since in most designed experiments used in RSM (central composite design or factorial designs) multicollinearity is minimized or eliminated, independence among the factors is not an issue when applying RSM techniques. For experimental designs where multicollinearity could be an issue, the reader is referred to Mason, Gunst, and Hess (1989) for techniques to eliminate multicollinearity.

## Non-Dominated Solutions

At the core of any multiobjective optimization problem is the ability to find non-dominated or Pareto-optimal solutions. As defined in Chapter One, *Pareto-optimal solutions* are those in which further improvement in one objective cannot be made without detriment to at least one other objective. A decision vector  $\mathbf{x}^* \in S$  is Pareto optimal if there does not exist another  $\mathbf{x} \in S$  such that  $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$  for all  $i = 1, \dots, m$  and  $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$  for at least one index  $j$ . An objective vector is Pareto optimal if the corresponding decision vector is Pareto optimal (Miettinen 2001). Pareto-optimal solutions may also be referred to as “noninferior, efficient, and non-dominated” interchangeably (Chankong and Haimes 1983). *Pareto-optimal or non-dominated solutions* may also be classified as those alternatives that are not dominated by another alternative where *dominated* is defined as:  $\mathbf{x}_1$  dominates  $\mathbf{x}_2$  if and only if  $v(f(\mathbf{x}_1)) \geq v(f(\mathbf{x}_2))$ , where  $v$  is the value function (Chankong and Haimes 1983). Decision-maker preferences, to be discussed later, are required to find the “most preferred” solution among the set of non-dominated solutions.

Multi-criteria decision-making literature provides three common approaches to developing non-dominated solutions for a multiobjective programming problem using vector optimization problem (VOP) theory (Chankong and Haimes 1983). These approaches are known as the weighting problem, the  $k$ th-objective Lagrangian problem, and the  $k$ th-objective  $\varepsilon$ -constraint problem. The formulation for each approach follows in standard format:

The weighting problem  $P(\mathbf{w})$ :

$$\text{Min} \sum_{j=1}^m w_j f_j(\mathbf{x})$$

Subject to:  $\mathbf{x} \in R$

where,  $w_j \geq 0$  and  $\sum_{j=1}^m w_j = 1$

The kth-objective Lagrangian problem  $P_k(\mathbf{u})$  :

$$\text{Min } f_k(\mathbf{x}) + \sum_{j \neq k} u_j f_j(\mathbf{x})$$

Subject to:  $\mathbf{x} \in R$

where,  $u_j \geq 0$  for all  $j \neq k$

The kth-objective  $\varepsilon$ -constraint problem  $P_k(\boldsymbol{\varepsilon})$  :

$$\text{Min } f_k(\mathbf{x})$$

Subject to:  $f_j(\mathbf{x}) \leq \varepsilon_j, j = 1, \dots, m, j \neq k$

$$\mathbf{x} \in R$$

Osborne (1997) discussed the association of these three methods as described by Chankong and Haimes (1983) including the theorems that prove that under the convexity assumption, all three approaches will provide the same optimal solution for some particular values of  $\mathbf{w}$ ,  $\mathbf{u}$ , and  $\boldsymbol{\varepsilon}$ .  $P(\mathbf{w})$  applies to the combined approaches for multiobjective optimization. Therefore, the reader is referred to these two references for additional details. For the purposes of exploring the conditions and assumptions associated with constrained problems, only the kth-objective Lagrangian and kth-objective  $\varepsilon$ -constraint approaches will be reviewed further. The kth-objective Lagrangian approach has the appearance of a combined method and Osborne (1997) classified it that way. However, for this research, the approach is classified as constrained because a priori weights are not used to combine the responses.



## The kth-Objective Lagrangian Problem $P_k(\mathbf{u})$

The necessary conditions for noninferiority for the Lagrangian approach are summarized by Chankong and Haimes (1983) as:

Assuming convexity, if  $\mathbf{x}^* \in \mathbf{X}^*$  and, for any given  $k$ ,

- i.  $P_k(\boldsymbol{\varepsilon}^*)$  is stable;
- ii. all  $f_j$ ,  $j = 1, \dots, m$ , and  $g_i$ ,  $i = 1, \dots, p$ , are faithfully convex with  $\mathbf{X} = \{\mathbf{x} \in R^n \mid g_i(\mathbf{x}) \leq 0, i = 1, \dots, p\}$ ;
- iii. the constraint of  $P_k(\boldsymbol{\varepsilon}^*)$  satisfies Karlin's constraint qualification; or
- iv. the constraints of  $P_k(\boldsymbol{\varepsilon}^*)$  satisfy the regularity assumption of  $\mathbf{x}^*$ ;

then there exists  $\mathbf{u}^* \in U_k$  such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ .

The sufficient conditions for noninferiority for the Lagrangian approach are summarized by Chankong and Haimes (1983) as:

$\mathbf{x}^*$  is a noninferior solution of VOP if for some  $k$  there exists  $\mathbf{u}^* \in U_k$  such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ , and if either

- i.  $u_j > 0$  for all  $j \neq k$ , or
- ii.  $\mathbf{x}^*$  is the unique minimizer of  $P_k(\mathbf{u})$ .

*Definition:* A problem is said to be *stable* if  $w_k(\mathbf{0})$  is finite and there exists  $M > 0$  such that for all  $\mathbf{y} \neq \mathbf{0}$ ,  $[w_k(\mathbf{0}) - w_k(\mathbf{y})] / \|\mathbf{y}\| < M$ , where  $w_k(\mathbf{y}) \stackrel{\Delta}{=} \inf_{\mathbf{x} \in \mathbf{X}} \{f_k(\mathbf{x}) \mid f_j(\mathbf{x}) - \varepsilon_j \leq y_j, j \neq k\}$  (Chankong and Haimes 1983).

*Definition:* The *faithfully convex* assumption holds when all functions  $f_j$ ,  $j = 1, \dots, m$ , are either linear or nonlinear and contain no straight-line segment in their graphs (Chankong and Haimes 1983).

*Definition:* The constraints of the problem are said to satisfy *Karlin's constraint qualification* if  $S$  is a convex subset of  $R^N$ ;  $g_j$ ,  $j = 1, \dots, p$ , are all convex functions on  $S$ ; and there exists  $\mathbf{x} \in S$  such that  $g_j(\mathbf{x}) \leq 0$  for all  $j = 1, \dots, p$ . This guarantees a saddle point of  $L(\mathbf{x}, \mathbf{u})$  (Chankong and Haimes 1983).

*Definition:* The *regularity assumption* is defined as  $\mathbf{x}^*$  is a regular point of the constraint of the interested problem where a regular point is defined as one where  $h(\mathbf{x}^*) = \mathbf{0}$  and  $\nabla h_1(\mathbf{x}^*), \dots, \nabla h_l(\mathbf{x}^*)$  are linearly independent (Chankong and Haimes 1983).

To test the sufficiency conditions it is easy to see if all  $u_j > 0$ . Furthermore, if the primary objective function is shown to be strictly convex then the solution is unique.

### **The $k$ th-Objective $\epsilon$ -Constraint Problem $P_k(\epsilon)$**

The sufficient conditions for noninferiority for the  $\epsilon$ -constraint approach are summarized by Chankong and Haimes (1983) as:

Let  $Y_k$  be the set of all  $\epsilon$  such that  $P_k(\epsilon)$  is feasible. Given an  $\epsilon$  in  $Y_k$ , let  $\mathbf{x}^*$  be an optimal solution of  $P_k(\epsilon)$ . Then  $\mathbf{x}^*$  is a noninferior solution of VOP if

- i.  $\mathbf{x}^*$  solves  $P_k(\epsilon)$  for every  $k = 1, \dots, m$ ;
- ii.  $\mathbf{x}^*$  is the unique solution of  $P_k(\epsilon)$ ; or

iii. The optimal value of  $P_k(\boldsymbol{\varepsilon}^0)$  is strictly greater than  $f_k(\mathbf{x}^*)$  for any  $\boldsymbol{\varepsilon}^0 \leq \boldsymbol{\varepsilon}^*$ .

The necessary conditions for noninferiority are (i) and (iii) above. However, if condition (ii) is true then it is sufficient to prove that  $\mathbf{x}^*$  is a noninferior solution of VOP. This then implies that conditions (i) and (iii) are also true based on Chankong and Haimes implication diagram, proving the necessary conditions.

Non-dominated solutions can be found by solving  $P_k(\boldsymbol{\varepsilon})$  for some  $\boldsymbol{\varepsilon}$  and  $k$  and then checking to see if one of the sufficiency conditions hold. A subset of non-dominated solutions can be acquired by varying  $\boldsymbol{\varepsilon}$ . However, testing the sufficiency conditions can be difficult.

Condition (i) requires  $P_k(\boldsymbol{\varepsilon})$  to be solved for every  $k$  to confirm even one non-dominated solution. In other words, for the set of  $\boldsymbol{\varepsilon}$  found when solving objective  $k$  as the primary objective including the objective value as  $\boldsymbol{\varepsilon}$  for  $k$ , when rotating each objective into the primary objective position, the same solution is found. In large problems with hundreds of constraints this problem could be intractable. However, in MRSM problems the relatively small number of constraints found in most problems make proving condition (i) possible.

Condition (ii) only requires one  $P_k(\boldsymbol{\varepsilon})$  to be solved but uniqueness may be difficult to prove. If the primary objective function is strictly convex then the solution is unique. If the primary objective is convex (but not strictly) and twice continuously differentiable then the solution is unique if the second-order sufficiency conditions hold.

Condition (iii) indicates that within a local region, that any  $\boldsymbol{\varepsilon}^0 \leq \boldsymbol{\varepsilon}^*$  will result in a non-dominated solution. Although this condition is difficult to check numerically, Lin (1977) has established a procedure to do so.

Only condition (ii) relies on the convexity of the primary objective function. Therefore,  $P_k(\boldsymbol{\epsilon})$  problems can be solved for nonconvex functions using conditions (i) and (iii). This results in the ability to find every non-dominated solution for the constrained problem, regardless of convexity.

### **Association Between $P_k(\boldsymbol{\epsilon})$ and $P_k(\mathbf{u})$**

Chankong and Haimes (1983) provide four methods by which the solution to an  $\epsilon$ -constraint problem ( $P_k(\boldsymbol{\epsilon})$ ) is also a solution to the Lagrangian problem ( $P_k(\mathbf{u})$ ):

1. If i)  $\mathbf{x}^*$  solves  $P_k(\boldsymbol{\epsilon}^*)$ , ii)  $P_k(\boldsymbol{\epsilon}^*)$  is stable, and iii) the convexity assumption holds, then there exist  $u_j \geq 0$  for all  $j \neq k$  such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ .
2. Assume i) all  $f_j$ ,  $j = 1, \dots, m$ , and  $g_i$ ,  $i = 1, \dots, p$ , are faithfully convex; ii)  $\mathbf{X}$  is given by  $\mathbf{X} = \{\mathbf{x} | g_i(\mathbf{x}) \leq 0, i = 1, \dots, p; \mathbf{x} \in S\}$  when  $S = \emptyset$ ; and iii)  $\mathbf{x}^*$  solves  $P_k(\boldsymbol{\epsilon})$ . Then there exists  $u_j \geq 0$  for all  $j \neq k$  such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ .
3. Assume i) the convexity assumption and ii) the constraints of  $P_k(\boldsymbol{\epsilon}^*)$  satisfy Karlin's constraint qualification. Then if  $\mathbf{x}^*$  solves  $P_k(\boldsymbol{\epsilon})$ , there exists  $u_j \geq 0$ ,  $j \neq k$  such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ .
4. Assume that i) the convexity assumption and ii) the constraints of  $P_k(\boldsymbol{\epsilon}^*)$  satisfy the regularity assumption at  $\mathbf{x}^*$ . Then if  $\mathbf{x}^*$  solves  $P_k(\boldsymbol{\epsilon})$ , there exist  $u_j \geq 0$ ,  $j \neq k$ , such that  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$ .

Conversely, if  $\mathbf{x}^*$  solves  $P_k(\mathbf{u})$  for some  $\mathbf{u} \in U_k$ , then  $\mathbf{x}^*$  solves  $P_k(\boldsymbol{\varepsilon})$ . Moreover  $P_k(\boldsymbol{\varepsilon}^*)$  is stable if the convexity assumption is assumed.

### **Lagrange and Karush-Kuhn-Tucker Multipliers**

For the Lagrangian problem, the value of  $u_j$  for all  $j \neq k$  is a constant that is “adjusted” to solve one or more equality constraints. These constants are known as Lagrange multipliers. Lagrange multipliers are the shadow prices of the constraints and can be used for sensitivity analysis. For problems with both equality and inequality constraints, the result is Karush-Kuhn-Tucker (KKT) multipliers. Some sources use the Lagrange multipliers nomenclature for both types of problems. In both cases, the multipliers can be used to determine local trade-offs for the active constraints. The multiplier provides the rate of change of the value of the objective function per unit increase in the right-hand-side constant of a constraint (Reklaitis, Ravindran, and Ragsdell 1983).

The following conditions are required for the result of the kth-objective  $\varepsilon$ -constraint problem to be KKT multipliers:

Let  $f$ ,  $h_i$ , and  $g_j$  be twice continuously differentiable. In a problem

$$\text{Min } f(\mathbf{x})$$

$$\text{Subject to } h_i(\mathbf{x}) = c_i, i = 1, \dots, l,$$

$$g_j(\mathbf{x}) \leq d_j, j = 1, \dots, p,$$

let  $\mathbf{x}^*$  be a local solution when  $c_i = 0, i = 1, \dots, l$ , and  $d_j = 0, j = 1, \dots, p$ , and let  $\mathbf{x}^*$

satisfy the following conditions:

1.  $\mathbf{x}^*$  is a regular point of the constraints
2. Second-order sufficiency conditions are satisfied at  $\mathbf{x}^*$
3. There is no degenerate binding constraint at  $\mathbf{x}^*$ .

Then there is a continuously differentiable vector-valued function  $\mathbf{x}(\mathbf{c}, \mathbf{d})$  defined on a neighborhood of  $(\mathbf{0}, \mathbf{0})$  in  $R^l \times R^p$  such that  $\mathbf{x}(\mathbf{0}, \mathbf{0}) = \mathbf{x}^*$  and such that for every  $(\mathbf{c}, \mathbf{d})$  in a vicinity of  $(\mathbf{0}, \mathbf{0})$ ,  $\mathbf{x}(\mathbf{c}, \mathbf{d})$  is a strict local solution. Moreover

$$\partial f[\mathbf{x}(\mathbf{0}, \mathbf{0})] / \partial c_i = -\lambda_i, i = 1, \dots, l,$$

$$\partial f[\mathbf{x}(\mathbf{0}, \mathbf{0})] / \partial d_j = -\mu_j, j = 1, \dots, p, \text{ (Chankong and Haimes 1983).}$$

*Definition: Second-order sufficiency conditions* are satisfied for  $\mathbf{x}^*$  if

- $\mathbf{Ax}^* = \mathbf{b}$ ,
- $\mathbf{Z}^T \nabla f(x^*) = 0$ , and
- $\mathbf{Z}^T \nabla^2 f(x^*) \mathbf{Z}$  is positive definite, where  $\mathbf{Z}$  is a basis matrix for the null space of  $\mathbf{A}$ , then  $x^*$  is a strict local minimizer of  $f$  over  $\{x : \mathbf{Ax} = \mathbf{b}\}$  (Nash and Sofer 1996).

*Definition: A degenerate binding constraint* is defined as one for which the Karush-Kuhn-Tucker multiplier is not strictly positive (Miettinen 1999).

### **Summary of Noninferior Solutions**

The chart below summarizes the method to use for finding noninferior solutions based on the types of response functions (i.e., convexity). Furthermore, it indicates the type of multipliers to use to conduct trade-offs among the objectives.

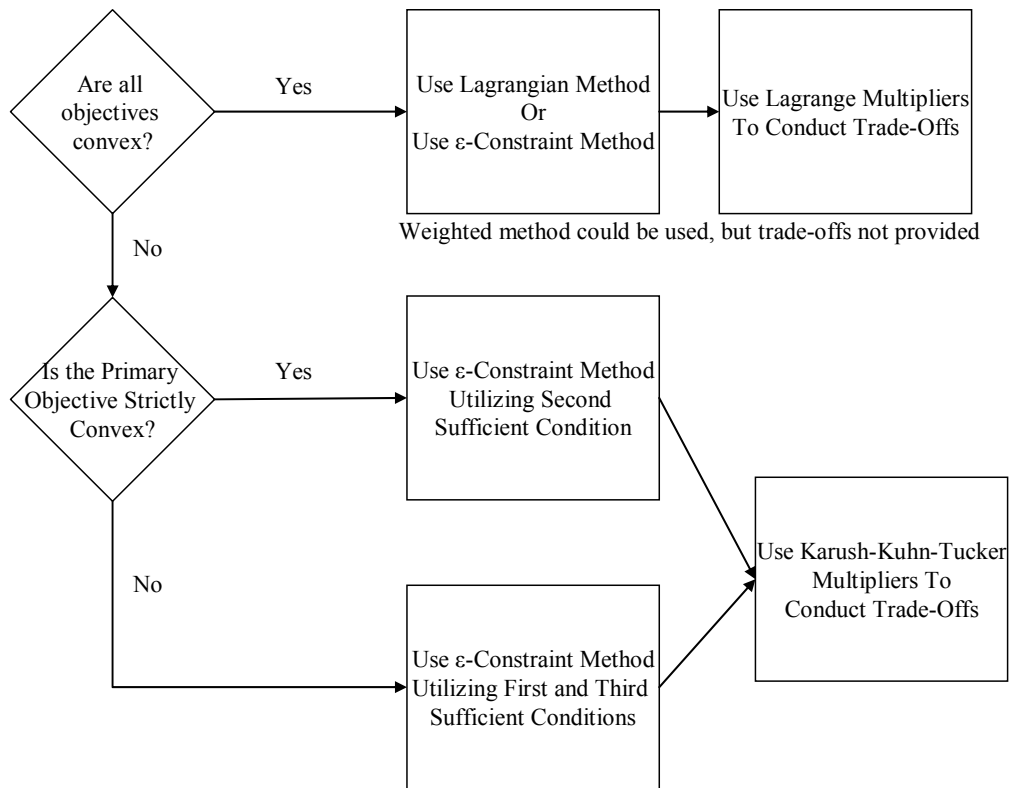


Figure 4-1 Methods for Finding Noninferior Solutions

### Transformation of the Response Functions

In combined approaches to multiobjective optimization, Osborne (1997) showed that individual response functions must be transformed to commensurable units prior to their combination. With constrained methods, the form of the response function is not an issue because multiple functions are not combined.

## Linear Dependence Between Response Functions

For the constraint method, consideration of linear dependence among the constraints needs to be considered. For linear programming problems, redundant constraints are usually removed (Nash and Sofer 1996). Miettinen (1999) defines an objective as redundant if “it is not in conflict with any other objective function.” It is suggested that linear dependence be checked for linear problems. Miettinen suggests that removing redundant objectives may make it easier on the decision-maker. However, determining the redundant constraint to be removed is based on the value of the right-hand-side. As described above, to find all non-dominated solutions, the value of the right-hand-side of each constraint objective would be systematically changed. Thus, the constraint that could be considered redundant may change as the values of the right-hand-side changes. The trade-offs that the decision-maker would be required to make would be impacted. To demonstrate, a counter-example is provided.

$$\begin{aligned} \text{Minimize} \quad & R_1 = x_1 + 3x_2 \\ \text{Subject to:} \quad & R_2 = 2x_1 + 2x_2 \geq 4 \\ & R_3 = 4x_1 + 4x_2 \geq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution (Scenario 1):  $x_1 = 2, x_2 = 0; R_1 = 2$ . In this scenario  $R_2$  is the binding constraint and  $R_3$  is the redundant constraint that would have been removed.

Now, let the value of the right hand side change to  $R_2 \geq 2$  and  $R_3 \geq 16$ . The solution to Scenario 2 is  $x_1 = 4, x_2 = 0; R_1 = 4$ . Conversely, in this scenario,  $R_3$  is the binding constraint and



$R_2$  is the redundant constraint that would have been removed. Figure 4-2 illustrates the two scenarios.

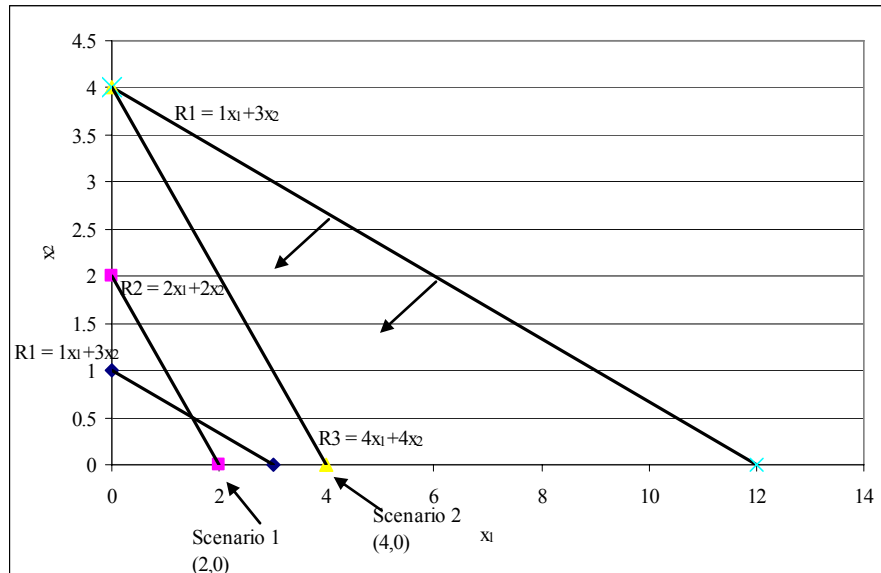


Figure 4-2 Linear Dependency

Given the above information about decision-maker preferences and the fact that the binding constraint is dependent on the right-hand-side values, it is not recommended to remove what may appear to be redundant responses.

### Correlation Between Response Functions

Miettinen (1999) suggested the correlation of the objective functions be checked for nonlinear problems. If objectives are strongly positively correlated then they may be redundant and should be considered for removal from the problem. Negatively correlated responses would not be removed. Miettinen suggests that removing redundant objectives may make it easier on the decision-maker. However, as shown above, determining the redundant constraint to be

removed would be based on the value of the right-hand-side. See the section above regarding linear dependency between response functions for a counter-example. The same argument can be made for nonlinear problems. Further, Osborne (1997) demonstrated that correlation between pairs of response functions varies dependent on an arbitrary selection of design parameter values. Therefore, it is not recommended to remove what may appear to be redundant responses based on correlation values in nonlinear problems.

### Scaling Factors

Since response functions are not combined in constrained methods, the original objectives do not need to be scaled into normalized units. Utilizing the same example as in the section above, it is shown that multiplying an individual objective by 1000 has no impact on the solution.

$$\text{Minimize} \quad R_1 = 1000x_1 + 3000x_2$$

$$\text{Subject to:} \quad R_2 = 2x_1 + 2x_2 \geq 4$$

$$R_3 = 4x_1 + 4x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

Solution:  $x_1 = 2, x_2 = 0; R_1 = 2000$ . When the primary objective to be minimized is multiplied by 1000, the solution remains the same and only the value of the objective function changes.

$$\begin{aligned} \text{Minimize} \quad & R_1 = 1x_1 + 3x_2 \\ \text{Subject to:} \quad & R_2 = 2000x_1 + 2000x_2 \geq 4000 \\ & R_3 = 4000x_1 + 4000x_2 \geq 2000 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Solution:  $x_1 = 2, x_2 = 0; R_1 = 2$ . When the constraint objectives are multiplied by 1000, the solution and the value of the primary objective function remains the same as in the original problem.

Therefore, it is concluded that the objectives can remain in their original units. Further, it is easier for decision-makers to make trade-offs when the objectives are in their original units.

### **Decision-Maker Preferences**

In order to determine the most preferred solution, decision-maker preferences must be considered. In constrained multiple objective problems, the decision-maker has two types of decisions that are usually made prior to the start of the optimization procedure. First, the decision-maker chooses the primary objective or objective to be optimized (in some methods all objectives must be ranked in importance). Second, the decision-maker puts some targets or bounds on all of the secondary, or constraint, objectives. The additional effort required by the decision-maker depends on the method used to elicit the decision-maker's preferences. This is discussed in the following section.

## **Eliciting Decision-Maker Preferences**

Somewhere in the process, the decision-maker's preferences must be taken into consideration to select one of the solutions in the Pareto optimal solution set as "best." A priori methods capture the preferences of the decision-maker before beginning the optimization method. A posteriori methods generate a set of Pareto optimal solutions from which the decision-maker chooses a preferred solution. Interactive methods provide a platform for decision-makers to provide input throughout the optimization process. The methods differ in the optimization function and the amount of information required of the decision-maker (Miettinen 1999).

The advantage to a priori methods is that the process tends to be simple because the problem is usually reduced to a single-objective problem using a combined approach. However, the disadvantage is that it is often difficult for the decision-maker to provide the necessary preference information. It has been argued that value functions and utility functions should not be used because the decision-maker is required to make hypothetical choices between alternatives. It is also difficult for the decision-maker to assess the interpretation of scaling constants. Furthermore, preferential independence and utility independence are required for the formulation of the optimization function (Mollaghasemi and Pet-Edwards 1997).

The advantage to a posteriori methods is that the decision-maker does not need to be able to provide preference information up front. However, the disadvantages are that most all or all of the non-dominated solutions should be found and then presented to the decision-maker to select the preferred solution, the algorithms are often complex for both the analyst and the decision-maker, many problems are too large for this method to be efficient, and problems with a

large number of non-dominated solutions may be difficult for the decision-maker to choose the “best” alternative. A posteriori methods are considered to place the most cognitive burden on the decision-maker and is the least popular class of methods (Mollaghasemi and Pet-Edwards 1997).

The advantages to interactive methods are that they can require less information from the decision-maker, the decision-maker is involved throughout the problem, and the information can be obtained incrementally. These methods require fewer assumptions than a priori methods. However, many of these procedures are more complicated and can be time consuming for the decision-maker (Mollaghasemi and Pet-Edwards 1997).

### **Preference Structure**

When utilizing a constrained approach, monotonicity of preference and preferential independence among the responses are not required since the response functions are not combined. Monotonicity is acquired if, for each response objective, an alternative having a smaller outcome is always preferred to a larger outcome (Osborne 1997). Preferential independence is acquired if the preference between outcomes is not dependent on the levels of the responses. For interactive or a posteriori methods, as well as a priori methods such as goal programming when lexicographic ordering is used, monotonicity of preference is not required.

### **Conclusions**

The following conditions and assumptions are required for a constrained method to find the most preferred non-dominated solution:

- Generate non-dominated solutions
- Generate solutions consistent with decision-maker preferences among the response objectives.

Additionally, if a Lagrangian constrained method is used, the following additional condition is required:

- Convexity must be preserved

The following conditions and assumptions are not required for constrained methods in order to find the most preferred non-dominated solution:

- Independence between the factors
- Independence of the regression errors
- Independence between the response functions
- Uncorrelated response functions
- Monotonicity of preference
- Mutual preferential independence
- Account for different magnitudes in the response objectives.

The following section compares these requirements against those required for combined methods.

### **Comparison of Conditions and Assumptions for Constrained vs. Combined Methods**

Table 4-1 below provides a summary of the conditions and assumptions necessary for constrained versus combined methods. The conditions and assumptions for combined methods

were obtained from the work of Osborne (1997). It is easily seen that constrained methods require fewer conditions and assumptions.

Table 4-1 Comparison of Required Conditions and Assumptions

<b>Conditions/Assumptions</b>	<b>Constrained Methods</b>		<b>Combined Methods</b>
	$\epsilon$ -Constraint	Lagrangian	Combined
Independence between the factors	No	No	No
Independence of the regression errors	No	No	No
Independence between the response functions	No	No	No
Uncorrelated response functions	No	No	No
Monotonicity of preference	No	No	Yes
Mutual preferential independence	No	No	Yes
Account for different magnitudes in the response objectives	No	No	Yes
Requires convexity to generate all non-dominated solutions	No	Yes	Yes
Generate non-dominated solutions	Yes	Yes	Yes
Generate solutions consistent with decision-maker preferences among the response objectives	Yes	Yes	Yes

### **Advantages and Disadvantages of a Constrained Approach**

Advantages to the constrained approach are that it can theoretically generate all of the non-dominated solutions and it can be easier for a decision-maker to rank order objectives and provide upper bounds or aspiration levels on the original objectives. With the  $\epsilon$ -constraint method it is possible to find every non-dominated solution by changing the value of the upper bounds, or right-hand-sides, of the constraints and rotating in each of the objectives as the primary objective. The disadvantage to this approach is that it can be labor intensive to find the non-dominated solutions since you must either show that the solution holds for all objectives or show that the solution is unique. It may also be difficult to determine appropriate values for the upper bounds (Chankong and Haimes 1983; Miettinen 1999). However an interactive version of the method may reduce the cognitive burden on the decision-maker.

Conversely, the combined approach does not guarantee all of the non-dominated solutions, particularly if the problem is nonconvex, and it may be difficult for the decision-maker to assign appropriate and consistent weights or preferences. As the number of responses increases, it would become increasingly more difficult for the decision-maker to apply consistent weights and increases the burden on the decision-maker as to the number of pairwise comparisons that would need to be made. Furthermore, with the weighting approach, the objectives may have to be scaled or normalized before being combined. This can be difficult for the decision-maker to interpret the results of a weighted and transformed objective, particularly when conducting trade-offs among the non-dominated solutions. As the number of responses increases, it would become even more difficult for the results of the weighting approach to be interpreted (Chankong and Haimes 1983; Miettinen 1999).

### **Evaluation of Current Constrained Methodologies**

The eight constrained methods discussed in Chapter Two will be evaluated against the conditions and assumptions defined above. A brief summary of the methodologies along with the evaluation of the methods follow.

#### **Lagrangian Methods**

Four of the current constrained methods utilized the Lagrangian constrained method: Dual Response, RPD Dual Response, Process Variance Dual Response, and Bias-Specified Robust Design.



## **Dual Response**

Dual response is a method introduced by Myers and Carter (1973) by which a set of optimal parameter settings are found by optimizing a primary response subject to the constraint of a secondary response. A secondary constraint may be applied to keep the solution within a region of interest. Lagrange multipliers are used to find the optimum values of the parameters.

Myers and Carter restrict this method to only quadratic functions of the responses. However, it is more appropriate to restrict the method to functions that are shown to be convex. Further, the region of interest must be selected appropriately, or the method may not generate non-dominated solutions. The method does allow for decision-maker preferences with the selection of the primary response and the target for the secondary response. However, the method provides for a single solution point and does not specify the use of trade-offs between the responses that may lead to a “better” solution for the decision-maker.

## **RPD Dual Response**

Vining and Myers (1990) utilized the dual response methodology to solve the three goals identified by Taguchi to achieve the mean response objective and minimize variance. For the target-is-best goal, the primary response is to minimize the variance and the mean target is the secondary response. For the other two goals, the mean is the primary response while the variance is the secondary response.

Similar to Myers and Carter, Vining and Myers restrict this method to only quadratic functions of the responses. However, it is more appropriate to restrict the method to functions

that are shown to be convex. Further, the region of interest must be selected appropriately, or the method may not generate non-dominated solutions. The method does allow for decision-maker preferences with the selection of the primary response and the target for the secondary response. The authors suggest running the optimization for multiple targets to provide for a set of solutions from which the decision-maker can select the best compromise solution. However, the method generates only a subset of solutions and does not discuss utilizing trade-offs. Therefore, the decision-maker may not select the most preferred solution.

### **Process Variance Dual Response**

Further, Miro-Quesada and Del Castillo (2004a) and Miro-Quesada and Del Castillo (2004b) provided a revised form of the variance response for the dual response problem that takes into account both the variability created by the randomness of the noise factors and the uncertainty in the parameter estimates.

This method is subject to the convexity assumption. Furthermore, this method is only used for problems where the primary response is to reduce the variance. The decision-maker only determines the bounds on the mean response and the bounds have to be normalized for this procedure. This makes it more difficult for the decision-maker to understand the results. Only one solution is generated.

## **Bias-Specified Robust Design**

Shin and Cho (2005) addressed the “target-is-best” goal by developing the BSRD model that allows for some process bias.

Similar to the Process Variance Dual Response Method, the BSRD method only addresses the problem where the primary objective is to minimize the variance. The decision-maker does specify a target for the mean and the amount that the mean can vary from the target. The authors do discuss the possibility of using Lagrange multipliers to allow for further trade-offs among the responses, providing additional flexibility for the decision-maker to find the best compromise solution. However, since the Lagrangian method is used, the response functions must be convex.

## **$\epsilon$ -Constraint Methods**

Four of the current constrained methods utilized the  $\epsilon$ -constraint method: Nonlinear Dual Response, Direct Function Minimization, Constrained Confidence Regions, and Goal Programming Satisficing Algorithm.

## **Nonlinear Dual Response**

Del Castillo and Montgomery (1993) developed a nonlinear programming approach to solve the same Taguchi goals that Vining and Myers addressed, as well as having the flexibility

to solve other types of problems including mixture experiments. Del Castillo and Montgomery suggested using the GRG algorithm to solve the problems.

This method is not restricted to the convexity condition and will generate non-dominated solutions. The decision-maker process includes determining the primary response and targets or bounds on the secondary responses. However, this method does not discuss utilizing trade-offs to find a potentially preferred solution to those generated based on the selections that the decision-maker defines a priori.

### **Constrained Confidence Regions**

To address the problem of sampling error, Del Castillo (1996) presented the idea of finding confidence regions for quadratic responses or confidence cones for linear responses. For quadratic responses, the Lagrangian function is maximized for each response, subject to the Karush-Kuhn-Tucker conditions utilizing the GRG algorithm. For linear responses, Box and Draper's methodology of finding confidence cones is utilized. Then a primary response is optimized subject to the confidence regions and confidence cones found in the previous step.

This method is not restricted to the convexity condition and will generate non-dominated solutions. The decision-maker determines the bounds on the secondary responses and can select the confidence level used to generate the solutions. The author suggests running the optimization at varying levels of confidence to generate multiple solutions that the decision-maker can choose from. However, this method only generates a subset of the solutions and does not guarantee the "most" preferred solution.

### **Direct Function Minimization**

Copeland and Nelson (1996) proposed a variation to the mean square error criterion, a direct function minimization, that places a restriction as to how far the mean can deviate from the target and recommended using the simplex procedure of Nelder and Mead (1965).

This method is not restricted to the convexity condition and will generate non-dominated solutions. The decision-maker process includes determining the primary response and targets or bounds on the secondary responses. However, this method only generates one solution and does not discuss utilizing trade-offs to find a potentially preferred solution.

### **Goal Programming Satisficing Algorithm**

Rees, Clayton, and Taylor (1985) combined the techniques of response surface methodology and goal programming by introducing the use of the satisficing algorithm. This methodology requires that the decision-maker rank the priority of the responses and provide a target or minimum level of attainment required for each response or goal. The satisficing algorithm, utilizing the path of steepest ascent, first optimizes the highest ranking response. Then the algorithm systematically attempts to find the optimal solution for each of the other responses within the region of interest that satisfies the lower priority goals without violating all higher priority goals.

This method is not restricted to the convexity condition and will generate non-dominated solutions. The decision-maker ranks the priority of the responses and selects target levels for the

responses. This method does not provide for evaluating trade-offs among the responses. It is assumed that the optimal solution generated by high priority goals is the preferred solution.

### **Summary**

The largest gap identified in the current set of constrained methods is the ability to find the decision-maker's preferred solution. All of the methods provide for decision-maker input. However, most methods were limited to only a priori input and did not allow for evaluation of trade-offs among the responses. Table 4-2 below provides a summary of the existing constrained methods.

Table 4-2 Summary of Existing Constrained Methods

Method	Problem Type	Requires Convexity	Generates non-dominated solutions	Type of DM process	Trade-offs	DM's best solution guaranteed
Dual Response (Myers and Carter 1973)	$P_k(\mathbf{u})$	Yes	Yes, if region of interest selected appropriately	A priori	No	No
RPD Dual Response (Vining and Myers 1990)	$P_k(\mathbf{u})$	Yes	Yes, if region of interest selected appropriately	A priori	Subset by modifying targets	No
Nonlinear Dual Response (Del Castillo and Montgomery 1993)	$P_k(\boldsymbol{\epsilon})$	No	Yes	A priori	No	No
Direct Function Minimization (Copeland and Nelson 1996)	$P_k(\boldsymbol{\epsilon})$	No	Yes	A priori	No	No
Constrained Confidence Regions (Del Castillo 1996)	$P_k(\boldsymbol{\epsilon})$	No	Yes	A priori	Subset by modifying confidence levels	No
Process Variance Dual Response (Miro-Quesada and Del Castillo 2004b)	$P_k(\mathbf{u})$	Yes	Yes	A priori	No	No
Bias-Specified Robust Design (Shin and Cho 2005)	$P_k(\mathbf{u})$	Yes	Yes	A priori	Could use Lagrange multipliers	No
Goal Satisficing Algorithm (Rees, Clayton, and Taylor 1985)	$P_k(\boldsymbol{\epsilon})$	No	Yes	A priori	No	No

**Suggested Guidelines for Methodology Development**

Although not required, the following section provides some guidelines from the literature to consider in the development of the new methodology. Two classifications for multi-criteria decision-making provide insight into problem and technique characteristics.

### **Classification Proposed by Mollaghasemi and Pet-Edwards**

Mollaghasemi and Pet-Edwards (1997) proposed a classification for multi-criteria decision-making problems based on the characteristics of the decision problem, the decision-maker, and the solution technique. Characteristics of the decision problem include the size and complexity of the problem (i.e., number of objectives) and the amount of uncertainty present in the problem. Characteristics of the decision-maker include the ability or desire of the decision-maker to articulate types and amounts of preference information. Characteristics of the solution technique include ease of use, in this situation the cognitive burden on the decision-maker; time required to solve the problem (interaction time with the decision-maker); accuracy (guarantees a non-dominated solution and whether it converges to an optimal or satisficing solution); restrictiveness of the assumptions which could include types/number of assumptions, conditions needed, and types of problems that can be solved using the method.

### **Classification Proposed by Chankong and Haimes**

Chankong and Haimes (1983) use a classification structure based on the decision situation or problem structure and the decision rule or preference modeling. Furthermore, they subclassify methods based on the inputs required and the output of the method. Specifically, the decision situation considers the types of decision variables, the level of measurement, state of nature, set of alternatives, and types of relationships. The decision rule can be defined by global preference functions, priorities, weights, goals, and ideals, or trade-offs and preferences. Inputs include data requirements such as types, amounts, and accuracy, the manner of the decision-



maker participation, and the number of decision-makers. The output can be a noninferior best-compromise solution, an inferior best-compromise solution, or a ranked list of alternatives.

Furthermore, Chankong and Haimes suggest an ex post evaluation that includes how the method is received by users including the ability of the method to produce a result that reflects the decision-maker's preferences, ability of the method to allow the decision-maker to learn more about the system and consequences of alternatives, knowledge required by the analyst, knowledge required from the decision-maker, and ease of use.

### **Other Potential Guidelines**

Additional characteristics found in the literature such as robustness, reliability, and ability to locate Pareto optimal solutions are discussed. Potential guidelines to address the shortcomings of current methods, such as measuring sensitivity to trade-offs and weights are also discussed.

### **Robustness**

A definition of robustness specifically for multiple response surface optimization was not found. In the response surface literature, two types of robustness are discussed: statistical robustness and robust experimental design. Two types of robustness are commonly found in statistics. The first relates to the importance of "robustness or insensitivity of many commonly used statistical procedures to deviations from theoretical normality" (Box, Hunter, and Hunter 1978) which Box and Tiao (1973) call criterion robustness. The second refers to the ability of

statistical techniques to “perform fairly well even if the actual probability law is not exactly the same as the one assumed” (Larson 1982) or in other words robustness to departures from assumptions which Box and Tiao (1973) call inference robustness.

Robust parameter design “entails designing the system so as to achieve robustness (insensitivity) to inevitable changes in the noise variables” (Myers and Montgomery 1995). Experimental design methods can be used to design or develop the best product or process. Montgomery defines best as “a product or process that is robust or insensitive to uncontrollable factors that will influence the product or process once it is in routine operation” (Montgomery 1991).

While the statistical definitions of robustness relate more to the theoretical assumptions, the application-oriented definitions such as robust design relate to the ability of changes, albeit noise factors or parameters, to have little or no impact on the final performance of the product.

Box (1999) indicated that applications of robust designs are to minimize the variation in the system or to minimize the impact of the variation of environmental variables on the system. He indicates split plot designs can be used for the second application. Further, Myers, et al. (2004) indicated that the areas of design that impact robustness are model misspecification, outliers, and errors in control. Goupy (2005) pointed out that finding robustness relates to finding an experimental region where the response is not sensitive to changes in the factors whereas checking robustness is to verify that a solution is not sensitive to changes in the factors.

## **Reliability**

Peterson (2004) proposed a Bayesian approach to analyze the reliability of models that can be used in conjunction with many of the existing response surface methods. The approach considers the correlation structure, the variability of the process, and the model parameter uncertainty. The process variability and model parameter uncertainty is dependent on the sample size and type of experimental design. Peterson suggested conducting a preliminary optimization and calculating the level of reliability to determine if an increase in sample size might reduce variability and thus increase the reliability.

## **Ability to Locate Pareto Optimal Solutions**

Recently, new metrics/performance measures for multiobjective optimization have been proposed. These include a measurement of the ability of the method to produce Pareto optimal solutions, the size of the largest hole in the distribution of solutions along the Pareto front, spacing metric which captures the ability of the method to provide a representative or well-diversified set of non-dominated solutions, a generalized distance measure that looks at how far one solution is compared to another, and a progress measure which is the ratio of the number of iterations taken divided by the number of non-dominated solutions found (Collette and Siarry 2005; Suman 2005).

## **Sensitivity to Trade-Offs and Weights**

For each set of trade-offs as defined by the primary response and bounds or targets on secondary responses in constrained approaches and the specific set of weights used in a combined approach, one solution is found. Similar to the discussion above concerning robustness, given all other preferences being equal, a decision-maker would prefer a solution that is less sensitive to variability to changes in the decision variables. In other words, a solution that provides the same objective value for a larger set of trade-offs or weights is preferred over a solution that has the same objective value for a smaller set of trade-offs or weights.

### **Summary of Suggested Methodology Characteristics**

Robustness and reliability, as defined above, are most relevant to the experimental design and development of the response functions. As the scope of this research is focused on the optimization process of response surface methodologies, these potential characteristics do not need to be considered.

However, the structure of the methodology and the preferences of the decision-maker are very relevant to the optimization process. Guidelines reflective of the decision-maker process, the structure of the methodology, and the solution are recommended. Table 4-3 below summarizes the recommended characteristics to consider in the development of the new methodology.

Table 4-3 Summary of Recommended Guidelines

<b>Measure</b>	<b>Target</b>
Decision-Maker Process	
Ease of use	High
Cognitive load	Low
Reflects decision-maker preferences	High
Explicit/implicit preferences	Explicit
Incorporates trade-offs among solutions	Yes
Structure of Methodology	
Response/objective limitations	Low
Ability to find all non-dominated solutions	High
Number of iterations to find the “best compromise solution”	Low

### Conclusions

This chapter discussed the development of conditions and assumptions associated with constrained multiple response surface optimization methodologies and the advantages of utilizing a constrained method versus combined method to solve multiple response surface optimization problems. Eight constrained multiple response surface methods were evaluated against the required conditions and assumptions. The largest gap existed in the ability to find the decision-maker’s preferred solution. Guidelines on characteristics to consider in the development of a new methodology were also provided.

The following chapter describes the development of a new constrained multiple response surface methodology to improve the decision making process and address the preferences of the decision-maker. This approach seeks to integrate relevant approaches from decision theory and constrained nonlinear optimization to generate a robust approach to MRSO that will truly capture the decision-maker preferences.

## **CHAPTER FIVE: DEVELOPMENT OF A NEW CONSTRAINED MULTIPLE RESPONSE SURFACE METHODOLOGY**

This chapter involves the development of a new constrained multiple response surface methodology that aligns with the required conditions and assumptions and considers the recommended characteristics.

### **Recommended Set of Conditions and Characteristics for Constrained Methods**

The following list provides a summary of the conditions and assumptions necessary for solving constrained multiple response surface optimization problems along with a set of recommended characteristics. This list will be used in the development of a proposed new constrained method for solving multiple response surface optimization problems.

Required conditions/assumptions:

- Generate non-dominated solutions
- Generate solutions consistent with decision-maker preferences among the response objectives
- Preserve convexity (if Lagrangian constrained method is used).

Recommended characteristics:

- High ease of use
- Low level of cognitive burden
- Reflects decision-maker preferences
  - Uses explicit preferences

- Allows for trade-offs among solutions
- Few limitations on the type and number of responses/objectives
- Ability to find all non-dominated solutions
- Low number of iterations to find the “best compromise solution”

This set of conditions and characteristics will be utilized in the development of a new constrained method. Ideally, the method would not require convexity, be able to generate all non-dominated solutions, and accurately reflect decision-maker preferences in an easy to use, low cognitive burden method. This chapter develops a new constrained multiple response surface optimization methodology. The use of the interactive surrogate worth trade-off is explored as a viable method to extend to multiple response surface optimization problems.

### **Interactive Surrogate Worth Trade-Off**

ISWT was identified in Chapter Three as a possible method that had yet to be extended to MRSO problems. The ISWT is an expansion of the surrogate worth trade-off (SWT) methodology. In the SWT method, a set of alternative solutions is generated and the decision-maker is asked a posteriori to determine the worth of the trade-offs between the responses for each solution. These worth values are used in a regression analysis to provide a final solution. The interactive surrogate worth trade-off (ISWT) method removes the need for generating a set of alternative solutions and incorporates the decision-maker throughout the process to conduct trade-offs that lead to the next solution point. The ISWT begins with the  $\epsilon$ -constraint method by having the decision-maker select one objective to optimize and provide upper bounds on the other objectives. The problem is solved and a Pareto optimal solution is presented to the

decision-maker. The decision-maker then conducts a “worth assessment” to determine how (much) the decision-maker would like to make a trade-off between the primary response and each secondary response where the value of the primary response changes by the value of the Lagrange multiplier for a one unit increase in value of the secondary response. An integer worth scale between -10 and 10 is used to indicate the relative strength of the preference, where 10 indicates a strong preference to make the trade-off, 0 indicates satisfaction with the current solution and the desire to make no trade-offs and -10 indicates a strong preference to make the opposite trade-off. The worth values are used to update the right-hand-side of the secondary response and then the problem is re-optimized. The process continues until the decision-maker is satisfied with the solution.

### **ISWT Algorithm (Chankong)**

The following outlines the steps utilized by the ISWT algorithm developed by Chankong (1977) to solve a multiobjective optimization problem that can be defined as follows:

$$\min_{x \in X} (f_1(x), \dots, f_m(x)), X = \{x \mid x \in E^N, g_i(x) \leq 0, i = 1, \dots, p\}$$

$$f_j : E^N \rightarrow R \text{ for all } j = 1, \dots, m \text{ and } g_i : E^N \rightarrow R \text{ for all } i = 1, \dots, p,$$

find  $x^*$ , the set of noninferior solutions, such that  $x^*$  solves  $\max_{c \in X^*} U(f_1, \dots, f_m)$

where  $U(\bullet)$  is a utility function defined by  $F = \{f(\mathbf{x}) \mid \mathbf{x} \in \mathbf{X}\}$

and is assumed to exist and is known only implicitly to the decision - maker.



Step 0: initialization. Select  $f_k$  as a primary objective. Guess an initial  $\boldsymbol{\varepsilon}^0$ .

- Select one objective to be the primary.
- For each  $j = 1, \dots, m$ , find  $f_{\min}^j = \min\{f_j(\mathbf{x}) \mid \mathbf{x} \in \mathbf{X}\}$ .
- Select an initial  $\boldsymbol{\varepsilon}_j^0 > f_{\min}^j$  for all  $j \neq k$ .

Step 1: local noninferior solution. With the current  $\boldsymbol{\varepsilon}^i$ , formulate  $P_k(\boldsymbol{\varepsilon}^i)$  and solve for (strict) local solution  $\mathbf{x}^i$ .

- Solve  $P_k(\boldsymbol{\varepsilon}^i)$ .
- Check regularity assumption and second-order sufficiency condition, if possible.
- Calculate  $\nabla f_j(\mathbf{x}^i)$  for all  $j = 1, \dots, m$  and  $\nabla g_s(\mathbf{x}^i)$  for  $s \in \mathbf{I}^i$  where  $\mathbf{I}^i = \{s \mid g_s(\mathbf{x}^i) = 0, s = 1, \dots, p\}$ .
- Find  $\bar{J}_k^i = \{j \mid 1 \leq j \leq m, j \neq k, \lambda_{kj}^i > 0\}$ .
- Estimate  $c_j^i = 1/|f_j(\mathbf{x}^i)|$  for all  $j \in J_k^i$ .

Step 2: trade-off information. Obtain all necessary trade-off information at this point.

- If  $J_k^i = J_k^{\Delta} = \{j \mid 1 \leq j \leq m, j \neq k\}$ , then go to Step 3(a).
- Otherwise, check whether  $\nabla f_j(\mathbf{x}^i)$  for all  $j = 1, \dots, m$  and  $\nabla g_s(\mathbf{x}^i)$  for  $s \in \mathbf{I}^i$  are linearly independent using the Gaussian elimination method.
  - If yes, then solve  $P_k(\bar{\boldsymbol{\varepsilon}}^i)$ , where  $\bar{\boldsymbol{\varepsilon}}_j^i = \boldsymbol{\varepsilon}_j^i$  for all  $j \in \bar{J}_j^i$  and  $\bar{\boldsymbol{\varepsilon}}_j^i = \boldsymbol{\varepsilon}_j^i - \delta \boldsymbol{\varepsilon}_j$  for some small  $\delta \boldsymbol{\varepsilon}_j > 0$  for all  $j \in J_k - \bar{J}_k^i$ , to get  $\bar{\mathbf{x}}^i = \mathbf{x}(\bar{\boldsymbol{\varepsilon}}^i)$  and  $\bar{\lambda}_{kj}^i > 0$  for all  $j \neq k$ . Go to Step 3(a).

- If no, then calculate  $\frac{\partial x(\mathcal{E}^i)}{\partial \mathcal{E}_\ell}$  for all  $\ell \in J_k^i$  and compute  $\nabla f_j(\mathbf{x}^i) * \frac{\partial x(\mathcal{E}^i)}{\partial \mathcal{E}_\ell}$

for each  $j \in J_k - J_k^i$  and  $\ell \in J_k^i$ . Go to Step 3(b).

Step 3: worth assessments. Exchange information obtained from step 1 with the decision-maker and in return obtain the worth values  $W_{kj}^i$  for  $j \neq k$  from the decision-maker.

(a) Case where  $\lambda_{kj}^i > 0$  for all  $j \neq k$ .

- For each  $\ell \neq k$ , ask the decision-maker, “Given that  $f_j = f_j(\mathbf{x}^i)$  for all  $j = 1, \dots, m$ , how much would you like to decrease  $f_k$  by  $\lambda_{kj}^i$  units for one unit increase in  $f_\ell$  and all other  $f_j$  remains unchanged?” Decision-maker should give  $W_{kj}^i$  for all  $j \neq k$ .

(b) Case where some  $\lambda_{kj}^i = 0$ .

- For each  $\ell \in J_k^i$ , ask the decision-maker, “Given that  $f_j = f_j(\mathbf{x}^i)$  for all  $j = 1, \dots, m$ , how much would you like to decrease  $f_k$  by  $\lambda_{kj}^i$  units for each  $j \in J_k - J_k^i$ ,  $f_j$  changes by  $\nabla f_j(\mathbf{x}^i) * \frac{\partial x(\mathcal{E}^i)}{\partial \mathcal{E}_\ell}$  units per one unit increase in  $f_\ell$ ?” Decision-maker should give  $W_{kj}^i$ ,  $j \in J_k^i$ .

(c) Worth assessments represent the preference of the decision-maker to make the trade-off in question. The decision-maker is asked to assign an integer between -10 and 10 where a large positive value indicates a strong preference to make the trade-off and a large negative number indicates a strong

preference to make the opposite trade-off. A value of 0 indicates indifference to the trade-off.

Step 4: step size.

- The step size determines how far we should move in the direction of improvement to find the next solution point.
- If  $W_{kj}^i \neq 0$  for some  $j \in J_k^i$  (or for some  $j \neq k$ ), then  $\alpha_M^i = \min\{\alpha_{M1}^i, \dots, \alpha_{Mm}^i\}$

where  $\alpha_{Mj} = (f_{j\min} - f_j(x^i)) / Q_j^i < 0$ ; any large number if  $Q_j^i \geq 0$

$$\text{and } Q_j^i = \begin{cases} W_{kj}^i / c_j^i & \text{for all } j \in J_k^i \\ \sum_{\ell \in J_k^i} (\nabla f_j(x^i) * \frac{\partial x(\mathcal{E}^i)}{\partial \mathcal{E}_\ell})(W_{k\ell}^i / c_\ell^i) & \text{for all } j \in J_k - J_k^i \\ \sum_{\ell \in J_k^i} (-\lambda_{k\ell}^i W_{k\ell}^i / c_\ell^i) & \text{for } j = k. \end{cases}$$

- Choose  $\alpha^i$  to be a small number between 0 and  $\alpha_M^i$  or by drawing  $f_j^i + \alpha Q_j^i$  against  $\alpha$  for all  $j = 1, \dots, m$ . Ask the decision-maker to estimate the best  $\alpha^i$  from the diagram so that  $\alpha^i$  solves  $\max_{0 < \alpha \leq \alpha_M^i} U(f_1^i + \alpha Q_1^i, \dots, f_m^i + \alpha Q_m^i)$  where  $U$  is a utility function defined on  $F = \{f(\mathbf{x}) \mid \mathbf{x} \in \mathbf{X}\}$  and is assumed to exist and is known only implicitly to the decision-maker.

Step 5: termination. Check stopping criteria and if not satisfied, proceed.

- If  $W_{kj}^i = 0$  for all  $j \neq k$  (or all  $j \in J_k^i$  if  $J_k \neq J_k^i$ ) ...STOP, or
- If  $W_{kj}^i \neq 0$  for some  $j \neq k$  (or some  $j \in J_k^i$ ), do the following:

- a. Check whether  $\alpha_M^i$  or the usable  $\alpha^i$  (i.e., one that makes  $P_k(\boldsymbol{\varepsilon}^{i+1})$  feasible) is less than  $\delta / \sum_{j \in J_k^i} |W_{kj}^i|$  where  $\delta$  is some small positive number. If NO, formulate  $P_k(\boldsymbol{\varepsilon}^{i+1})$  and continue to Step 6. If Yes, do (b),
- b. Select new slopes for some or all of  $\gamma_j^i(W_{kj}^i) = c^i W_{kj}^i / |f_j(x^i)|$  to get a new  $d^i$  and go back to (a). Repeat (a) or (b) at most 10 times (say) if no new usable direction can be found before then...STOP, a constrained local maximum has been found.

Step 6: update. Use  $W_{kj}^i$  to update  $\boldsymbol{\varepsilon}^i \rightarrow \boldsymbol{\varepsilon}^{i+1}$  and return to step 1.

- Put  $\varepsilon_j^{i+1} = \varepsilon_j^i + \alpha^i (W_{kj}^i / c_j^i)$  for all  $j \in J_k^i$  and

$$\varepsilon_j^{i+1} = \varepsilon_j^i + \alpha^i \sum_{\ell \in J_k^i} (\nabla f_j(x^i) * \frac{\partial x(\boldsymbol{\varepsilon}^i)}{\partial \varepsilon_\ell}) (W_{kj}^i / c_j^i) \text{ for all } j \in J_k - J_k^i.$$

- Go to step 1.

### **Reasons for Utilizing the ISWT Algorithm**

The reasons the ISWT algorithm was considered for the response surface problem can be summarized as follows:

- ISWT uses the  $\varepsilon$ -constraint methodology so it is possible to find all non-dominated solutions regardless of convexity of the response functions.
- The interactive nature of the algorithm means that all solutions do not need to be found. The process stops when the decision-maker is “satisfied” with the solution.

- Since ISWT is not an a priori method, it does not require a constant preference structure nor does the decision-maker need to completely define their preference structure in advance.
- Characteristics of the ISWT solution technique as classified by Mollaghasemi and Pet-Edwards (1997) are:
  - Ease of Use: low learning curve and low to moderate cognitive burden on the decision-maker
  - Time and Effort: Moderate interaction time, depending on problem size
  - Accuracy: Non-dominated solution is guaranteed
  - Assumptions: Nonrestrictive
- Decision-makers can make partial trade-offs requiring only that the decision-maker can determine the worth of a trade-off for one “constraint” objective compared to the primary objective at a time.
- Objectives can stay in original units making it easier for decision-makers to understand the trade-offs they are making.

### **Modified ISWT Method for Multiple Response Surface Optimization Problems**

This section develops a new multiple response surface methodology that addresses gaps identified in previous methodologies, in particular, incorporating the explicit preferences of decision-makers and allowing for explicit trade-offs among the responses. The methodology satisfies the conditions set forth in the previous chapter, as well as considers the recommended characteristics. The methodology utilizes the ISWT algorithm as a basis for incorporating

decision-maker trade-offs. The specific characteristics of multiple response surface problems include the definition of a region of interest based on the experimental design. This uniquely sets up the problem to allow for some simplification of the ISWT algorithm. Specifically, the region of interest allows for the new methodology to require binding constraints which allows for the necessary information needed for decision-makers to make informed trade-offs among the responses.

The following provides a complete outline of the process incorporating both the experimental design aspects of MRSM problems along with the MCDM solution of ISWT, as well as the guidelines for decision-maker involvement.

### **Guidelines for Finding the Best Compromise Solution**

The following section provides guidelines for selecting the region of interest, selecting the primary response, selecting bounds or targets on the secondary responses, and how to analyze trade-offs.

#### **Selecting the Region of Interest**

The region of interest needs to be defined in order to place a constraint on the solution set. Research, particularly that of Del Castillo and Montgomery (1993), discussed defining regions of interest and compared results using a cuboidal region versus a spherical region. The authors indicate in some situations a cuboid region can be a better choice of defining the region.

For illustrative purposes, a cuboid region of interest based on the space defined by the independent variables is used throughout this research.

### **Selecting the Primary Response**

Historically it is the most important response that is optimized in multiple response surface methodologies. However, the trade-off literature recommends using the response that is easiest for the decision-maker to make trade-offs (e.g., cost) as the primary response. Another consideration is the ability of the decision-maker to place bounds or targets on the responses. It is recommended that the responses for which a target can be confidently established should be considered for a secondary response.

If there are only two responses, select the response with the better defined target as secondary. If there are more than two responses, select the response easiest for the decision-maker to make trade-offs as the primary.

### **Selecting Targets or Bounds**

First, each response is optimized individually to find the minimum and maximum values within the region of operability as defined by the settings used for the independent variables in the experimental design (e.g., coded as -1.414 to 1.414). This method will be called the Extreme Values Method (EVM).

If the decision-maker has bounds defined that lie within the experimental region of interest then those are used as the initial  $\epsilon$  in the optimization. When a target is defined, if it is a

maximization constraint, the target is used as the lower bound and the upper extreme value associated with the constraint is used for the upper bound; if it is a minimization constraint, the target is used as the upper bound and the lower extreme value associated with the constraint is used for the lower bound. If the decision-maker does not have targets or bounds identified then the extreme values will be used. Worth trade-offs can then be used along with the Lagrange multipliers to find alternative solutions for the decision-maker to consider.

### **Analyzing Trade-Offs**

Analyzing trade-offs is comprised of three steps: collecting the Lagrange multipliers, having the decision-maker assess the worth of the partial trade-offs, and updating the right-hand-side values (or  $\epsilon$ ).

Lagrange multipliers indicate the impact that changing the constraint's right-hand-side value has on the primary objective. For response surface problems this is interpreted as the amount of trade-off that is made on the primary response for a one unit change in the secondary response. Chankong (1977) explains that partial trade-offs are only valid when the Lagrange multipliers are positive. For those responses where all of the associated Lagrange multipliers are zero, a small change is made in  $\epsilon$  and the problem is resolved. The change needs to be large enough so that at least one the Lagrange multiplier associated with the response becomes positive.

In Miettinen's (1999) discussion of ISWT, Tarvainen (1984) is referenced as a possible alternative scale for the worth trade-off step. Tarvainen recommends using the alternative scale of -2 to 2 (rather than the original scale of -10 to 10) to reduce the number of alternative values



that the decision-maker must deal with, thus making it easier for the decision-maker to maintain consistency while conducting the worth assessments. The five point scale is sufficient to represent the direction and rough degree of preference or satisfaction for the trade-off. The decision-maker selects an integer between -2 and 2 to indicate preference for making the trade-offs where:

- 2 means that the decision-maker definitely wants to make the trade-off,
- 1 means that the decision-maker somewhat wants to make the trade-off,
- 0 means that the decision-maker is satisfied at the current point and does not want to make a trade-off,
- -1 means that the decision-maker somewhat wants to make the opposite trade-off,
- and -2 means that the decision-maker definitely wants to make the opposite trade-off.

The value of the right-hand-side ( $\epsilon$ ) is updated based on the worth assessment made by the decision-maker. In general for each partial trade-off assessment, a step size is multiplied by the worth from the decision-maker of the trade-off and the current value of the constraint response. This is multiplied by either the difference between the current  $\epsilon$  and the lower bound for a minimization objective or between the current  $\epsilon$  and the upper bound for a maximization objective. This value is added to the current  $\epsilon$  and the problem is updated with new right-hand-size values and re-optimized. The step size is a constant value raised to the power of the iteration to aid in convergence multiplied by  $\beta^i$  where  $\beta^i = 1 / \sqrt{\sum_{j \neq k} (W_{kj}^i |f_j(x^i)|)^2}$ . A common step size method in optimization based on the golden section search sets  $c$  equal to .618.

Figure 5-1 below illustrates the movement of  $\epsilon$  for the update process. From the diagram it can be seen that in order to improve a response objective,  $\epsilon$  needs to be moved within the region between the current  $\epsilon$  and the minimum or maximum value of  $\epsilon$ , depending on the type of constraint.

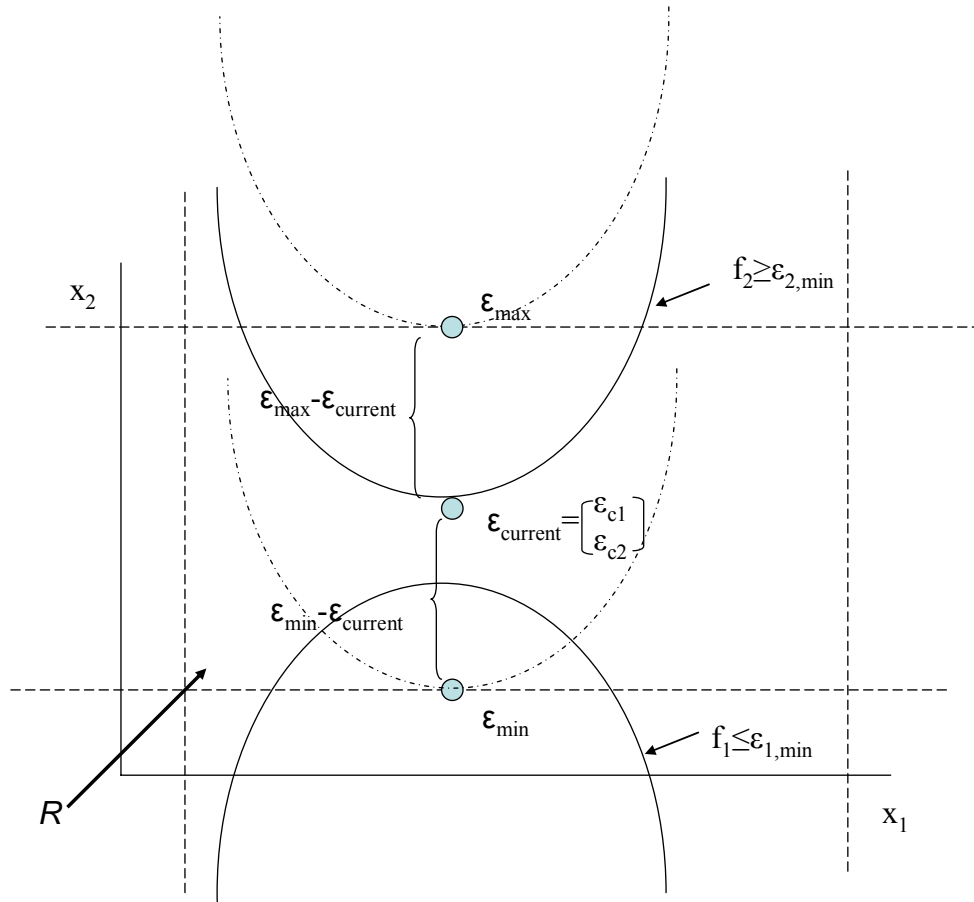


Figure 5-1 Updating Procedure

**Terminating Criteria**

Theoretically, the only stopping criteria desired are those based on the decision-maker satisfaction and interest in exploring additional non-dominated solutions. Step 8 terminates the

process if the decision-maker is satisfied with the current point. Step 10 terminates the process if all worth assessments equal zero, indicating that the decision-maker does not want to make any further trade-offs.

Therefore, contingencies have been incorporated into the methodology for three specific situations that were encountered during the testing phase. If a solution converges to a point and the decision-maker is not satisfied, either modify the step size and attempt to find another non-dominated solution or go back to a previous solution and change the direction of the worth assessment in order to move in a different direction. If the problem becomes infeasible, again, either modify the step size and attempt to find another non-dominated solution or go back to a previous solution and change the direction of the worth assessment in order to move in a different direction. If  $\epsilon$  is updated to a point outside the extreme values, use the associated extreme value and re-solve.

### **MISWT Algorithm**

This section outlines the new MISWT algorithm. The algorithm is demonstrated step by step in the Silicon Wafer problem in Chapter Six.

Step 1: Problem Definition. The responses and factors of interest must be defined.

Step 2: Experimental Design.

- a. Conduct the screening experiment.
- b. Fit single order models.
- c. Find the direction for improvement.

- d. Conduct additional experiments if additional data points are needed to fit higher order models.

Step 3: Develop the Response Functions.

- Fit higher order models. Usually multiple regression is used to fit each response.

Step 4: Identify the Region of Interest.

- The region of interest needs to be defined in order to place a constraint on the solution set. For illustrative purposes, a cuboid region of interest is used throughout this research.

Step 5: Identify Primary Response.

- If the problem only has two responses then use the response with the better defined target (based on the decision-maker's confidence in the target) as the secondary response. This could reduce the number of iterations required.
- If there are more than two responses, select the response that is easiest for the decision-maker to make trade-off comparisons as the primary response.

Step 6: Identify the Targets or Bounds for the Secondary Responses.

- a) Minimize and maximize each response individually, unconstrained within the region of operability, to find the extreme upper and lower bounds for each response. This will be referred to as the Extreme Values Method (EVM) and will define the widest possible range for the bounds within the region of interest. These extreme values will also be utilized in Step 11 to update  $\epsilon$ .
- b) Ask the decision-maker to place a bound or target on the secondary responses.

- If the decision-maker desires to set a range around the possible response values for each of the secondary responses, he/she specifies an upper and lower bound for each constrained response.
  - If the decision-maker has a specific target to reach on a secondary response then, if it is a maximization constraint, the target is used as the lower bound and the upper extreme value associated with the constraint is used for the upper bound; if it is a minimization constraint, the target is used as the upper bound and the lower extreme value associated with the constraint is used for the lower bound.
- c) Use the narrower of the bounds defined by step 6a and step 6b above in the optimization problem. The tighter the bounds, the quicker the convergence may be to a “preferred” solution.

Step 7: Run the Optimization.

- Utilizing the parameters defined in the higher order models and the selection of the region of interest, primary response, and target or bounds on the secondary responses, formalize the problem and run the optimization procedure.
- This dissertation will use Frontline’s Premium Solver in Excel to conduct the optimizations. This software uses the GRG nonlinear programming algorithm.
- Note that the method only guarantees local optima. If the problem is non-convex, it is necessary to use alternative starting points to test for local optima.

- If after running the optimization in subsequent iterations a solution converges to a point and the decision-maker is not satisfied or the problem becomes infeasible, either modify the step size and re-solve in an attempt to find another non-dominated solution or go back to a previous solution and change the direction of the worth assessment in order to move in a different direction.

#### Step 8: Decision-Maker's Satisfaction with the Current Solution

- The decision-maker is asked if satisfied with the current solution. If yes, then the procedure stops. If no, then proceed to Step 9.

#### Step 9: Collect Trade-Off Information (Lagrange multipliers (LM) or Karush-Kuhn-Tucker multipliers (KKT)).

- For purposes of this research, Lagrange multipliers and Karush-Kuhn-Tucker multipliers terminology will be used interchangeably for this step.
- Chankong (1977) explains that partial trade-offs are only valid when the Lagrange multipliers are positive. Therefore, at least one Lagrange multiplier associated with each of the secondary responses must be greater than 0, indicating it is an active or binding constraint, to conduct trade-offs with the decision-maker.
- If at least one Lagrange multiplier associated with each secondary response is greater than 0, then collect trade-off information and move to Step 10.
  - In Frontline's Premium Solver in Excel, this information can be located on the sensitivity report.

- If there are any secondary responses with no Lagrange multipliers greater than 0, then perturb at least one of the bounds associated with the response and return to Step 7.
  - In order to achieve a positive Lagrange multiplier, the value of one of the bounds for the constraint can be perturbed by adding (or subtracting) the slack of the constraint, using the answer report in Excel's Solver, and some small constant. In other words, by using the slack value in the answer report,  $\varepsilon$  is modified so that constraint becomes binding.

Step 10: Collect Decision-Maker Worth Values.

- Have the decision-maker make comparisons between the primary response and each secondary response.
- If the primary response is a maximization problem, ask, "Given that the primary response value equals  $Y_1$  and the secondary response (max, min) value equals  $Y_2$ , please indicate your preference for decreasing the primary response by LM units for one unit (increase, decrease) in the secondary response?"
- If the primary response is a minimization problem, ask, "Given that the primary response value equals  $Y_1$  and the secondary response (max, min) value equals  $Y_2$ , please indicate your preference for increasing the primary response by LM units for one unit (increase, decrease) in the secondary response?"
- Use Tarvainen's (1984) integer scale from -2 to 2 to evaluate the partial trade-offs and assess the decision-maker's preference in making the specified trade-

offs. The decision-maker selects an integer worth value (W) between -2 and 2 to indicate preference for making the trade-offs where:

- 2 means that the decision-maker definitely wants to make the trade-off,
  - 1 means that the decision-maker somewhat wants to make the trade-off,
  - 0 means that the decision-maker is satisfied at the current point and does not want to make a trade-off,
  - -1 means that the decision-maker somewhat wants to make the opposite trade-off,
  - and -2 means that the decision-maker definitely wants to make the opposite trade-off.
- If all W=0, then stop, otherwise continue to Step 11.

Step 11: Update  $\epsilon$  and return to Step 7.

- In order to move to another non-dominated solution, the value of  $\epsilon$  is updated to reflect a direction of movement and a step size, or distance, to move.
- Equation: In general the formula is New RHS = Old RHS + Step Size\*Worth (Current Value) \* Direction. The specific equations to update  $\epsilon$  follow:

a. For minimization constraints:

$$\epsilon_j^{i+1} = \epsilon_j^i + \alpha^i (W_{kj}^i |f_j(x^i)|) * (\epsilon_{j_{\min}} - \epsilon_j^i) \text{ for all } j \neq k,$$

where  $\alpha$  is the step size =  $(c)^i (\beta^i)$ .

b. For maximization constraints:

$$\epsilon_j^{i+1} = \epsilon_j^i + \alpha^i (W_{kj}^i |f_j(x^i)|) * (\epsilon_{j_{\max}} - \epsilon_j^i) \text{ for all } j \neq k,$$

where  $\alpha$  is the step size =  $(c)^i (\beta^i)$ .



- If  $\varepsilon$  is updated to a point outside the extreme values, use the associated extreme value to keep the problem within the region of interest.
- Direction: As discussed above and seen in Figure 5-1, the use of the extreme values minus the current point,  $(\varepsilon_{j_{\min}} - \varepsilon_j^i)$  and  $(\varepsilon_{j_{\max}} - \varepsilon_j^i)$ , provide a direction of movement: minimization constraints are moving  $\varepsilon$  in the direction of the lower bound while maximization constraints are moving  $\varepsilon$  in the direction of the upper bound.
- Worth:  $\text{Worth} = (W_{kj}^i |f_j(x^i)|)$ . The worth value in the equation indicates the strength of the decision-maker's preference and the direction. If the worth value is negative then the new  $\varepsilon$  will move in a direction opposite of the direction of optimization. When there are only two responses, only the sign matters. However, when there are more than two responses, the strength of the worth assessment is normalized in the step size equation to incorporate the relative importance of the response trade-offs in the distance and direction of movement.
- Step Size: The step size provides the amount by which the search will move to find the next non-dominated solution. The step size is multiplied by the current value of the constraint, thereby moving some distance as a proportion of the current value. The step size is equal to  $(c)^i (\beta^i)$ . Summarizing Chankong's (1977) proof,  $\beta^i = 1 / \sqrt{\sum_{j \neq k} (W_{kj}^i |f_j(x^i)|)^2}$ . This research will utilize a common step size method in optimization based on the golden section

search to set  $c$  equal to .618. In order to ensure that the iterations result in convergence,  $c$  is raised to a power equal to the number of the iteration,  $i$ .

- Return to Step 7 to re-optimize at the new  $\epsilon$ .

### **Summary**

This chapter has developed a new algorithm to solve multiple response surface optimization problems. Utilizing the ISWT multi-criteria decision-making algorithm as a basis for development, the specific characteristics of the multiple response surface optimization problem and the conditions and assumptions developed in Chapter Four for constrained methods, a new multiple response surface methodology was created. Figure 5-2 below provides a flow diagram of the new Modified ISWT (MISWT) MRSO process. Chapter Six will demonstrate the new methodology using test problems from the literature.

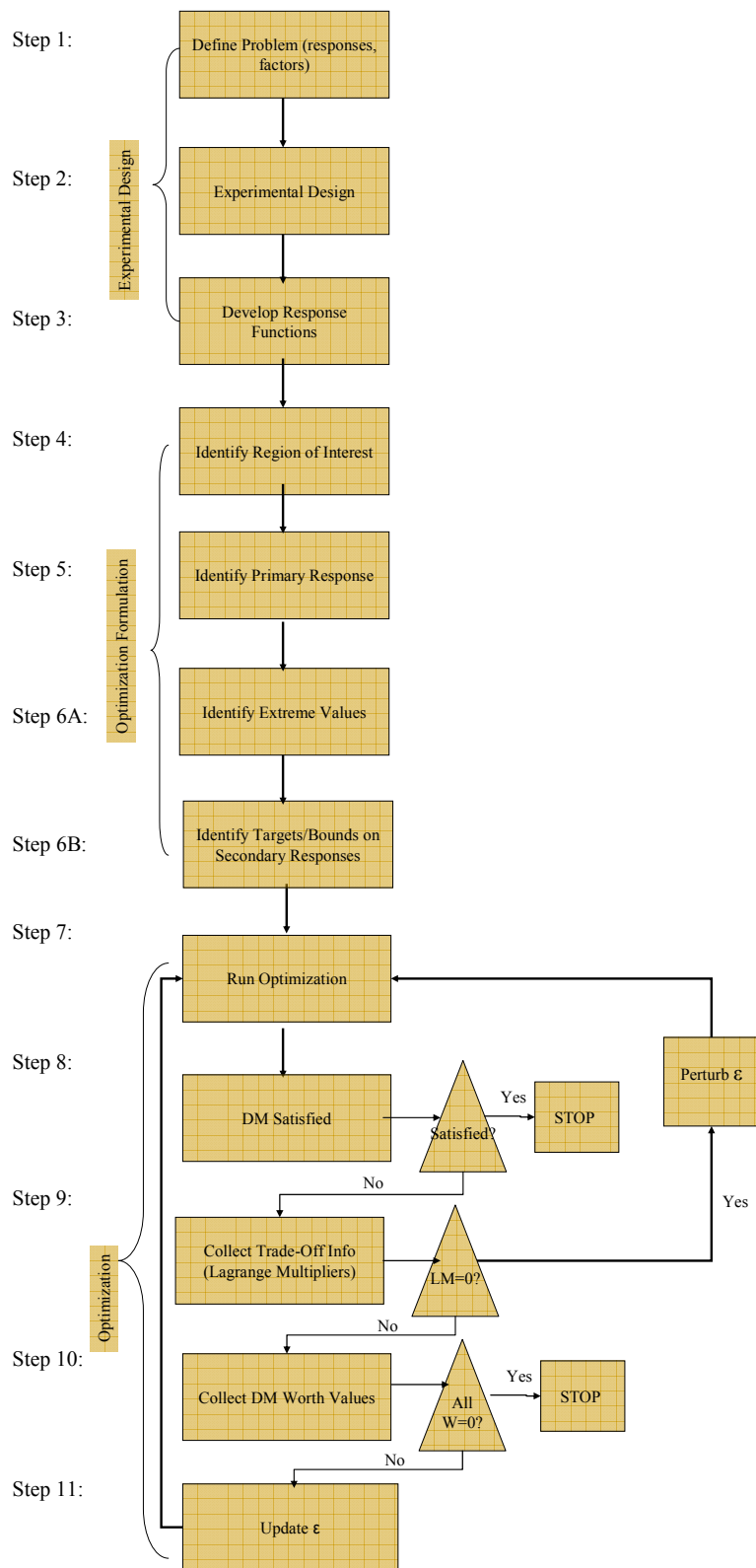


Figure 5-2 Modified ISWT for MRSO Problems

## **CHAPTER SIX: DEMONSTRATION OF THE NEW CONSTRAINED MULTIPLE RESPONSE SURFACE OPTIMIZATION METHOD**

In order to demonstrate the new Modified ISWT Method for MRSO problems, results of the new method will be compared to existing constrained MRSM techniques using five test problems from the literature. The decision-maker preferences will be simulated using a random number generator. Finally, the method will be evaluated against the recommended set of conditions and characteristics discussed in Chapter Five.

### **Demonstration of New Method to Existing Constrained Methods Using Test Problems**

Results of the new method, MIWST, will be compared to existing constrained MRSM techniques using five test problems from the literature. The test problems have been chosen such that results can be compared to existing constrained methods for different types of MRSO problems. The problems reflect a variety of number of factors and responses, and types of responses. A more complete list of potential test problems can be found in Appendix A. Frontline System's Excel Premium Solver was used to conduct the optimizations for the new method in an easily implemented spreadsheet environment. This represents an easily accessible computer based implementation of the ISWT.

For purposes of this research, a random number generator is used to select an integer between -2 to 2 to simulate a decision-maker for each partial trade-off. Although MCDM methods are based on the assumption of a rational decision-maker, using a "random" worth value allows the research to explore what happens when a decision-maker selects a direction and continues to move in the same direction as well as when a decision-maker changes direction. In

reality a decision-maker may not know until they evaluate partial trade-offs that they are willing to move in a different direction (e.g., willing to trade-off a small decrease in standard deviation for a much larger increase in the mean). Therefore, this testing method will demonstrate the robustness of the proposed method to manage many types of decision-makers including a traditional “rational” decision-maker.

For each problem, a table summarizing the process used is included with a map to the Figure in the Appendix where Excel screenshots with the details are provided. Each problem also contains a results table that outlines for each iteration the solution point, response values, original  $\epsilon$ , the values of the Lagrange multipliers, the values of the worth assessments, and the updated  $\epsilon^{i+1}$ . For some problems, multiple scenarios were tested.

### **Test Problem 1: Silicon Wafers**

The silicon wafer problem was analyzed by Shin and Cho (2005). Silicon wafers are produced by a subcontractor for a large motor corporation. There were problems maintaining the dimensions, surface quality and flatness of the housing. The coating thickness of the wafers was considered by the customer to be the most important characteristic. A central composite design was conducted with two factors (mould temperature and injection flow rate) and the responses were the mean and standard deviation of the coating thickness of the wafers. The target value of the coating thickness was 71.14, the maximum allowable process bias was .9, and the variability of the thickness was targeted to be as small as possible. This problem demonstrates a typical two quadratic response problem, trading off the mean and the standard deviation. The data for

the experimental design and screen shots of the Excel spreadsheets can be found in Appendix B.

Details of the process for the new method are described below.

Step 1: Determine the Responses and Factors. Based on the problem defined by Shin and Cho, the customer selected the mean and standard deviation of the coating thickness as the responses of interest. Mould temperature and injection flow rate were identified as the factors.

Step 2: Conduct Experimental Design. The results of a central composite design conducted by Shin and Cho will be used for comparison purposes. Figure 6-1 below provides the design and results of the experiment.

<b>X1</b>	<b>X2</b>	<b>Y1</b>	<b>Y2</b>	<b>Y3</b>	<b>Y4</b>	<b>Y (avg)</b>	<b>Y (std)</b>
-1	-1	76.3	73.5	68.8	74.2	73.20	3.17
1	-1	70.4	81.2	76.7	79.6	76.98	4.76
-1	1	76.6	72.0	77.7	78.5	76.20	2.91
1	1	72.3	67.5	75.7	72.7	72.05	3.39
-1.414	0	70.6	75.8	69.9	71.5	71.95	2.65
1.414	0	74.1	80.2	76.2	77.1	76.90	2.53
0	-1.414	78.5	68.7	76.2	75.3	74.68	4.21
0	1.414	70.2	76.3	79.2	75.9	75.40	3.77
0	0	74.1	71.8	72.5	71.9	72.58	1.06
0	0	72.1	70.4	73.3	74.2	72.50	1.64
0	0	74.2	69.8	71.2	72.2	71.85	1.85
0	0	70.1	69.3	71.6	72.5	70.88	1.44
0	0	69.8	70.6	71.6	74.1	71.53	1.87

X1=mould temperature (99.6, 301.4)  
X2=injection flow rate (40, 70)  
Y=quality of printing process (4 replicates)

Figure 6-1 Experimental Design

Step 3: Develop Response Functions. For comparison purposes, the response functions Shin and Cho obtained through regression will be used here.

$$\mu = 71.87 + 0.83x_1 - 0.11x_2 + 1.25x_1^2 + 1.55x_2^2 - 1.98x_1x_2$$

$$\sigma = 1.57 + 0.24x_1 - 0.28x_2 + 0.58x_1^2 + 1.28x_2^2 - 0.28x_1x_2$$

Figure 6-2 illustrates the convex response functions.

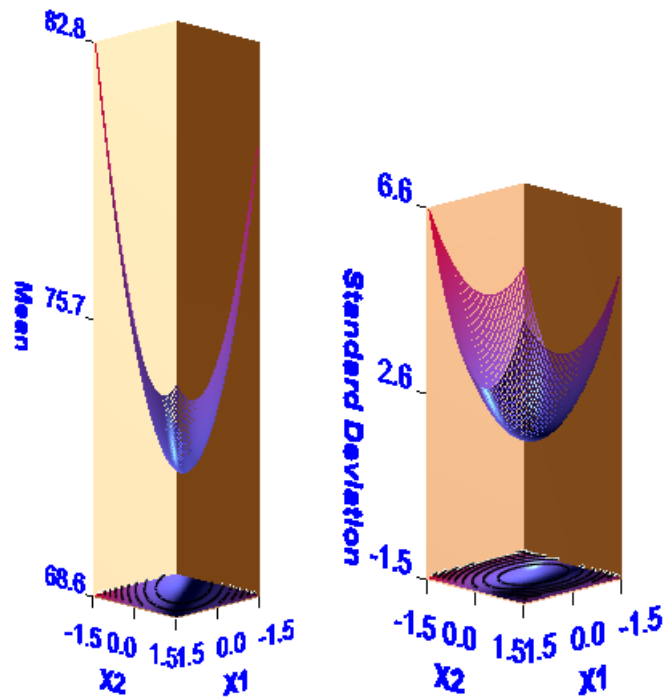


Figure 6-2 Response Surface and Contour Plots for Silicon Wafer Problem

Step 4: Determine the Region of Interest. Throughout this research, a cuboid region of interest is used based on the value of the  $x(s)$  used in the design of experiments. In this case, the cuboid region is defined by -1.414 and 1.414.

Step 5: Determine the Primary Response. The decision-maker is asked to select which of the responses to use as the primary response. In the article by Shin and Cho, minimizing the variance was chosen as the primary response.

Step 6: Determine the Bounds/Targets on the Secondary Responses.

6A: The mean was optimized, unconstrained, to find the extreme values.

	C	D	E	F	G	H	I	J	K
1		Constant	x1	x2	x1sq	x2sq	x1x2		Response
2	Settings		1.41	-1.41	2.00	2.00	-2.00		Value
3	$\mu$ (Y1)	71.87	0.83	-0.11	1.25	1.55	-1.98		82.76
4	$\sigma$ (Y2)	1.57	0.24	-0.28	0.58	1.28	-0.28		6.58
5									
6	Constraints								
7		x1	>=		-1.414				
8		x1	<=		1.414				
9		x2	>=		-1.414				
10		x2	<=		1.414				
11									
12		Lower Bound			71.63				
13		Upper Bound			82.76				
14									
15		Case	>=		70.24				
16			<=		72.04				

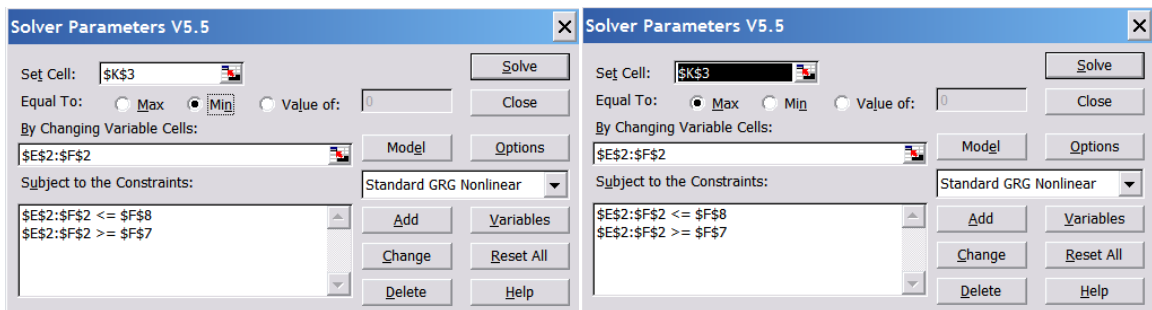


Figure 6-3 Extreme Values Method

6B: The decision-maker was asked to place bounds or a target on the mean response. For comparison with Shin and Cho, we assume that the decision-maker has set bounds on the mean of 70.24 and 72.04.

6C: The narrower of the two sets of bounds are used. In this case, the specified lower bound is below the EVM of 71.63. The method would recommend using the EMV lower bound and the decision-maker upper bound. However for comparison purposes with Shin and Cho, the decision-maker defined bounds will be used.

Step 7: Run the optimization for the problem as defined by the region of interest, primary response, and bounds on the secondary responses as determined above:



Minimize  $\sigma(x_1, x_2)$

Subject to :

$$\mu \geq 70.24$$

$$\mu \leq 72.04$$

$$-1.414 \leq (x_1, x_2) \leq 1.414$$

Figure 6-4 below provides a screenshot of Excel's Solver.

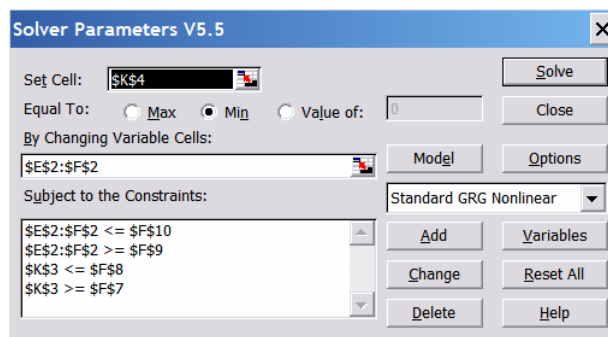


Figure 6-4 Sample Solver Screenshot

Figure 6-5 provides the solution for the optimization of the problem above. The initial solution point was found to be (-.19, .09) with a mean value of 71.79 and standard deviation of 1.54.

	A	B	C	D	E	F	G	H	I	J	K
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response
2		Settings			-0.19	0.09	0.03	0.01	-0.02		Value
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.79
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54
5											
6		Constraints									
7				$\mu$	$\geq$	70.24					=ISWT!F15
8				$\mu$	$\leq$	72.04					=ISWT!F16
9				x1, x2	$\geq$	-1.41					
10				x1, x2	$\leq$	1.41					
11											
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No
13											
14	Step 9:	Lagrange multiplier=									0.00

Figure 6-5 Optimization Sample

Step 8: Decision-Maker Satisfaction. The decision-maker is asked if satisfied with the above solution. If yes, then stop. If no, as in this case, continue to Step 9.

Step 9: Collect Trade-Off Information. The Lagrange multipliers are collected from the “Sensitivity Report” output from Excel Solver, Figure 6-6. For this problem, the Lagrange multiplier is equal to 0. Therefore,  $\epsilon$  must be perturbed and the problem re-optimized. Since the mean is a maximization objective, the  $\epsilon$  for the lower bound of the mean is perturbed by adding the slack collected from the “Answer Report” output from Excel Solver (Figure 6-7) plus some small constant. As can be seen from Figure 6-8,  $\epsilon$  moves from a value of 70.24 to 71.80 ( $70.24+1.55+.01=71.80$ ).

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 1				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.19	0.00
10	\$F\$2	Settings x2		0.09	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.79	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.79	0.00

Figure 6-6 Trade-Off Results from Solver

	X	Y	Z	AA	AB	AC	AD
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Silicon Wafers.xls]ISWT 1						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Min)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$K\$4	$\sigma$ (Y2) Value	1.57	1.54		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-0.19		
16		\$F\$2	Settings x2	0.00	0.09		
17							
18							
19	Constraints						
20		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
21		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3<=\$F\$8	Not Binding	0.2457027
22		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3>=\$F\$7	Not Binding	1.55
23		\$E\$2	Settings x1	-0.19	\$E\$2<=\$F\$10	Not Binding	1.5993901
24		\$F\$2	Settings x2	0.09	\$F\$2<=\$F\$10	Not Binding	1.324902
25		\$E\$2	Settings x1	-0.19	\$E\$2>=\$F\$9	Not Binding	1.23
26		\$F\$2	Settings x2	0.09	\$F\$2>=\$F\$9	Not Binding	1.50

Figure 6-7 Answer Report Sample

	A	B	C	D	E	F	G	H	I	J	K	L
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response	
2		Settings			-0.17	0.10	0.03	0.01	-0.02		Value	
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.80	
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54	
5												
6		Constraints										
7				$\mu$	$\geq$	71.80					=ISWT 1!F7+ISWT 1!AD22+0.01	
8				$\mu$	$\leq$	72.04					=ISWT 1!F8	
9				x1, x2	$\geq$	-1.41						
10				x1, x2	$\leq$	1.41						

Figure 6-8 Perturb Sample

Step 7: Run Optimization. The problem was re-optimized using the new  $\epsilon$ . The new solution point found was (-.17, .10) with a mean value of 71.8 and standard deviation of 1.54. See Figure 6-8 above.

Step 8: Decision-Maker Satisfaction. The decision-maker is asked if satisfied with the above solution. If yes, then stop. If no, as in this case, continue to Step 9.

Step 9: Collect Trade-Off Information. The Sensitivity Report provided by Solver for the second iteration returned a Lagrange multiplier value of .06 as seen in Figure 6-9 below.

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 2</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.17	0.00
10	\$F\$2	Settings x2		0.10	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.80	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.80	0.06

Figure 6-9 Trade-Off Example, Iteration 2

Step 10: Conduct Worth Assessments. The Lagrange multiplier found in the step above was used in collecting worth assessments with the decision-maker. The decision-maker was

asked, “Given that the mean equals 71.8 and the standard deviation equals 1.54, please indicate your preference for increasing the standard deviation by .06 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.?” In this test a random number generator returned an integer value of 1 indicating that the decision-maker is interested in making this trade-off. Since the worth assessment did not equal 0, the process continues to Step 11.

Step 11: Update  $\epsilon$ . Using a worth assessment of 1, the step size is calculated as  $(.618^1)/(\text{SQRT}((1*71.8)^2)) = .0086$ . This is used to calculate the updated  $\epsilon^{i+1}$  as  $71.8 + (.0086 * 1 * |71.8|) * (72.04 - 71.8) = 71.95$ .

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:		Constant	x1	x2	x1sq	x2sq	x1x2			Response		
2		Settings		-0.17	0.10	0.03	0.01	-0.02			Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.80		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7			$\mu$	$\geq$	71.80						=ISWT I!F7+ISWT I!AD22+0.01		
8			$\mu$	$\leq$	72.04						=ISWT I!F8		
9			x1, x2	$\geq$	-1.41								
10			x1, x2	$\leq$	1.41								
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.06		
15													
16	Step 10:	Given that the mean equals 71.8 and the standard deviation equals 1.54,											
17		please indicate your preference for increasing the standard deviation by 0.06 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									1.00		
22													
23	Step 11:	Update $\epsilon$	c=				0.618						
24			iteration=				1						
25			step size=				0.00861				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27			$\epsilon$ =				71.95				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

Figure 6-10 Silicon Wafer, Iteration 2,  $\epsilon$  Update

Return to Step 7. The problem is re-optimized using this new right-hand-side bound for the mean. Screen shots of additional iterations can be found in Appendix B. Table 6-1 provides a summary of the process used for solving the Silicon Wafer problem. The process stopped when the solution point returned to the same solution from a previous iteration.

Table 6-1 Silicon Wafers Process, Scenario 1

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the coating thickness Factors: mould temperature and injection flow rate	
Step 2:	A Central Composite Design with 4 replicates	B-1
Step 3:	$\mu = 71.87 + 0.83x_1 - 0.11x_2 + 1.25x_1^2 + 1.55x_2^2 - 1.98x_1x_2$ $\sigma = 1.57 + 0.24x_1 - 0.28x_2 + 0.58x_1^2 + 1.28x_2^2 - 0.28x_1x_2$	B-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.414, 1.414.	B-1
Step 5:	Primary response: minimize standard deviation.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the mean: 71.63, 82.76. Shin and Cho set the upper bound=72.04 and the lower bound=70.24. Since the specified lower bound is below the EVM of 71.63, the method would recommend using the EMV lower bound and the decision-maker upper bound. However for comparison purposes, the decision-maker defined bounds will be used.	B-3
Step 7:	Minimize $\sigma(x_1, x_2)$ Subject to : $\mu \geq 70.24$ $\mu \leq 72.04$	B-4, B-5
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	B-5
Step 9:	Lagrange multiplier = 0 from the "Sensitivity Report". Using the slack value of 1.55 from the "Answer Report", perturb the value of $\epsilon = 70.24 + 1.55 + .01$ for the lower bound.	B-5
Step 7:	Re-optimize.	B-6
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	B-6
Step 9:	Lagrange multiplier=.06.	B-6
Step 10:	Given that the mean equals 71.8 and the standard deviation equals 1.54, please indicate your preference for increasing the standard deviation by .06 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 1.	B-6
Step 11:	Update $\epsilon^j$ to $\epsilon^{j+1} = 71.95 = 71.8 + (((.618^1)/\text{SQRT}((1.0*\text{ABS}(71.80))^2)))*1.0*\text{ABS}(71.8))*(72.04-71.80)$ .	B-6
Step 7-11:	Repeat steps 7-11 for Iterations 3 – 8.	B-7 to B-12
Step 7:	Repeat step 7. Iteration 9. Solution and responses same as Iteration 6. Stop.	B-13

Table 6-2 below summarizes the results of the experiment. The initial solution found by the MISWT method was the same found by Shin and Cho. As seen by the results table, the simulated decision-maker preferred to move in a positive direction, increasing the mean, for iterations 2-4. At the next point, the decision-maker determined that the trade-off was too much and provided a negative worth assessment, causing the solution to show a decrease in the mean. The analyst stopped the process when there was no substantive improvement seen and the solution returned to a point that was obtained at a previous iteration. The final solution was at a point (.15, .12) with a mean value of 71.99 and standard deviation of 1.60. However, the methodology suggests that when the process converges to a solution point that either a different step size is used to move the solution to a different non-dominated solution or return to a previous iteration and have the decision-maker re-evaluate the worth to move in a new direction. Given a real-world decision-maker, if the solution point obtained was not satisfactory, one of these two methods would be implemented to find a solution point that satisfies the decision-maker.

Table 6-2 Silicon Wafers Results, Scenario 1

Method	Iteration	x1	x2	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Shin & Cho		-0.19	0.09	71.79	1.54				
MISWT	1	-0.19	0.09	71.79	1.54	70.24	0.00	Perturb	
	2	-0.17	0.10	71.80	1.54	71.80	0.06	1.00	71.95
	3	0.10	0.14	71.95	1.58	71.95	0.39	1.00	71.98
	4	0.14	0.13	71.98	1.59	71.98	0.39	1.00	72.00
	5	0.15	0.12	72.00	1.60	72.00	0.39	-1.00	71.99
	6	0.15	0.12	71.99	1.60	71.99	0.39	-1.00	71.99
	7	0.14	0.12	71.99	1.60	71.99	0.39	2.00	71.99
	8	0.14	0.12	71.99	1.60	71.99	0.39	1.00	71.99
	9	0.15	0.12	71.99	1.60				

An additional scenario, Scenario 2, demonstrates what happens when the decision-maker does not know what to set for the bounds. Table 6-3 summarizes the process used for Scenario 2. The main difference is that in Step 6, the extreme values found in Step 4 were used. Table 6-4 summarizes the results of this scenario. The same initial solution found in Scenario 1 and by Shin and Cho was found in Scenario 2. With the wider bounds, the simulated decision-maker opted to continue to increase the mean at the expense of the standard deviation until a mean of 80.78 was reached. The decision-maker chose to move back (smaller mean) at the final iteration and settled on a point (1.41, -1.08) with a mean of 80.49 and a standard deviation of 5.29. In this iteration, the decision-maker provided a worth assessment of 0 indicating satisfaction with the solution and the process stopped.



Table 6-3 Silicon Wafers Process, Scenario 2

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the coating thickness Factors: mould temperature and injection flow rate	
Step 2:	A Central Composite Design with 4 replicates	B-1
Step 3:	$\mu = 71.87 + 0.83x_1 - 0.11x_2 + 1.25x_1^2 + 1.55x_2^2 - 1.98x_1x_2$ $\sigma = 1.57 + 0.24x_1 - 0.28x_2 + 0.58x_1^2 + 1.28x_2^2 - 0.28x_1x_2$	B-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.414, 1.414.	B-1
Step 5:	Primary response: minimize standard deviation.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the mean: 71.63, 82.76. It is assumed that the decision-maker does not know where to set the bounds. Therefore, the bounds defined by the extreme values will be used: 71.63 and 82.76.	B-3
Step 7:	Minimize $\sigma(x_1, x_2)$ Subject to : $\mu \geq 71.63$ $\mu \leq 82.76$	B-14, B-15
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	B-15
Step 9:	Lagrange multiplier = 0 from the "Sensitivity Report". Using the slack value of 10.96 from the "Answer Report", perturb the value of $\epsilon = 71.80$ for the lower bound.	B-15, B-16
Step 7:	Re-optimize.	B-16
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	B-16
Step 9:	Lagrange multiplier=.06.	B-16
Step 10:	Given that the mean equals 71.8 and the standard deviation equals 1.54, please indicate your preference for increasing the standard deviation by .06 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 1.	B-16
Step 11:	Update $\epsilon^j$ to $\epsilon^{j+1} = 78.57$ .	B-16
Step 7-11:	Repeat steps 7-11 for Iterations 3 – 5.	B-17 to B-19
Step 7-11:	Repeat steps 7-10. Iteration 6. Worth assessment = 0. Stop.	B-20

Table 6-4 Silicon Wafers Results, Scenario 2

Method	Iteration	x1	x2	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
MISWT	1	-0.19	0.09	71.79	1.54	71.63	0.00	Perturb	
	2	-0.17	0.10	71.80	1.54	71.80	0.06	1.00	78.57
	3	1.41	-0.75	78.57	4.28	78.57	0.49	1.00	80.17
	4	1.41	-1.03	80.17	5.12	80.17	0.54	1.00	80.78
	5	1.41	-1.13	80.78	5.45	80.78	0.56	-1.00	80.49
	6	1.41	-1.08	80.49	5.29	80.49	0.55	0.00	Stop

Scenario 3 demonstrates what happens when the secondary response is substituted as the primary response (i.e., the mean becomes the primary response and the standard deviation becomes the secondary response). Table 6-5 summarizes the process used for Scenario 3 and Table 6-6 summarizes the results of this scenario. Letting the worth assessment equal 2 at each iteration, the solution converged to a point (.76, -.10) with a mean of 73.4 and a standard deviation of 2.15. The same solution was reached for two iterations, so the analyst stopped the process. Similar to Scenario 1, if a real decision-maker was available, one of the alternatives would be used to move off of the converged solution until a solution was found that satisfied the decision-maker.

Table 6-5 Silicon Wafers Process, Scenario 3

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the coating thickness Factors: mould temperature and injection flow rate	
Step 2:	A Central Composite Design with 4 replicates	B-1
Step 3:	$\mu = 71.87 + 0.83x_1 - 0.11x_2 + 1.25x_1^2 + 1.55x_2^2 - 1.98x_1x_2$ $\sigma = 1.57 + 0.24x_1 - 0.28x_2 + 0.58x_1^2 + 1.28x_2^2 - 0.28x_1x_2$	B-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.414, 1.414.	B-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 1.54, 6.58. The decision-maker did not specify a target or bounds. Therefore, the extreme values will be used.	B-21
Step 7:	Maximize $\mu(x_1, x_2)$ Subject to : $\sigma \geq 1.54$ $\sigma \leq 6.58$	B-22, B-23
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	B-23
Step 9:	Lagrange multiplier = 1.70 from the "Sensitivity Report".	B-23
Step 10:	Given that the mean equals 82.75 and the standard deviation equals 6.58, please indicate your preference for decreasing the mean by 1.70 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	B-23
Step 11:	Update $\epsilon^i$ to $\epsilon^{i+1} = 3.47$ .	B-23
Step 7-11:	Repeat steps 7-11 for iterations 2-12.	B-24 to B-34
Step 7	Repeat step 7 for iteration 13. Converged to the same point for the last two iterations. Stop.	B-35

Table 6-6 Silicon Wafers Results, Scenario 3

Method	Iteration	x1	x2	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
MISWT	1	1.41	-1.41	82.75	6.58	6.58	1.70	2.00	3.47
	2	1.41	-0.35	76.76	3.47	3.47	2.54	2.00	2.73
	3	1.10	-0.23	74.88	2.73	2.73	2.55	2.00	2.45
	4	0.95	-0.17	74.16	2.45	2.45	2.55	2.00	2.32
	5	0.87	-0.14	73.83	2.32	2.32	2.55	2.00	2.25
	6	0.82	-0.13	73.65	2.25	2.25	2.55	2.00	2.21
	7	0.80	-0.12	73.55	2.21	2.21	2.55	2.00	2.18
	8	0.78	-0.11	73.49	2.18	2.18	2.55	2.00	2.17
	9	0.77	-0.11	73.45	2.17	2.17	2.55	2.00	2.16
	10	0.77	-0.11	73.43	2.16	2.16	2.55	2.00	2.16
	11	0.76	-0.11	73.42	2.16	2.16	2.55	2.00	2.15
	12	0.76	-0.10	73.41	2.15	2.15	2.55	2.00	2.15
	13	0.76	-0.10	73.41	2.15	Stop			

## **Test Problem 2: Chemical Experiment**

The chemical experiment was conducted by Del Castillo (1996) and has been referenced in the research of Gheware (2003) and Osborne (1997). In this experiment there were two factors (temperature and reaction time) and two responses (the mean and variance of the process yield). A rotatable CCD experimental design was used with three replicates. This problem provides an example using a combination of quadratic and linear responses. Appendix C provides the data as well as the Excel and Solver formulations for the problem.

Table 6-7 provides a summary of the process used for solving the chemical experiment problem. In this problem, the mean is maximized subject to treating the standard deviation as a constraint. The bounds were not defined by the decision-maker so the extreme values were used. The process terminated when the decision-maker rated the worth of the current trade-off as 0.

Table 6-7 Chemical Experiment Process, Scenario 1

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of process yield. Factors: temperature and reaction time.	
Step 2:	Rotatable Central Composite Design with three replicates.	C-1
Step 3:	$\mu = 760.46 + 43.71x_1 + 154.84x_2 - 12.05x_1^2 - 73.42x_2^2 - 24.70x_1x_2$ $\sigma = 19.26 + 6.31x_1 + 6.28x_2$ (Osborne's response functions used).	C-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.414, 1.414.	C-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 1.46, 37.06. The decision-maker did not specify bounds on the secondary response. Therefore, the bounds are defined by the extreme values.	C-3
Step 7:	Maximize $\mu(x_1, x_2)$ Subject to : $\sigma \geq 1.46$ $\sigma \leq 37.06$	C-4, C-5
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	C-5
Step 9:	Lagrange multiplier = 0 from the "Sensitivity Report". Using the slack value of 6.53 from the "Answer Report", perturb the value of $\epsilon = 30.52$ .	C-5, C-6
Step 7:	Re-optimize.	C-6
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	C-6
Step 9:	Lagrange multiplier = .01.	C-6
Step 10:	Given that the mean equals 849.92 and the standard deviation equals 30.52, please indicate your preference for decreasing the mean by .01 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	C-6
Step 11:	Update $\epsilon^1$ to $\epsilon^{i+1} = 12.56$ .	C-6
Step 7-11:	Repeat steps 7-11 for iteration 3.	C-7
Step 7-10:	Repeat steps 7-10 for iteration 4. Worth assessment = 0. Stop.	C-8

Table 6-8 below summarizes the results of the experiment, along with the results reported by Osborne for the Del Castillo problem. The initial solution found with the new method was the same as that found by Osborne (.89, .91) with a mean of 849.92 and a standard deviation of 30.53. The simulated decision-maker was satisfied with the point at (-1.3, .92) with a mean of 792.85 and a standard deviation of 16.80. The iterative process allowed the decision-maker to find a more satisficing point by decreasing the mean by 57 units for a trade-off of a decrease of 14 units of standard deviation.

Table 6-8 Chemical Experiment Results, Scenario 1

Method	Iteration	x1	x2	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Del Castillo		0.89	0.91	849.92	30.53				
MISWT	1	0.89	0.91	849.92	30.53	37.06	0.00		Perturb
	2	0.88	0.91	849.92	30.52	30.52	0.01	2.00	12.56
	3	-1.41	0.35	732.55	12.56	12.56	21.94	-2.00	16.80
	4	-1.30	0.92	792.85	16.80	16.80	8.31	0.00	Stop

### Test Problem 3: Printing Press

The printing press problem has been referenced in many multiple response surface methods research including Box and Draper (1987), Vining and Myers (1990), Del Castillo and Montgomery (1993), Lin and Tu (1995), Copeland and Nelson (1996), Tang and Xu (2002), Koksoy and Doganaksoy (2003), Gheware (2003), and Osborne (1997).

The printing press problem was analyzed to determine the impact of three factors (speed, pressure, and distance) and the quality of the printing process (ability of machine to apply colored inks to package labels). The two responses of interest were the mean and standard deviation of the quality of the printing process. A  $3^3$  factorial design with 3 replicates was used to develop the quadratic functions for the two responses. The data for the experimental design as well as the final quadratic functions to be used in the problem can be found in Appendix D. The Excel and Solver formulations for the problem are also included.

The printing press problem demonstrates responses with three independent variables. Table 6-9 provides a summary of the process used for solving the printing press problem. The initial scenario seeks to minimize the standard deviation subject to a target mean of 500. The process ended when the decision-maker reached a preferred solution ( $W=0$ ).

Table 6-9 Printing Press Process, Scenario 1

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A $3^3$ factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$  $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: minimize standard deviation.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the mean: 74.9, 911.1. The decision-maker specified a target of 500 for the mean. Therefore, the bounds are defined as 500, 911.1.	D-3
Step 7:	Minimize $\sigma(x_1, x_2, x_3)$ Subject to: $\mu \geq 500$ $\mu \leq 911.1$	D-4, D-5
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-5
Step 9:	Lagrange multiplier = .12 from the "Sensitivity Report".	D-5
Step 10:	Given that the mean equals 500 and the standard deviation equals 45.1, please indicate your preference for increasing the standard deviation by .12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	D-5
Step 11:	Update $\epsilon^1$ to $\epsilon^{+1} = 754.06$ .	D-5
Step 7-11:	Repeat steps 7-11 for iterations 2-6.	D-6 to D-10
Step 7-10:	Repeat steps 7-10 for iteration 7. Worth assessment = 0. Stop.	D-11

Table 6-10 below summarizes the results of the experiment. The initial solution was the same point that was found by Del Castillo and Montgomery when they used a cuboid region of interest. The simulated decision-maker made trade-offs for six iterations before indicating satisfaction with the point at (1.0, 1.0, .24) with a mean of 747.88 and a standard deviation of 84.84. The iterative process allowed the decision-maker to find a more satisficing point by increasing the mean by 248 units for a trade-off of an increase of 40 units of standard deviation.

Table 6-10 Printing Press Results, Scenario 1

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Vining & Myers			0.61	0.23	0.10	500.00	51.80				
Del Castillo & Montgomery	cuboid		1.00	0.12	-0.26	500.00	45.10				
Del Castillo & Montgomery	$\rho_2=1$		0.98	0.03	-0.18	500.00	45.32				
Del Castillo & Montgomery	$\rho_2=1.5$		1.19	-0.22	-0.19	500.00	43.60				
Del Castillo & Montgomery	$\rho_2=2$		1.34	-0.43	-0.15	500.00	42.45				
Del Castillo & Montgomery	$\rho_2=3$		0.95	1.25	-0.73	500.00	46.98				
Del Castillo & Montgomery	$\rho_2 \leq 3$		1.57	-0.72	-0.09	500.00	40.65				
MISWT	cuboid	1	1.00	0.12	-0.26	500.00	45.10	500.00	0.12	2.00	754.06
		2	1.00	1.00	0.27	754.06	86.33	754.06	0.24	-2.00	694.08
		3	1.00	1.00	0.02	694.08	73.17	694.08	0.20	1.00	745.30
		4	1.00	1.00	0.23	745.30	84.23	745.30	0.24	-1.00	721.12
		5	1.00	1.00	0.13	721.12	78.75	721.12	0.22	1.00	738.25
		6	1.00	1.00	0.20	738.25	82.58	738.25	0.23	1.00	747.88
		7	1.00	1.00	0.24	747.88	84.84	747.88	0.24	0.00	Stop

Scenario 2 provides another example of a decision-maker that did not know where to set the bounds for the mean. Therefore, the extreme values were used. In this problem, the wider bound found other local optima, very different from the original solution in Scenario 1. The initial point in scenario 2 was (-.2, -1.0, -.42) with a mean of 139.38 and a standard deviation of 14.76. For this scenario, worth assessments were made with the goal of reaching the same mean target of 500 that was used in Scenario 1. After 12 iterations, a solution point was reached (1.0, .12, -.26) with a mean of 500.38 and a standard deviation of 45.14. Thus, it has been illustrated that using the extreme values as bounds can still achieve the target mean; however it is not as efficient as when the decision-maker has a satisficing target in mind. See Table 6-11 and Table 6-12.



Table 6-11 Printing Press Process, Scenario 2

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A 3 <sup>3</sup> factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$ $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: minimize standard deviation.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the mean: 74.9, 911.1. Assume that the decision-maker does not have a target or bounds in mind. Therefore, the extreme values will be used.	D-3
Step 7:	Minimize $\sigma(x_1, x_2, x_3)$ Subject to: $\mu \geq 74.9$ $\mu \leq 911.1$	D-4, D-12
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-12
Step 9:	Lagrange multiplier = 0 from the "Sensitivity Report". Using the slack value of 64.48 from the "Answer Report", perturb the value of $\epsilon = 149.38$ .	D-12, D-13
Step 7:	Re-optimize.	D-13
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-13
Step 9:	Lagrange multiplier = .01.	D-13
Step 10:	Given that the mean equals 149.38 and the standard deviation equals 14.83, please indicate your preference for increasing the standard deviation by .01 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	D-13
Step 11:	Update $\epsilon^j$ to $\epsilon^{j+1} = 620.12$ .	D-13
Step 7-11:	Repeat steps 7-11 for iterations 3-11.	D-14 to D-22
Step 7-10:	Repeat steps 7-10 for iteration 12. Worth assessment = 0. Stop.	D-23

Table 6-12 Printing Press Results, Scenario 2

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
MISWT	cuboid	1	-0.20	-1.00	-0.42	139.38	14.76	74.90	0.00		Perturb
		2	-0.09	-1.00	-0.39	149.38	14.83	149.38	0.01	2.00	620.12
		3	1.00	1.00	-0.27	620.12	60.50	620.12	0.15	-2.00	508.99
		4	1.00	0.20	-0.27	508.99	46.18	508.99	0.12	-1.00	414.08
		5	1.00	-0.52	-0.14	414.08	34.97	414.08	0.12	2.00	486.58
		6	1.00	0.01	-0.24	486.58	43.49	486.58	0.12	1.00	524.85
		7	1.00	0.34	-0.30	524.85	48.10	524.85	0.12	-1.00	503.33
		8	1.00	0.15	-0.26	503.33	45.50	503.33	0.12	-1.00	489.29
		9	1.00	0.03	-0.24	489.29	43.82	489.29	0.12	1.00	498.27
		10	1.00	0.10	-0.26	498.27	44.89	498.27	0.12	1.00	503.70
		11	1.00	0.15	-0.27	503.70	45.54	503.70	0.12	-1.00	500.38
		12	1.00	0.12	-0.26	500.38	45.14	500.38	0.12	0.00	Stop

Table 6-13 through Table 6-20 below demonstrate scenarios where the mean becomes the primary response. Scenarios 3, 4, and 5 demonstrate a comparison to Vining and Myers for targets set on the standard deviation. The initial solution point for Scenario 3 was the same as that found by Del Castillo and Montgomery when a cuboid region of interest was used. All three scenarios obtained an initial solution equal to the target standard deviation and ended when the decision-maker provided a worth assessment = 0 indicating a preferred solution was found. Furthermore, Scenario 5 demonstrates a situation where the decision-maker is satisfied with the initial solution.

Table 6-13 Printing Press Process, Scenario 3

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A 3 <sup>3</sup> factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$ $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 12.50, 137.5. The decision-maker specified a target of 60 for the standard deviation. Therefore, the bounds are defined as 12.50, 60.	D-24
Step 7:	Maximize $\mu(x_1, x_2, x_3)$ Subject to : $\sigma \geq 12.5$ $\sigma \leq 60$	D-25, D-26
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-26
Step 9:	Lagrange multiplier = 6.86 from the "Sensitivity Report".	D-26
Step 10:	Given that the mean equals 616.7 and the standard deviation equals 60, please indicate your preference for decreasing the mean by 6.86 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	D-26
Step 11:	Update $\epsilon^i$ to $\epsilon^{i+1} = 30.65$ .	D-26
Step 7-11:	Repeat steps 7-11 for iterations 2-4.	D-27 to D-29
Step 7-10:	Repeat steps 7-10 for iteration 5. Worth assessment = 0. Stop.	D-30

Table 6-14 Printing Press Results, Scenario 3

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Vining & Myers			0.77	0.31	0.20	557.90	60.00				
Del Castillo & Montgomery	cuboid		1.00	1.00	-0.28	616.70	60.00				
Del Castillo & Montgomery	$\rho_2=1$		0.95	0.31	0.09	594.08	60.00				
Del Castillo & Montgomery	$\rho_2=1.5$		1.19	0.28	-0.03	626.87	60.00				
Del Castillo & Montgomery	$\rho_2=2$		1.40	0.19	-0.11	647.64	60.00				
Del Castillo & Montgomery	$\rho_2=3$		1.72	-0.10	-0.13	672.50	60.00				
MISWT	cuboid	1	1.00	1.00	-0.28	616.70	60.00	60.00	6.86	2.00	30.65
		2	1.00	-0.75	-0.09	376.75	30.65	30.65	8.66	2.00	23.71
		3	1.00	-1.00	-0.15	314.39	23.71	23.71	11.23	1.00	21.07
		4	0.88	-1.00	-0.25	278.13	21.07	21.07	14.98	-1.00	22.32
		5	0.98	-1.00	-0.23	296.46	22.32	22.32	14.38	0.00	Stop

Table 6-15 Printing Press Process, Scenario 4

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A 3 <sup>3</sup> factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$ $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 12.50, 137.5. The decision-maker specified a target of 75 for the standard deviation. Therefore, the bounds are defined as 12.50, 75.	D-24
Step 7:	Maximize $\mu(x_1, x_2, x_3)$ Subject to : $\sigma \geq 12.5$ $\sigma \leq 75$	D-31, D-32
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-32
Step 9:	Lagrange multiplier = 4.92 from the "Sensitivity Report".	D-32
Step 10:	Given that the mean equals 703.23 and the standard deviation equals 75, please indicate your preference for decreasing the mean by 4.92 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	D-32
Step 11:	Update $\epsilon^j$ to $\epsilon^{j+1} = 36.38$ .	D-32
Step 7-11:	Repeat steps 7-11 for iterations 2-5.	D-33 to D-36
Step 7-10:	Repeat steps 7-10 for iteration 6. Worth assessment = 0. Stop.	D-37

Table 6-16 Printing Press Results, Scenario 4

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Vining & Myers			1.05	0.44	0.29	687.5	75				
MISWT	cuboid	1	1.00	1.00	0.05	703.23	75.00	75.00	4.92	2.00	36.38
		2	1.00	-0.44	-0.15	426.12	36.38	36.38	8.57	-1.00	45.49
		3	1.00	0.15	-0.26	503.30	45.49	45.49	8.33	-1.00	53.28
		4	1.00	0.73	-0.35	566.60	53.28	53.28	7.87	1.00	47.33
		5	1.00	0.28	-0.29	518.54	47.33	47.33	8.25	-2.00	50.47
		6	1.00	0.52	-0.32	544.19	50.47	50.47	8.08	0.00	Stop

Table 6-17 Printing Press Process, Scenario 5

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A 3 <sup>3</sup> factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$ $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 12.50, 137.5. The decision-maker specified a target of 90 for the standard deviation. Therefore, the bounds are defined as 12.50, 90.	D-24
Step 7:	Maximize $\mu(x_1, x_2, x_3)$ Subject to : $\sigma \geq 12.5$ $\sigma \leq 90$	D-38, D-39
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-39
Step 9:	Lagrange multiplier = 3.90 from the "Sensitivity Report".	D-39
Step 10:	Given that the mean equals 768.73 and the standard deviation equals 90, please indicate your preference for decreasing the mean by 3.90 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 0. Stop.	D-39

Table 6-18 Printing Press Results, Scenario 5

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
Vining & Myers			1.33	0.56	0.37	813.9	90				
MISWT	cuboid	1	1.00	1.00	0.33	768.73	90.00	90.00	3.90	0.00	Stop

A final scenario with the printing press problem, Scenario 6, illustrates when the decision-maker did not have a target or bound in mind for the standard deviation, so the extreme values were used to establish the bounds on the constraint response. This scenario demonstrates a situation where the decision-maker chooses a negative trade-off that updates  $\epsilon$  to a value outside the extreme value. So, the extreme value is used instead in order to keep the problem

within the region of interest. The decision-maker then selects a different worth value moving in the opposite direction. The problem is resolved and a new solution is found. The decision-maker is satisfied with the new solution so the process stops.

Table 6-19 Printing Press Process, Scenario 6

Step #	Description	Figure #
Step 1:	Responses: maximize mean and minimize standard deviation of the quality of the printing press. Factors: speed, pressure, and distance.	
Step 2:	A 3 <sup>3</sup> factorial design with 3 replicates.	D-1
Step 3:	$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$ $\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$	D-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -1.0, 1.0.	D-1
Step 5:	Primary response: maximize mean.	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the standard deviation: 12.50, 137.5. Assume the decision-maker did not have specified target or bounds for the standard deviation. Therefore, the bounds are defined as 12.50, 137.5.	D-24
Step 7:	Maximize $\mu(x_1, x_2, x_3)$ Subject to : $\sigma \geq 12.5$ $\sigma \leq 137.5$	D-40, D-41
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	D-41
Step 9:	Lagrange multiplier = 2.35 from the "Sensitivity Report".	D-41
Step 10:	Given that the mean equals 911.1 and the standard deviation equals 137.5, please indicate your preference for decreasing the mean by 2.35 units for one unit decrease in the standard deviation, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of -1.	D-41
Step 11:	Update $\epsilon^1$ to $\epsilon^{i+1} = 214.75$ . This is outside of the extreme value so $\epsilon$ is set at the extreme value.	D-41 to D-42
Step 7-11:	Repeat steps 7-11 for iteration 3.	D-43
Step 7-10:	Repeat steps 7-10 for iteration 4. Worth assessment = 0. Stop.	D-44

Table 6-20 Printing Press Results, Scenario 6

Method	Region of Interest	Iteration	x1	x2	x3	$\mu$	$\sigma$	$\epsilon^i$	LM	W	$\epsilon^{i+1}$
MISWT	cuboid	1	1.00	1.00	1.00	911.10	137.50	137.50	2.35	-1.00	214.75
		2						214.75	Outside	Extreme Value	
		3	1.00	1.00	1.00	911.10	137.50	137.50	2.35	2.00	89.76
		4	1.00	1.00	0.32	767.80	89.76	89.76	3.91	0.00	Stop

### **Test Problem 4: Propellant Mixture**

The propellant mixture problem was analyzed by Del Castillo and Montgomery (Del Castillo and Montgomery 1993). This problem moves away from the typical dual response problem. In this experiment there were three factors or components to mix (fuel, oxidizer, and binder) with the objective of maximizing the primary response (burning rate) subject to satisfactory level of two secondary minimization responses (variability of burning rate and cost). The satisfactory levels indicated were variability less than or equal to 4.5 and cost less than or equal to 20. The experimental design along with the Excel and Solver formulations are provided in Appendix E.

Table 6-21 provides a summary of the process used for solving the propellant mixture problem with the decision-maker defined bounds on the secondary responses.

Table 6-21 Propellant Mixture Process, Scenario 1

Step #	Description	Figure #
Step 1:	Responses: maximize burning rate, minimize variability and cost. Factors: fuel, oxidizer, and binder.	
Step 2:	Conduct an experimental design.	E-1
Step 3:	$Y1 = 35.4945x_1 + 42.7755x_2 + 70.3612x_3 + 16.0204x_1x_2 + 36.3347x_1x_3 + 136.8204x_2x_3 + 854.9818x_1x_2x_3$ $Y2 = 3.8815x_1 + 9.0387x_2 + 13.6339x_3 - 0.1904x_1x_2 - 16.6195x_1x_3 - 27.6761x_2x_3$ $Y3 = 23.1333x_1 + 19.7333x_2 + 14.7333x_3$	E-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: 0, 1.	E-1
Step 5:	Primary response: maximize Y1 (burning rate).	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the secondary responses. Y1: 0, 106.65; Y2: 0, 13.63; Y3: 0, 21.13. The decision-maker specified upper bounds on Y2 and Y3 as 4.5 and 20, respectively. These are within the extreme value bounds, so will be used.	E-3
Step 7:	Maximize $Y1(x_1, x_2, x_3)$ Subject to : $0 \leq Y2 \leq 4.5$ $0 \leq Y3 \leq 20$ $x_1 + x_2 + x_3 = 1$	E-4, E-5
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	E-5
Step 9:	Lagrange multipliers $Y2 = 0$ and $Y3 = 0$ from the "Sensitivity Report". Using the slack values for Y2 of 0.32 and Y3 of 1.76 from the "Answer Report", perturb the values of $\epsilon$ ( $Y2$ ) = 4.17 and $\epsilon$ ( $Y2$ ) = 18.23.	E-5, E-6
Step 7:	Re-optimize.	E-6
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	E-6
Step 9:	Lagrange multiplier $Y2 = 5.02$ and $Y3 = 2.63$ .	E-6
Step 10:	Given that the Y1 equals 106.61 and Y2 equals 4.17, please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of -1.  Given that the Y1 equals 106.61 and Y3 equals 18.23, please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	E-6
Step 11:	Update $\epsilon^i$ to $\epsilon^{i+1}$ ( $Y2$ ) = 4.46; $\epsilon^i$ to $\epsilon^{i+1}$ ( $Y3$ ) = 7.04.	E-6
Step 7:	Repeat steps 7 for iteration 3. The problem is infeasible. Return to Iteration 2 and re-evaluate worth assessment.	E-7
Step 7-11	Repeat steps 7-11 for iteration 4. Worth assessment for $Y2=0$ ; $Y3 = 2$ , therefore update $\epsilon^i$ to $\epsilon^{i+1}$ only for $Y3 = 6.96$ and return to step 7.	E-8
Step 7-10:	Repeat steps 7-9 for iteration 5. $W=0$ for both responses. Stop.	E-9



The initial solution found with the new method was the same as that found by Del Castillo and Montgomery (.21, .34, .44) with Y1=106.65, Y2=4.18, and Y3=18.24. This problem provides an example of a situation where  $\epsilon$  modified the problem so that it became infeasible. An alternative worth assessment was provided to modify  $\epsilon$  to a feasible solution. The decision-maker provided a worth assessment = 0, so the process stopped.

Table 6-22 Propellant Mixture Results, Scenario 1

Method	Iteration	$x_1$	$x_2$	$x_3$	Y1	Y2	Y3	$\epsilon_2^i$	$\epsilon_3^i$	LM <sub>2</sub>	LM <sub>3</sub>	W <sub>2</sub>	W <sub>3</sub>	$\epsilon^{i+1}_2$	$\epsilon^{i+1}_3$
Del Castillo & Montgomery		0.21	0.34	0.44	106.65	4.17	18.23								
MISWT	1	0.21	0.34	0.44	106.65	4.18	18.24	4.50	20.00	0.00	0.00			Perturb	
	2	0.20	0.35	0.44	106.61	4.17	18.23	4.17	18.23	5.02	2.63	-1.00	2.00	4.46	7.04
	3	0.00	0.11	0.33	32.85	4.46	7.04	4.46	7.04	Infeasible					
	4	Return to Iteration 2						4.17	18.23	5.02	2.63	0.00	2.00	4.17	6.96
	5	0.20	0.35	0.44	106.61	4.17	18.23	4.17	18.23	5.02	2.63	0.00	0.00	Stop	

A second scenario was run assuming that the decision-maker was not able to provide bounds for the secondary responses. The extreme value method was used to set the bounds. The process was similar to that of Scenario 1 ending in a solution where the decision-maker is satisfied. The solution point was the same found by Del Castillo and Montgomery as well as the initial solution found in Scenario 1 of the new method.

Table 6-23 Propellant Mixture Process, Scenario 2

Step #	Description	Figure #
Step 1:	Responses: maximize burning rate, minimize variability and cost. Factors: fuel, oxidizer, and binder.	
Step 2:	Conduct an experimental design.	E-1
Step 3:	$Y1 = 35.4945x_1 + 42.7755x_2 + 70.3612x_3 + 16.0204x_1x_2 + 36.3347x_1x_3 + 136.8204x_2x_3 + 854.9818x_1x_2x_3$ $Y2 = 3.8815x_1 + 9.0387x_2 + 13.6339x_3 - 0.1904x_1x_2 - 16.6195x_1x_3 - 27.6761x_2x_3$ $Y3 = 23.1333x_1 + 19.7333x_2 + 14.7333x_3$	E-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: 0, 1.	E-1
Step 5:	Primary response: maximize Y1 (burning rate).	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the secondary responses. Y1: 0, 106.65; Y2: 0, 13.63; Y3: 0, 21.13. Scenario 2 assumes that the decision-maker does not know where to set the bounds on the secondary responses. Therefore, the extreme values will be used.	E-3
Step 7:	Maximize $Y1(x_1, x_2, x_3)$ Subject to : $0 \leq Y2 \leq 4.5$ $0 \leq Y3 \leq 20$ $x_1 + x_2 + x_3 = 1$	E-10, E-11
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	E-11
Step 9:	Lagrange multipliers $Y2 = 0$ and $Y3 = 0$ from the “Sensitivity Report”. Using the slack values for Y2 of 9.45 and Y3 of 4.90 from the “Answer Report”, perturb the values of $\epsilon$ ( $Y2$ ) = 4.17 and $\epsilon$ ( $Y2$ ) = 18.23.	E-11, E-12
Step 7:	Re-optimize.	E-12
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	E-12
Step 9:	Lagrange multiplier $Y2 = 5.02$ and $Y3 = 2.63$ .	E-12
Step 10:	Given that the Y1 equals 106.61 and Y2 equals 4.17, please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of -1.  Given that the Y1 equals 106.61 and Y3 equals 18.23, please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2.	E-12
Step 11:	Update $\epsilon^i$ to $\epsilon^{i+1}$ ( $Y2$ ) = 4.46; $\epsilon^i$ to $\epsilon^{i+1}$ ( $Y3$ ) = 7.04.	E-12
Step 7:	Repeat steps 7 for iteration 3. The problem is infeasible. Return to Iteration 2 and re-evaluate worth assessment.	E-13
Step 7-11	Repeat steps 7-11 for iteration 4. Worth assessment for $Y2=0$ ; $Y3 = 2$ , therefore update $\epsilon^i$ to $\epsilon^{i+1}$ only for $Y3 = 6.96$ and return to step 7.	E-14
Step 7-10:	Repeat steps 7-9 for iteration 5. $W=0$ for both responses. Stop.	E-15

Table 6-24 Propellant Mixture Results, Scenario 2

Method	Iteration	$x_1$	$x_2$	$x_3$	Y1	Y2	Y3	$\epsilon_2^i$	$\epsilon_3^i$	LM <sub>2</sub>	LM <sub>3</sub>	W <sub>2</sub>	W <sub>3</sub>	$\epsilon^{i+1}_2$	$\epsilon^{i+1}_3$
MISWT	1	0.21	0.34	0.44	106.65	4.18	18.24	13.63	23.13	0.00	0.00			Perturb	
	2	0.20	0.35	0.44	106.61	4.17	18.23	4.17	18.23	5.02	2.63	-1.00	2.00	4.46	7.04
	3	0.00	0.11	0.33	32.85	4.46	7.04	4.46	7.04	Infeasible					
	4	Return to Iteration 2						4.17	18.23	5.02	2.63	0.00	2.00	4.17	6.96
	5	0.20	0.35	0.44	106.61	4.17	18.23	4.17	18.23	5.02	2.63	0.00	0.00	Stop	

### Test Problem 5: Foaming Properties of Dialyzed Whey Protein

Numerous researchers have conducted variations of the whey protein concentrates problem. Khuri and Conlon (1981) conducted an experiment utilizing five factors (heating temperature, pH level, redox potential, sodium oxalate, and sodium lauryl sulfate) to investigate their impact on the foaming properties of whey protein concentrates. There were four maximization responses: maximum overrun, time at first drop, indented protein, and soluble protein. The data for the experimental design and Excel screen shots can be found in Appendix F.

The whey protein problem provides an example with four responses. Table 6-25 provides a summary of the process and Table 6-26 provides the results for the whey protein problem. Multiple perturbations were required before the process ended in a point where the decision-maker indicated worth assessments = 0 for all partial trade-offs. Khuri and Conlon used a combined method. The solution points found by the constrained method were different as the responses were not equally weighted. The final preferred solution had a higher Y1 and a lower value for Y2, Y3, and Y4 than the solution returned by Khuri and Conlon. This process demonstrated how more than three responses can be solved allowing for the decision-maker to trade-off multiple responses.

Table 6-25 Whey Protein Process, Scenario 1

Step #	Description	Figure #
Step 1:	Responses: maximizing maximum overrun, time at first drop, indented protein, and soluble protein. Factors: heating temperature, pH level, redox potential, sodium oxalate, and sodium lauryl sulfate.	
Step 2:	A central composite design with 6 center points.	F-1
Step 3:	$Y1 = 1176.98 - 176.08x_1 - 18.17x_2 + 57.58x_3 + 21.83x_4 + 19.08x_5 - 56.97x_1^2 - 23.84x_2^2 - 44.84x_3^2 - 34.09x_4^2 - 9.22x_5^2$ $- 25.00x_1x_2 - 48.88x_1x_3 - 12.25x_1x_4 - 36.75x_1x_5 - 2.88x_2x_3 + 16.75x_2x_4 + 11.00x_2x_5 - 4.38x_3x_4 + 2.38x_3x_5 - 1.00x_4x_5$ $Y2 = 9.44 + 1.08x_1 + 3.69x_2 + 1.60x_3 - 0.21x_4 - 1.50x_5 + 0.56x_1^2 - 0.13x_2^2 + 0.24x_3^2 + 0.37x_4^2 + 1.06x_5^2$ $+ 2.15x_1x_2 - 0.46x_1x_3 - 2.75x_1x_4 - 2.13x_1x_5 + 0.19x_2x_3 - 0.85x_2x_4 - 2.35x_2x_5 - 0.96x_3x_4 - 1.21x_3x_5 + 0.63x_4x_5$ $Y3 = 77.79 - 10.12x_1 - 8.68x_2 - 0.10x_3 - 0.71x_4 + 1.37x_5 - 4.16x_1^2 - 4.77x_2^2 - 2.36x_3^2 - 0.34x_4^2 - 0.33x_5^2$ $- 6.21x_1x_2 + 2.77x_1x_3 + 1.77x_1x_4 + 0.71x_1x_5 - 1.68x_2x_3 - 0.26x_2x_4 + 0.48x_2x_5 + 0.04x_3x_4 + 1.51x_3x_5 + 0.03x_4x_5$ $Y4 = 103.81 - 8.25x_1 + 7.51x_2 + 2.38x_3 + 1.13x_4 + 1.66x_5 - 7.37x_1^2 - 4.66x_2^2 - 2.58x_3^2 - 2.01x_4^2 + 1.07x_5^2$ $- 0.11x_1x_2 + 0.47x_1x_3 + 2.09x_1x_4 + 0.77x_1x_5 - 0.78x_2x_3 + 0.29x_2x_4 - 0.16x_2x_5 + 1.44x_3x_4 + 4.09x_3x_5 + 0.29x_4x_5$	F-2
Step 4:	Region of interest is cuboid, defined by the values of the x(s) in the experimental design: -2, 2.	F-1
Step 5:	Primary response: maximize Y1 (maximum overrun).	
Step 6:	The region of interest is used in a single response optimization to find the extreme values of the secondary responses. Y1: -44.86, 1640.54; Y2: -22.44, 85.04; Y3: 42.45, 97.79; Y4: -13.41, 127.15. No bounds were specified. It is assumed that the decision-maker does not know where to set the bounds on the secondary responses. Therefore, the extreme values will be used.	F-3
Step 7:	Maximize $Y1(x_1, x_2, x_3, x_4, x_5)$ Subject to : $0 \leq Y2 \leq 85.04$ $0 \leq Y3 \leq 97.79$ $0 \leq Y4 \leq 127.15$	F-4, F-5
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	F-5
Step 9:	Lagrange multipliers $Y2 = 0$ , $Y3 = 0$ , and $Y4 = 0$ from the "Sensitivity Report". Using the slack values for Y2 of 68.2, Y3 = 39.2, and Y4 of 24.3 from the "Answer Report", perturb the values of $\epsilon$ (Y2) = 16.74, $\epsilon$ (Y3) = 58.36, and $\epsilon$ (Y4) = 102.79. (Note: larger constants were subtracted in order to return Lagrange multipliers > 0.	F-5, F-6
Step 7:	Re-optimize.	F-6
Step 8:	Is the decision-maker satisfied with the solution found in Step 7? No.	F-6
Step 9:	Lagrange multiplier $Y2 = .09$ , $Y3 = .02$ , and $Y4 = .01$ .	F-6
Step 10:	Given that the Y1 equals 1640.54 and Y2 equals 16.74, please indicate your preference for decreasing Y1 by 0.09 units for one unit increase in Y2, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 2. Given that the Y1 equals 1640.54 and Y3 equals 58.36, please indicate your preference for decreasing Y1 by 0.02 units for one	F-6

Step #	Description	Figure #
	unit increase in Y3, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of -2.  Given that the Y1 equals 1640.54 and Y4 equals 102.79, please indicate your preference for decreasing Y1 by 0.01 units for one unit increase in Y4, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off. Decision-maker indicates a worth assessment of 1.	
Step 11:	Update $\epsilon^1$ to $\epsilon^{i+1}$ (Y2) = 25.62; $\epsilon^1$ to $\epsilon^{i+1}$ (Y3) = 40.48; $\epsilon^1$ to $\epsilon^{i+1}$ (Y4) = 112.52.	F-6
Step 7-9:	Repeat steps 7-9 for iteration 3. Lagrange multiplier for Y2 = 0 and Y4 = 0 so perturb the values of $\epsilon$ for Y2 to 15.69 and Y4 to 93.54 using the slack values of 9.83 and 18.87 respectively from the Answer Report and return to Step 7.	F-7, F-8
Step 7-9:	Repeat steps 7-9 for iteration 4. Lagrange multiplier for Y2 = 0 so perturb the value of $\epsilon$ for Y2 to 11.29 using the slack value of 4.3 from the Answer Report and return to Step 7.	F-8, F-9
Step 7-11:	Repeat steps 7-11 for iteration 5. Worth assessment for Y3 = 0, therefore update $\epsilon^1$ to $\epsilon^{i+1}$ only for Y2 = 12.99 and Y4 = 106.36 and return to step 7.	F-9
Step 7-9:	Repeat steps 7-9 for iteration 6. Lagrange multiplier for Y4 = 0 so perturb the value of $\epsilon$ for Y4 to 100.39 using the slack value of 5.87 from the Answer Report and return to Step 7.	F-10, F-11
Step 7-10:	Repeat steps 7-10 for iteration 7. All worth assessments = 0. Stop.	F-11

Table 6-26 Whey Protein Results, Scenario 1

Method	Iteration	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	Y1	Y2	Y3	Y4	$\epsilon_2^i$	$\epsilon_3^i$	$\epsilon_4^i$	LM <sub>2</sub>	LM <sub>3</sub>	LM <sub>4</sub>	W <sub>2</sub>	W <sub>3</sub>	W <sub>4</sub>	$\epsilon_2^{i+1}$	$\epsilon_3^{i+1}$	$\epsilon_4^{i+1}$	
Khuri & Conlon		-1.31	-0.16	0.30	0.46	1.72	1433.55	16.96	81.63	106.64													
MISWT	1	-2.00	1.33	1.70	0.87	2.00	1640.54	16.84	58.56	102.89	85.04	97.79	127.15	0.00	0.00	0.00	Perturb						
	2	-2.00	1.35	1.70	0.87	2.00	1640.54	16.74	58.36	102.79	16.74	58.36	102.79	0.09	0.02	0.01	2.00	-2.00	1.00	25.62	40.48	112.52	
	3	-2.00	2.00	2.00	1.62	2.00	1613.32	15.79	40.48	93.64	25.62	40.48	112.52	0.00	7.25	0.00	Perturb						
	4	-0.04	2.00	-0.52	2.00	0.76	1018.57	11.39	40.48	93.54	15.69	40.48	93.54	0.00	11.03	9.73	Perturb						
	5	-0.04	2.00	-0.53	2.00	0.80	1018.56	11.29	40.48	93.54	11.29	40.48	93.54	0.35	11.11	9.52	1.00	0.00	2.00	12.99	40.48	106.36	
	6	-1.53	2.00	2.00	1.61	1.68	1492.04	12.99	40.48	100.49	12.99	40.48	106.36	42.66	32.25	0.00	Perturb						
	7	-1.54	2.00	2.00	1.60	1.67	1492.04	12.99	40.48	100.39	12.99	40.48	100.39	43.08	32.29	0.08	0.00	0.00	0.00	Stop			

## Conclusions

The variety of test problems in this section demonstrated how the proposed modified interactive surrogate worth trade-off (MISWT) method can successfully be utilized to solve multiple response surface optimization problems. Furthermore, it demonstrated an accessible computer-based implementation of the algorithm.

Unlike many of the current constrained methods that focus on two responses, the MISWT method was tested on problems ranging from two (silicon wafer, chemical, and printing press) to four responses (whey protein).

Many of the current approaches also focused on the dual response problem. However, the MISWT was demonstrated on dual response (silicon wafer, chemical, and printing press), mixture (propellant), and multiple maximization response (whey protein) problems. The complexity of the responses also varied demonstrating all quadratic responses as well as a combination of quadratic and linear (chemical) responses. The number of independent variables also varied from two (silicon wafer and chemical) to five (whey protein).

The examples demonstrated that the method can handle situations where the decision-maker has a target (printing press) or bound (silicon wafer, propellant) in mind as well as when the decision-maker does not know (chemical, whey protein). Furthermore, the problems with decision-maker specified targets or bounds were also run using the extreme values, assuming that the decision-maker did not know how to set the bounds. The silicon wafer and propellant problems demonstrated a situation where the scenario using the extreme values, or wider bounds, produced the same initial solution point as that found when the decision-maker had a narrower bound specified. The printing press problem (scenario 2) demonstrated a situation where the

methodology does provide the ability to get to the same 500 target starting from the extreme values if the decision-maker does not know ahead of time the goal. However, it does demonstrate that if a target is known, the solution can be reached quicker when it is used.

Swapping out the primary objective was demonstrated with the silicon wafer and printing press problems. Iteration 3 of Scenario 4 of the printing press problem demonstrates that maximizing the mean can return the same or similar solution as that reached by minimizing the standard deviation (Iteration 11, Scenario 2).

In order to test for local optima, it is recommended that alternative initial starting points are used. All problems were tested using multiple initial starting points. The only problem to return alternative optima was the printing press problem. This was also observed in scenario 2 where the initial solution returned a mean of 139 rather than around 500 where the other scenarios were. The printing press problem is non-convex.

Various potential terminating criteria were demonstrated including worth assessments equal to 0 (silicon wafer scenario 2, chemical experiment, printing press scenarios, and whey protein), convergence to same point (silicon wafer scenarios 1 and 3), infeasibility (propellant) and when  $\epsilon$  is updated to a point outside of the extreme values (printing press scenario 6). It was shown how the process can manage these situations.

Finally, it was demonstrated that the proposed MISWT method can return solutions that compare to existing constrained methods (Shin and Cho, Del Castillo, Del Castillo and Montgomery). All problems demonstrated how when the decision-maker preferences are explicitly taken into consideration an alternative solution may be preferred over the initial solution. The decision-maker may be willing to make trade-offs that cannot be captured with implicit methods.

Therefore, it is concluded that MISWT is a reasonable alternative to existing constrained methods, and it can be argued that MISWT is a better method due to the ability of the method to explicitly consider decision-maker trade-offs and allow the decision-maker to find preferred solutions.

### **Evaluation of New Method Compared to Conditions and Characteristics**

The MISWT method successfully meets the conditions and assumptions required for constrained methods to find the most preferred non-dominated solution:

- Generate non-dominated solutions, as guaranteed by the use of the  $\epsilon$ -constraint approach.
- Generate solutions consistent with decision-maker preferences among the response objectives, as guaranteed by the interactive process of the methodology. Decision-maker preferences are explicitly considered in the methodology and the method allows for the decision-maker to make trade-offs among the responses.

Furthermore, the method addresses the following recommended characteristics:

- High ease of use: the method was easily implemented using an Excel add-in, Premium Solver. No programming was required.
- Low level of cognitive burden: Tarvainen's 5-point scale was utilized in an effort to reduce the moderate level of cognitive burden of the ISWT method. Furthermore, it was demonstrated that if the decision-maker cannot specify bounds or targets, the method can still find the decision-maker's preferred solution.
- Reflects decision-maker preferences



- Uses explicit preferences: The method explicitly uses the decision-maker's preference for the primary response and bounds or targets on secondary responses.
- Allows for trade-offs among solutions: The method allows for explicit trade-offs among responses by the decision-maker to determine the most preferred solution.
- Few limitations on the type and number of responses/objectives: The method was demonstrated on both linear and quadratic responses as well as a variety of problems with the number of responses ranging from two to four.
- Ability to find all non-dominated solutions: As guaranteed by the use of the  $\epsilon$ -constraint approach, it is possible to find all non-dominated solutions. The interactive nature of the method allows for the preferred non-dominated solution, regardless of response convexity, to be found without having to generate all of the non-dominated solutions. However, multiple starting points should be used due to the possibility of obtaining local optima.
- Low number of iterations to find "best compromise solution:" It was demonstrated that the number of iterations to find the preferred solution ranged from 1 to 13 with not all of the iterations requiring decision-maker involvement.

The modified interactive surrogate worth trade-off methodology successfully addresses the required and recommended conditions for multiple response surface methods. Furthermore, it provides distinct advantages over current MRSM approaches in that it does not require convexity to find all non-dominated solutions and explicitly takes into consideration decision-maker preferences.

## CHAPTER SEVEN: CONCLUSIONS

### Summary and Conclusions

This research has developed a new constrained multiple response surface methodology based on the multi-criteria decision-making technique, interactive surrogate worth trade-off. The theoretical contribution is the development of the conditions and assumptions necessary for constrained multiple response surface optimization methods. The methodological contribution is the development of a new constrained multiple response surface method that explicitly takes into account the preferences of the decision-maker as well as the conditions and assumptions identified through this research. The practical contribution is a constrained multiple response surface methodology that is reasonable for the average decision-maker to use that explicitly takes into account the preferences of the decision-maker as demonstrated with the test problems.

The current multiple response surface methods were summarized and categorized. A methods tree was developed to depict the evolution of multiple response surface methodologies. Gaps in the research were identified, particularly with respect to methods that explicitly consider decision-maker preferences and allow for explicit trade-offs among the responses. The larger area of multi-criteria decision-making was explored and the interactive surrogate worth trade-off method was identified for modification to apply to response surface optimization problems.

The conditions and assumptions required for constrained methods to find the most preferred non-dominated solution are that they must generate non-dominated solutions and must generate solutions consistent with decision-maker preferences. If a Lagrangian method is used,

then convexity is also required in order to generate all non-dominated solutions. Far fewer assumptions and conditions are required for constrained methods compared to those for combined methods.

Recommended characteristics identified for development of a multiple response method included a high ease of use, low level of cognitive burden, reflects decision-maker preferences including explicit use of preferences and allows for trade-offs among solutions, few limitations on the type and number of responses/objectives, the ability to find all non-dominated solutions, and a low number of iterations to find the “most preferred solution.”

A new constrained multiple response surface methodology was developed based on the multi-criteria decision-making technique, interactive surrogate worth trade-off. The new methodology maintains the essence of ISWT while simplifying the process based on the particular specifications of multiple response surface problems. The MISWT methodology was demonstrated on five test problems from the literature. The variety of test problems in this section demonstrated how the proposed modified interactive surrogate worth trade-off (MISWT) method can successfully be utilized to solve multiple response surface optimization problems. Furthermore, it demonstrates the accessibility of a computer-based implementation of the algorithm.

The MISWT method successfully meets the conditions and assumptions required for constrained methods to find the most preferred non-dominated solution:

- Generate non-dominated solutions, as guaranteed by the use of the  $\epsilon$ -constraint approach. And the method does not require convexity.
- Generate solutions consistent with decision-maker preferences among the response objectives, as guaranteed by the interactive process of the methodology. Decision-

maker preferences are explicitly considered in the methodology and allows for the decision-maker to make trade-offs among the responses.

The MISWT addresses the recommended characteristics as follows:

- High ease of use: the method was easily implemented using an Excel add-in, Premium Solver. No programming was required.
- Low level of cognitive burden: Tarvainen's 5-point scale was utilized in an effort to reduce the moderate level of cognitive burden of the ISWT method. Furthermore, it was demonstrated that if the decision-maker cannot specify bounds or targets, the method can still find the decision-maker's preferred solution.
- Reflects decision-maker preferences
  - Uses explicit preferences: The method explicitly uses the decision-maker's preference for the primary response and bounds or targets on secondary responses.
  - Allows for trade-offs among solutions: The method allows for explicit trade-offs among responses by the decision-maker to determine the most preferred solution.
- Few limitations on the type and number of responses/objectives: The method was demonstrated on both linear and quadratic responses as well as a variety of problems with the number of responses ranging from two to four.
- Ability to find all non-dominated solutions: As guaranteed by the use of the  $\epsilon$ -constraint approach, it is possible to find all non-dominated solutions. The interactive nature of the method allows for the preferred non-dominated solution, regardless of response convexity, to be found without having to generate all of the non-dominated solutions.

- Low number of iterations to find the “best compromise solution:” It was demonstrated that the number of iterations to find the preferred solution ranged from 1 to 13 with not all of the iterations requiring decision-maker involvement.

It was demonstrated that the proposed MISWT method can return solutions that compare to existing constrained methods (Shin and Cho, Del Castillo, Del Castillo and Montgomery). Furthermore, the test problems demonstrated that when the decision-maker preferences are explicitly taken into consideration an alternative solution may be preferred over the initial solution. The decision-maker may be willing to make trade-offs that cannot be captured with implicit methods. Therefore, it is concluded that MISWT is a reasonable alternative to existing constrained methods. Furthermore, it can be argued that MISWT is a better method due to the ability of the method to explicitly consider decision-maker trade-offs and allow the decision-maker to find preferred solutions.

### **Recommendations for Future Research**

An important area for future research is to examine the use of the MISWT method with a real decision-maker. Specifically, the methodology should be further evaluated by decision-makers on ease of use, cognitive burden, and level of interaction. A suggested problem and questions are included in Appendix G.

An alternative approach utilizing the surrogate worth trade-off (SWT) approach first and then employing the interactive portion of the process should be explored. The SWT method generates a set of alternative optima and then the decision-maker is asked to assess each point and a regression is run to find the optimal solution based on this feedback. This approach may

provide for a method to find a starting solution when the decision-maker does not have targets or bounds in mind for the secondary response and may reduce the number of iterations required to reach a preferred solution. Another alternative would be to generate a set of alternative optima and have the decision-maker select one preferred solution to start the interactive portion from that point. These alternative methodologies should be compared to determine if one is easier and reduces the cognitive burden on the decision-maker.

Another area of future research is the interpretation of “units” in decision-maker trade-offs. This research assumes that the decision-maker can make reasonable trade-offs among responses with different units (e.g., time vs. standard deviation). Furthermore, when move in a direction outside of the region of operability, whether or not to interact with the decision-maker in original or coded units for the decision to expand the region should also be addressed.

Another area of future development is to further automate the MISWT process to make the analysis easier on the decision-maker and/or analyst. Macros could be utilized to provide the functionality within Excel with the Solver add-in to automate the transition between iterations and provide a more interactive, user-friendly interface.

## **APPENDIX A: TEST PROBLEMS FOUND IN MRSM LITERATURE**

Table A-1 Test Problems in the Literature

Use	Test Problem	Reference	# Factors	# Responses
	Beef stew military field ration	(Wurl and Albin 1999), (Tsui, et al. 2001)	5	4 (2 $\mu$ , 2 $\sigma$ )
	Bonding process	(Khoo and Chen 2001)	3	2
	Catapult experiment	(Luner 1994), (Del Castillo 1996)	3	2 ( $\mu$ , $\sigma$ )
Y	Chemical experiment	(Del Castillo 1996), (Gheware 2003), (Osborne 1997)	2	2 ( $\mu$ , $\sigma$ )
	Colloidal gas aphrons study	(Jauregi, Gilmour, and Varley 1997), (Kim and Lin 2006)	3	6 (3 $\mu$ , 3 $\sigma$ )
	Force transducer	(Romano, Varetto, and Vicario 2004)	5	2
	Hard disk driver	(Su and Tong 1997), (Liao 2004)	6	4
Y	Silicon wafers	(Shin and Cho 2005)	2	2 ( $\mu$ , $\sigma$ )
	Integrated circuit	(Yang and Tseng 2002), (Yang, Kuo, and Chou 2005)	5	2 ( $\mu$ , $\sigma$ )
	Inventory model simulation	(Rees, Clayton, and Taylor 1985)	2	3
	Liquid chromatography assay	(Peterson 2004), (Miro-Quesada and Del Castillo 2004a)	3	4
	Mechanical properties of steel	(Kim and Lin 2000) (Xu, et al. 2004)	6	3
	Mixed resolution design	(Borrer and Montgomery 2000), (Miro-Quesada and Del Castillo 2004b)	7	2 ( $\mu$ , $\sigma$ )
	Numerical example	(Myers, Khuri, and Vining 1992), (Koksoy and Doganaksoy 2003)	4	2 ( $\mu$ , $\sigma$ )
	Polymer experiment	(Myers and Montgomery 1995), (Vining 1998)	3	2
	Printed circuit boards	(Plante 2001)	7	5
Y	Printing machine	(Box and Draper 1987), (Vining and Myers 1990), (Del Castillo and Montgomery 1993), (Lin and Tu 1995), (Copeland and Nelson 1996), (Tang and Xu 2002), (Koksoy and Doganaksoy 2003), (Gheware 2003), (Osborne 1997)	3	2 ( $\mu$ , $\sigma$ )
Y	Propellant mixture problem	(Montgomery and Voth 1991), (Del Castillo and Montgomery 1993)	3	3 ( $\mu$ , $\sigma$ , \$)
	Pseudolatex formation	(Frisbee and McGinity 1994), (Peterson 2004)	3	2
	Semiconductor manufacturing facility	(Montgomery 2001), (Miro-Quesada and Del Castillo 2004b), (Koksoy and Doganaksoy 2003)	5	2 ( $\mu$ , $\sigma$ )
	Sheet metal hydroforming process	(Kuhnt and Erdbrugge 2004)	4	2
	Solder experiment	(Lee and Nelder 2003), (Myers, Brenneman, and Myers 2005)	8	2 ( $\mu$ , $\sigma$ )
	Tire tread compound	(Derringer and Suich 1980), (Xu, et al. 2004), (Ames, et al. 1997)	3	4
	Vehicle side impact	(Youn and Choi 2004)	11	11
	VLSI device design	(Young, Teplik, Tweed, Tracht, and Alvarez 1991), (Plante 2001)	4	9



Use	Test Problem	Reference	# Factors	# Responses
	Wheel cover component experiment	<b>(Harper, Kosbe, and Peyton 1987)</b> , (Kim and Lin 2006), (Kuhnt and Erdbrugge 2004)	7	4 (2 $\mu$ , 2 $\sigma$ )
	Whey protein concentrates	<b>(Khuri and Conlon 1981)</b> , (Gheware 2003), (Osborne 1997), (Diaz-Garcia, Ramos-Quiroga, and Cabrera-Vicencio 2005)	2	4
Y	Foaming properties of whey protein	<b>(Richert, Morr, and Cooney 1974)</b> (Khuri and Conlon 1981), (Xu, et al. 2004)	5	4
	Foaming properties of whey protein (different responses used)	<b>(Khuri and Cornell 1987)</b> , ( <i>Miro-Quesada and Del Castillo 2004a</i> )	5	3
	Wire bonding process	<b>(Del Castillo, Montgomery, and McCarville 1996)</b>	3	6

\*Bold type face indicates the earliest reference identified for the problem. Italics indicate constrained methods.

## **APPENDIX B: SILICON WAFER PROBLEM**

Silicon Wafers (Shin and Cho, 2005)

<b>X1</b>	<b>X2</b>	<b>Y1</b>	<b>Y2</b>	<b>Y3</b>	<b>Y4</b>	<b>Y (avg)</b>	<b>Y (std)</b>
-1	-1	76.3	73.5	68.8	74.2	73.20	3.17
1	-1	70.4	81.2	76.7	79.6	76.98	4.76
-1	1	76.6	72.0	77.7	78.5	76.20	2.91
1	1	72.3	67.5	75.7	72.7	72.05	3.39
-1.414	0	70.6	75.8	69.9	71.5	71.95	2.65
1.414	0	74.1	80.2	76.2	77.1	76.90	2.53
0	-1.414	78.5	68.7	76.2	75.3	74.68	4.21
0	1.414	70.2	76.3	79.2	75.9	75.40	3.77
0	0	74.1	71.8	72.5	71.9	72.58	1.06
0	0	72.1	70.4	73.3	74.2	72.50	1.64
0	0	74.2	69.8	71.2	72.2	71.85	1.85
0	0	70.1	69.3	71.6	72.5	70.88	1.44
0	0	69.8	70.6	71.6	74.1	71.53	1.87

X1=mould temperature (99.6, 301.4)

X2=injection flow rate (40, 70)

Y=quality of printing process (4 replicates)

Figure B-1 Silicon Wafers, Experimental Design

Fitted Responses:

$$\mu = 71.87 + 0.83 x_1 - 0.11 x_2 + 1.25 x_1^2 + 1.55 x_2^2 - 1.98 x_1 x_2$$

$$\sigma = 1.57 + 0.24 x_1 - 0.28 x_2 + 0.58 x_1^2 + 1.28 x_2^2 - 0.28 x_1 x_2$$

Figure B-2 Silicon Wafers, Fitted Responses

“ISWT”

	C	D	E	F	G	H	I	J	K
1		Constant	x1	x2	x1sq	x2sq	x1x2		Response
2	Settings		1.41	-1.41	2.00	2.00	-2.00		Value
3	$\mu$ (Y1)	71.87	0.83	-0.11	1.25	1.55	-1.98		82.76
4	$\sigma$ (Y2)	1.57	0.24	-0.28	0.58	1.28	-0.28		6.58
5									
6	Constraints								
7		x1	>=	-1.414					
8		x1	<=	1.414					
9		x2	>=	-1.414					
10		x2	<=	1.414					
11									
12		Lower Bound		71.63					
13		Upper Bound		82.76					
14									
15		Case	>=	70.24					
16			<=	72.04					

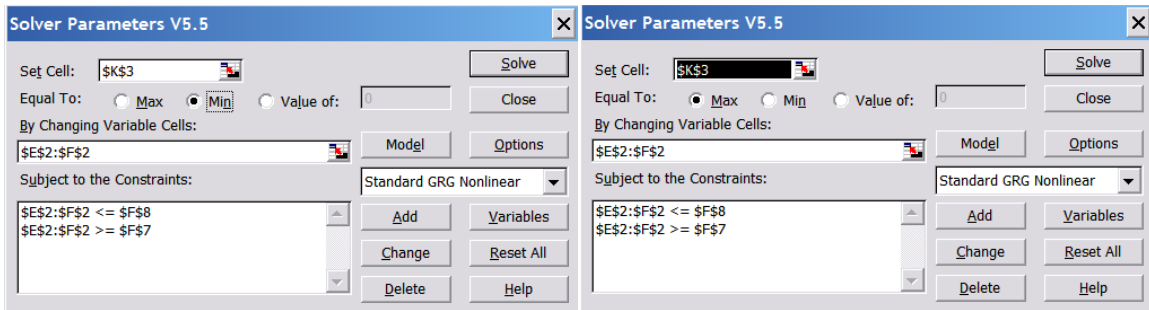


Figure B-3 Silicon Wafers, Scenario 1, Extreme Values

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.19	0.09	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.79		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	70.24					=ISWT!F15		
8				$\mu$	$\leq$	72.04					=ISWT!F16		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.00		

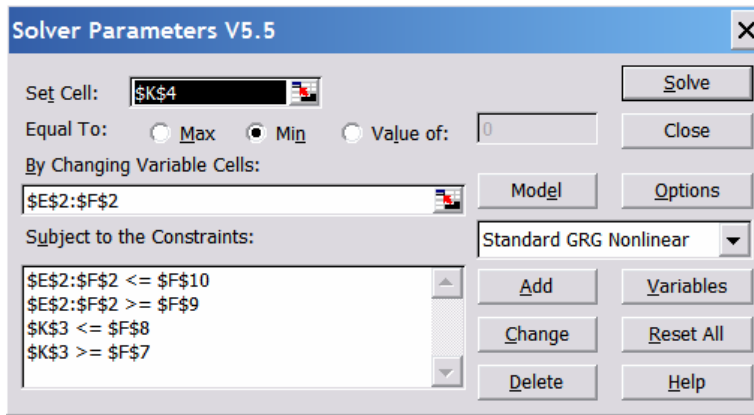


Figure B-4 Silicon Wafers, Scenario 1, Solver Parameters

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.19	0.09	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.79		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	70.24					=ISWT!F15		
8				$\mu$	$\leq$	72.04					=ISWT!F16		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"											No
13													
14	Step 9:	Lagrange multiplier=											0.00

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 1</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.19	0.00
10	\$F\$2	Settings x2		0.09	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.79	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.79	0.00

	X	Y	Z	AA	AB	AC	AD
3	<b>Microsoft Excel 11.0 Answer Report</b>						
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 1</b>						
5	<b>Result: Solver found a solution. All constraints and optimality conditions are satisfied.</b>						
6	<b>Engine: Standard GRG Nonlinear</b>						
7							
8	Target Cell (Min)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$K\$4	$\sigma$ (Y2) Value	1.57	1.54		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-0.19		
16		\$F\$2	Settings x2	0.00	0.09		
17							
18							
19	Constraints						
20		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
21		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3<=\$F\$8	Not Binding	0.2457027
22		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3>=\$F\$7	Not Binding	1.55
23		\$E\$2	Settings x1	-0.19	\$E\$2<=\$F\$10	Not Binding	1.5993901
24		\$F\$2	Settings x2	0.09	\$F\$2<=\$F\$10	Not Binding	1.324902
25		\$E\$2	Settings x1	-0.19	\$E\$2>=\$F\$9	Not Binding	1.23
26		\$F\$2	Settings x2	0.09	\$F\$2>=\$F\$9	Not Binding	1.50

Figure B-5 Silicon Wafers, Scenario 1, Iteration 1

"ISWT 2"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.17	0.10	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.80		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	71.80					=ISWT 1!F7+ISWT 1!AD22+0.01		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									<input type="text" value="No"/>		
13													
14	Step 9:	Lagrange multiplier=									<input type="text" value="0.06"/>		
15													
16	Step 10:	Given that the mean equals 71.8 and the standard deviation equals 1.54,											
17		please indicate your preference for increasing the standard deviation by 0.06 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									<input type="text" value="1.00"/>		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			1						
25				step size=			0.00861				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			<input type="text" value="71.95"/>				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 2				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		-0.17	0.00
10	\$F\$2	Settings x2		0.10	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value		71.80	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.80	0.06

Figure B-6 Silicon Wafers, Scenario 1, Iteration 2

"ISWT 3"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.10	0.14	0.01	0.02	0.01		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.95		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.58		
5													
6		Constraints											
7				$\mu$	$\geq$	71.95					=ISWT 2!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 71.95 and the standard deviation equals 1.58,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			2						
25				step size=			0.00531				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			71.98				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 3				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.10	0.00	
10	\$F\$2	Settings x2	0.14	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value	71.95	0.00	
16	\$K\$3	$\mu$ (Y1) Value	71.95	0.39	

Figure B-7 Silicon Wafers, Scenario 1, Iteration 3



"ISWT 4"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.14	0.13	0.02	0.02	0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.98		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.59		
5													
6		Constraints											
7				$\mu$	$\geq$	71.98					=ISWT 3!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 71.98 and the standard deviation equals 1.59,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			3						
25				step size=			0.00328				=G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			72.00				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 4				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.14	0.00	
10	\$F\$2	Settings x2	0.13	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value	71.98	0.00	
16	\$K\$3	$\mu$ (Y1) Value	71.98	0.39	

Figure B-8 Silicon Wafers, Scenario 1, Iteration 4

"ISWT 5"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.15	0.12	0.02	0.01	0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		72.00		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.60		
5													
6		Constraints											
7				$\mu$	$\geq$	72.00					=ISWT 4!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 72 and the standard deviation equals 1.6,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									-1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			4						
25				step size=			0.00203				=G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			71.99				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 5				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.15	0.00	
10	\$F\$2	Settings x2	0.12	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value	72.00	0.00	
16	\$K\$3	$\mu$ (Y1) Value	72.00	0.39	

Figure B-9 Silicon Wafers, Scenario 1, Iteration 5

"ISWT 6"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.15	0.12	0.02	0.02	0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.99		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.60		
5													
6		Constraints											
7				$\mu$	$\geq$	71.99					=ISWT 5!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 71.99 and the standard deviation equals 1.6,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									-1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			5						
25				step size=			0.00125				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			71.99				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 6</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.15	0.00
10	\$F\$2	Settings x2		0.12	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.99	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.99	0.39

Figure B-10 Silicon Wafers, Scenario 1, Iteration 6

“ISWT 7”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.14	0.12	0.02	0.02	0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.99		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.60		
5													
6		Constraints											
7				$\mu$	$\geq$	71.99					=ISWT 6!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 71.99 and the standard deviation equals 1.6,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			6						
25				step size=			0.00039				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			71.99				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 7				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.14	0.00	
10	\$F\$2	Settings x2	0.12	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value	71.99	0.00	
16	\$K\$3	$\mu$ (Y1) Value	71.99	0.39	

Figure B-11 Silicon Wafers, Scenario 1, Iteration 7

"ISWT 8"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.14	0.12	0.02	0.02	0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.99		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.60		
5													
6		Constraints											
7				$\mu$	$\geq$	71.99					=ISWT 7!G27		
8				$\mu$	$\leq$	72.04					=ISWT 1!F8		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.39		
15													
16	Step 10:	Given that the mean equals 71.99 and the standard deviation equals 1.6,											
17		please indicate your preference for increasing the standard deviation by 0.39 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			7						
25				step size=			0.00048				=G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			71.99				=F7+((G25*G21*ABS(K3))*(ISWT!F16-F7))		
28													

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 8</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.14	0.00
10	\$F\$2	Settings x2		0.12	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.99	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.99	0.39

Figure B-12 Silicon Wafers, Scenario 1, Iteration 8

“ISWT 9”

	A	B	C	D	E	F	G	H	I	J	K
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response
2		Settings			0.15	0.12	0.02	0.02	0.02		Value
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.99
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.60
5											
6		Constraints									
7				$\mu$	$\geq$	71.99					=ISWT 8!G27
8				$\mu$	$\leq$	72.04					=ISWT 1!F8
9				x1, x2	$\geq$	-1.41					
10				x1, x2	$\leq$	1.41					

Figure B-13 Silicon Wafers, Scenario 1, Iteration 9

“ISWT 1b”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.19	0.09	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.79		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	71.63					=ISWT!F12		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.00		

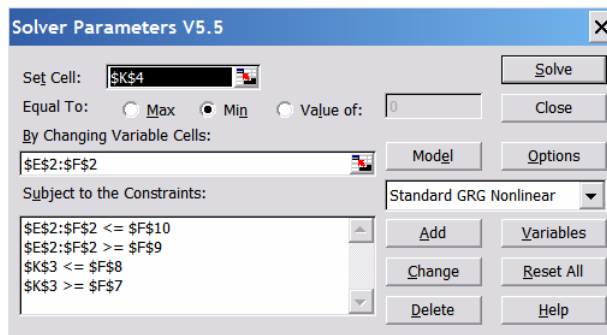


Figure B-14 Silicon Wafers, Scenario 2, Solver Parameters

"ISWT 1b"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.19	0.09	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.79		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	71.63					=ISWT!F12		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"											No
13													
14	Step 9:	Lagrange multiplier=											0.00

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 1b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.19	0.00
10	\$F\$2	Settings x2		0.09	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$3	$\mu$ (Y1) Value		71.79	0.00
16	\$K\$3	$\mu$ (Y1) Value		71.79	0.00

	X	Y	Z	AA	AB	AC	AD
3	<b>Microsoft Excel 11.0 Answer Report</b>						
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 1b</b>						
5	<b>Result: Solver found a solution. All constraints and optimality conditions are satisfied.</b>						
6	<b>Engine: Standard GRG Nonlinear</b>						
7							
8	Target Cell (Min)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$K\$4	$\sigma$ (Y2) Value	1.57	1.54		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-0.19		
16		\$F\$2	Settings x2	0.00	0.09		
17							
18							
19	Constraints						
20		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
21		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3<=\$F\$8	Not Binding	10.961976
22		\$K\$3	$\mu$ (Y1) Value	71.79	\$K\$3>=\$F\$7	Not Binding	0.16
23		\$E\$2	Settings x1	-0.19	\$E\$2<=\$F\$10	Not Binding	1.5993901
24		\$F\$2	Settings x2	0.09	\$F\$2<=\$F\$10	Not Binding	1.324902
25		\$E\$2	Settings x1	-0.19	\$E\$2>=\$F\$9	Not Binding	1.23
26		\$F\$2	Settings x2	0.09	\$F\$2>=\$F\$9	Not Binding	1.50

Figure B-15 Silicon Wafers, Scenario 2, Iteration 1

“ISWT 2b”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-0.17	0.10	0.03	0.01	-0.02		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		71.80		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		1.54		
5													
6		Constraints											
7				$\mu$	$\geq$	71.80					=ISWT!F12+ISWT 1b!AD22+0.01		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.06		
15													
16	Step 10:	Given that the mean equals 71.8 and the standard deviation equals 1.54,											
17		please indicate your preference for increasing the standard deviation by 0.06 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			1						
25				step size=			0.00861				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			78.57				=F7+((G25*G21*ABS(K3))*(ISWT!F13-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 2b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	-0.17	0.00	
10	\$F\$2	Settings x2	0.10	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value	71.80	0.00	
16	\$K\$3	$\mu$ (Y1) Value	71.80	0.06	

Figure B-16 Silicon Wafers, Scenario 2, Iteration 2



"ISWT 3b"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-0.75	2.00	0.56	-1.05		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		78.57		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		4.28		
5													
6		Constraints											
7				$\mu$	$\geq$	78.57					=ISWT 2b!G27		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"											No
13													
14	Step 9:	Lagrange multiplier=											0.49
15													
16	Step 10:	Given that the mean equals 78.57 and the standard deviation equals 4.28,											
17		please indicate your preference for increasing the standard deviation by 0.49 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=											1.00
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			2						
25				step size=			0.00486						=(G23^G24)/(SQRT((G21*ABS(K3))^2))
26													
27				$\epsilon$ =			80.17						=F7+((G25*G21*ABS(K3))*(ISWT!F13-F7))
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 3b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.41	-0.80
10	\$F\$2	Settings x2		-0.75	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value		78.57	0.00
16	\$K\$3	$\mu$ (Y1) Value		78.57	0.49

Figure B-17 Silicon Wafers, Scenario 2, Iteration 3

"ISWT 4b"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-1.03	2.00	1.06	-1.45		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		80.17		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		5.12		
5													
6		Constraints											
7				$\mu$	$\geq$	80.17					=ISWT 3b!G27		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"											No
13													
14	Step 9:	Lagrange multiplier=											0.54
15													
16	Step 10:	Given that the mean equals 80.17 and the standard deviation equals 5.12,											
17		please indicate your preference for increasing the standard deviation by 0.54 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=											1.00
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			3						
25				step size=			0.00294						= (G23^G24)/(SQRT((G21*ABS(K3))^2))
26													
27				$\epsilon$ =			80.78						=F7+((G25*G21*ABS(K3))*(ISWT!F13-F7))
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 4b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.41	-1.30
10	\$F\$2	Settings x2		-1.03	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value		80.17	0.00
16	\$K\$3	$\mu$ (Y1) Value		80.17	0.54

Figure B-18 Silicon Wafers, Scenario 2, Iteration 4

“ISWT 5b”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-1.13	2.00	1.27	-1.59		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		80.78		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		5.45		
5													
6		Constraints											
7				$\mu$	$\geq$	80.78					=ISWT 4b!G27		
8				$\mu$	$\leq$	82.76					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.56		
15													
16	Step 10:	Given that the mean equals 80.78 and the standard deviation equals 5.45,											
17		please indicate your preference for increasing the standard deviation by 0.56 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									-1.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			4						
25				step size=			0.00181				=(G23^G24)/(SQRT((G21*ABS(K3))^2))		
26													
27				$\epsilon$ =			80.49				=F7+((G25*G21*ABS(K3))*(ISWT!F13-F7))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 5b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.41	-1.47
10	\$F\$2	Settings x2		-1.13	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value		80.78	0.00
16	\$K\$3	$\mu$ (Y1) Value		80.78	0.56

Figure B-19 Silicon Wafers, Scenario 2, Iteration 5

"ISWT 6b"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-1.08	2.00	1.17	-1.53		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		80.49		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		5.29		
5													
6		Constraints											
7				$\mu$	>=	80.49				=ISWT 5b!G27			
8				$\mu$	<=	82.76				=ISWT!F13			
9				x1, x2	>=	-1.41							
10				x1, x2	<=	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.55		
15													
16	Step 10:	Given that the mean equals 80.49 and the standard deviation equals 5.29,											
17		please indicate your preference for increasing the standard deviation by 0.55 units for one unit increase in the mean,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									0.00		

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 6b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.41	-1.39
10	\$F\$2	Settings x2		-1.08	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$3	$\mu$ (Y1) Value		80.49	0.00
16	\$K\$3	$\mu$ (Y1) Value		80.49	0.55

Figure B-20 Silicon Wafers, Scenario 2, Iteration 6

“ISWT c”

	C	D	E	F	G	H	I	J	K
1		Constant	x1	x2	x1sq	x2sq	x1x2		Response
2	Settings		-0.19	0.09	0.03	0.01	-0.02		Value
3	$\mu$ (Y1)	71.87	0.83	-0.11	1.25	1.55	-1.98		71.79
4	$\sigma$ (Y2)	1.57	0.24	-0.28	0.58	1.28	-0.28		1.54
5									
6	Constraints								
7		x1	$\geq$	-1.414					
8		x1	$\leq$	1.414					
9		x2	$\geq$	-1.414					
10		x2	$\leq$	1.414					
11									
12		Lower Bound		1.54					
13		Upper Bound		6.58					

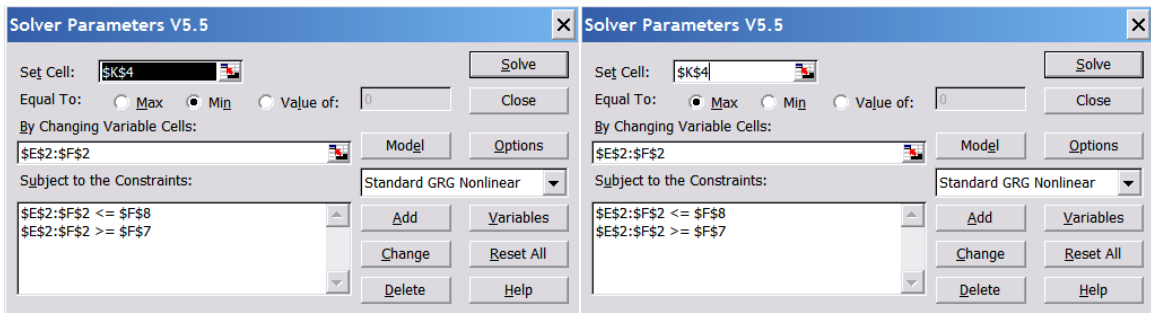


Figure B-21 Silicon Wafers, Scenario 3, Extreme Values

“ISWT 1c”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-1.41	2.00	2.00	-2.00		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		82.75		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		6.58		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c!F12		
8				$\sigma$	$\leq$	6.58					=ISWT c!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									1.70		
15													
16	Step 10:	Given that the mean equals 82.75 and the standard deviation equals 6.58,											
17		please indicate your preference for decreasing the mean by 1.7 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			1						
25				step size=			0.04696				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			3.47				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

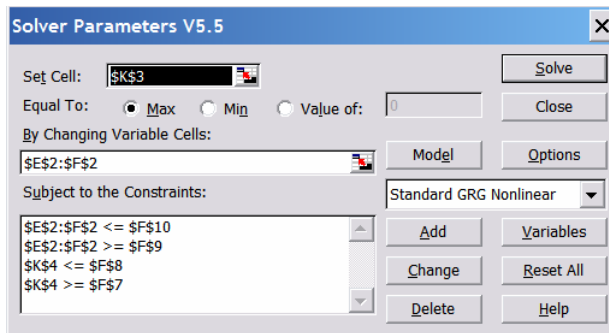


Figure B-22 Silicon Wafers, Scenario 3, Solver Parameters

"ISWT 1c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-1.41	2.00	2.00	-2.00		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		82.75		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		6.58		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c!F12		
8				$\sigma$	$\leq$	6.58					=ISWT c!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									1.70		
15													
16	Step 10:	Given that the mean equals 82.75 and the standard deviation equals 6.58,											
17		please indicate your preference for decreasing the mean by 1.7 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			1						
25				step size=			0.04696				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			3.47				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 1c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	1.41	3.30	
10	\$F\$2	Settings x2	-1.41	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	6.58	1.70	
16	\$K\$4	$\sigma$ (Y2) Value	6.58	0.00	

Figure B-23 Silicon Wafers, Scenario 3, Iteration 1

“ISWT 2c”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.41	-0.35	2.00	0.12	-0.50		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		76.76		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		3.47		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'!F12		
8				$\sigma$	$\leq$	3.47					=ISWT 1c'!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.54		
15													
16	Step 10:	Given that the mean equals 76.76 and the standard deviation equals 3.47,											
17		please indicate your preference for decreasing the mean by 2.54 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			2						
25				step size=			0.05511				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.73				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 2c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	1.41	0.04	
10	\$F\$2	Settings x2	-0.35	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	3.47	2.54	
16	\$K\$4	$\sigma$ (Y2) Value	3.47	0.00	

Figure B-24 Silicon Wafers, Scenario 3, Iteration 2



"ISWT 3c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			1.10	-0.23	1.20	0.05	-0.25		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		74.88		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.73		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'!F12		
8				$\sigma$	$\leq$	2.73					=ISWT 2c'!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 74.88 and the standard deviation equals 2.73,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			3						
25				step size=			0.04323				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.45				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 3c</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.10	0.00
10	\$F\$2	Settings x2		-0.23	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$4	$\sigma$ (Y2) Value		2.73	2.55
16	\$K\$4	$\sigma$ (Y2) Value		2.73	0.00

Figure B-25 Silicon Wafers, Scenario 3, Iteration 3

“ISWT 4c”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.95	-0.17	0.90	0.03	-0.16		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		74.16		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.45		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c!F12		
8				$\sigma$	$\leq$	2.45					=ISWT 3c!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 74.16 and the standard deviation equals 2.45,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			4						
25				step size=			0.02978				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.32				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 4c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.95	0.00	
10	\$F\$2	Settings x2	-0.17	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.45	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.45	0.00	

Figure B-26 Silicon Wafers, Scenario 3, Iteration 4

"ISWT 5c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.87	-0.14	0.75	0.02	-0.12		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.83		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.32		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'F12		
8				$\sigma$	$\leq$	2.32					=ISWT 4c'G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.83 and the standard deviation equals 2.32,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			5						
25				step size=			0.01946				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.25				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 5c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.87	0.00	
10	\$F\$2	Settings x2	-0.14	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.32	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.32	0.00	

Figure B-27 Silicon Wafers, Scenario 3, Iteration 5

"ISWT 6c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.82	-0.13	0.68	0.02	-0.11		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.65		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.25		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c!F12		
8				$\sigma$	$\leq$	2.25					=ISWT 5c!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.65 and the standard deviation equals 2.25,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			6						
25				step size=			0.0124				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.21				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 6c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.82	0.00	
10	\$F\$2	Settings x2	-0.13	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.25	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.25	0.00	

Figure B-28 Silicon Wafers, Scenario 3, Iteration 6

"ISWT 7c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.80	-0.12	0.64	0.01	-0.09		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.55		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.21		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'!F12		
8				$\sigma$	$\leq$	2.21					=ISWT 6c'!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.55 and the standard deviation equals 2.21,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			7						
25				step size=			0.0078				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.18				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 7c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.80	0.00	
10	\$F\$2	Settings x2	-0.12	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.21	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.21	0.00	

Figure B-29 Silicon Wafers, Scenario 3, Iteration 7

"ISWT 8c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.78	-0.11	0.61	0.01	-0.09		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.49		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.18		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c!F12		
8				$\sigma$	$\leq$	2.18					=ISWT 7c!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.49 and the standard deviation equals 2.18,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			8						
25				step size=			0.00487				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.17				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 8c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.78	0.00	
10	\$F\$2	Settings x2	-0.11	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.18	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.18	0.00	

Figure B-30 Silicon Wafers, Scenario 3, Iteration 8

“ISWT 9c”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.77	-0.11	0.60	0.01	-0.08		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.45		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.17		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'F12		
8				$\sigma$	$\leq$	2.17					=ISWT 8c'G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.45 and the standard deviation equals 2.17,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			9						
25				step size=			0.00303				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.16				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 9c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.77	0.00	
10	\$F\$2	Settings x2	-0.11	0.00	
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.17	2.55	
16	\$K\$4	$\sigma$ (Y2) Value	2.17	0.00	

Figure B-31 Silicon Wafers, Scenario 3, Iteration 9

"ISWT 10c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.77	-0.11	0.59	0.01	-0.08		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.43		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.16		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'!F12		
8				$\sigma$	$\leq$	2.16					=ISWT 9c'!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.43 and the standard deviation equals 2.16,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			10						
25				step size=			0.00188				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.16				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Silicon Wafers.xls]ISWT 10c</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.77	0.00
10	\$F\$2	Settings x2		-0.11	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$4	$\sigma$ (Y2) Value		2.16	2.55
16	\$K\$4	$\sigma$ (Y2) Value		2.16	0.00

Figure B-32 Silicon Wafers, Scenario 3, Iteration 10



"ISWT 11c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.76	-0.11	0.58	0.01	-0.08		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.42		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.16		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'!F12		
8				$\sigma$	$\leq$	2.16					=ISWT 10c'!G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.42 and the standard deviation equals 2.16,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			11						
25				step size=			0.00116				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.15				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 11c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.76		0.00
10	\$F\$2	Settings x2	-0.11		0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.16		2.55
16	\$K\$4	$\sigma$ (Y2) Value	2.16		0.00

Figure B-33 Silicon Wafers, Scenario 3, Iteration 11

"ISWT 12c"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.76	-0.10	0.58	0.01	-0.08		Value		
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.41		
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.15		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.54					=ISWT c'F12		
8				$\sigma$	$\leq$	2.15					=ISWT 11c'G27		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									2.55		
15													
16	Step 10:	Given that the mean equals 73.41 and the standard deviation equals 2.15,											
17		please indicate your preference for decreasing the mean by 2.55 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			12						
25				step size=			0.00072				=G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			2.15				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Silicon Wafers.xls]ISWT 12c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.76		0.00
10	\$F\$2	Settings x2	-0.10		0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name	Value	Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value	2.15		2.55
16	\$K\$4	$\sigma$ (Y2) Value	2.15		0.00

Figure B-34 Silicon Wafers, Scenario 3, Iteration 12

“ISWT 13c”

	A	B	C	D	E	F	G	H	I	J	K
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response
2		Settings			0.76	-0.10	0.58	0.01	-0.08		Value
3		$\mu$ (Y1)		71.87	0.83	-0.11	1.25	1.55	-1.98		73.41
4		$\sigma$ (Y2)		1.57	0.24	-0.28	0.58	1.28	-0.28		2.15
5											
6		Constraints									
7				$\sigma$	$\geq$	1.54				=ISWT c!F12	
8				$\sigma$	$\leq$	2.15				=ISWT 12c!G27	
9				x1, x2	$\geq$	-1.41					
10				x1, x2	$\leq$	1.41					

Figure B-35 Silicon Wafers, Scenario 3, Iteration 13

## **APPENDIX C: CHEMICAL EXPERIMENT**

Chemical Experiment (Del Castillo, 1996)

<b>X1</b>	<b>X2</b>	<b>Y (avg)</b>	<b>Y (std)</b>
-1	-1	456.50	8.76
1	-1	595.60	21.96
-1	1	808.70	21.21
1	1	849.00	40.77
1.41	0	794.10	30.82
-1.41	0	673.70	18.32
0	1.41	835.00	16.79
0	-1.41	387.40	3.38
0	0	757.90	9.18
0	0	760.70	24.14
0	0	761.30	22.31
0	0	757.50	10.16
0	0	764.90	22.62

X1=temperature  
X2=reaction time  
Y=yield

Figure C-1 Chemical Experiment, Experimental Design

Fitted Responses:

$$\mu = 760.46 + 43.71x_1 + 154.84x_2 - 12.05x_1^2 - 73.42x_2^2 - 24.70x_1x_2$$

$$\sigma = 19.26 + 6.31x_1 + 6.28x_2$$

Figure C-2 Chemical Experiment, Fitted Responses

“ISWT”

	C	D	E	F	G	H	I	J	K
1		Constant	x1	x2	x1sq	x2sq	x1x2		Response
2	Settings		-1.41	-1.41	2.00	2.00	2.00		Value
3	$\mu$ (Y1)	760.46	43.71	154.84	-12.05	-73.42	-24.7		259.44
4	$\sigma$ (Y2)	19.26	6.31	6.28	0	0	0		1.46
5									
6	Constraints								
7		x1	$\geq$	-1.414					
8		x1	$\leq$	1.414					
9		x2	$\geq$	-1.414					
10		x2	$\leq$	1.414					
11									
12		Lower Bound		1.46					
13		Upper Bound		37.06					

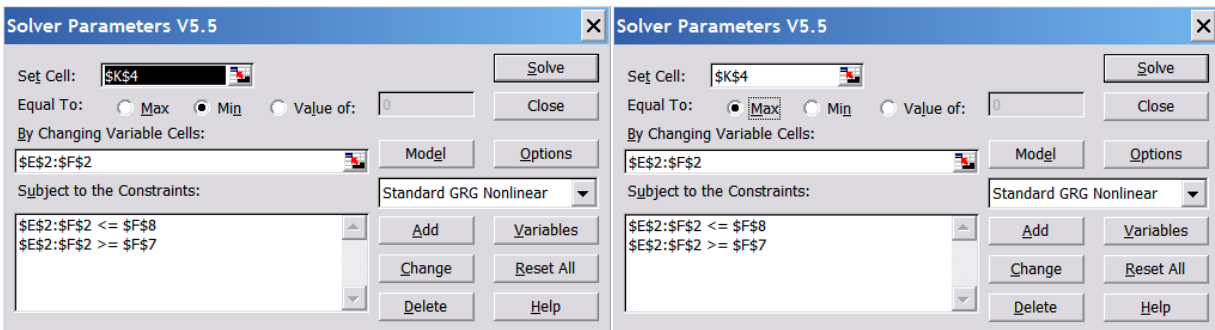


Figure C-3 Chemical Experiment, Extreme Values

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.89	0.91	0.78	0.82	0.80		Value		
3		$\mu$ (Y1)		760.46	43.71	154.84	-12.05	-73.42	-24.7		849.92		
4		$\sigma$ (Y2)		19.26	6.31	6.28	0	0	0		30.53		
5													
6		Constraints											
7				$\sigma$	$\geq$	1.46					=ISWT!F12		
8				$\sigma$	$\leq$	37.06					=ISWT!F13		
9				x1, x2	$\geq$	-1.41							
10				x1, x2	$\leq$	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.00		

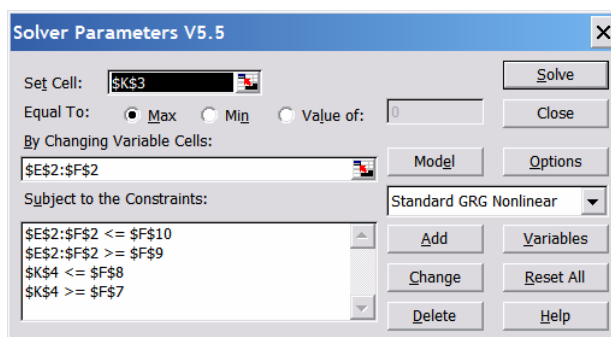


Figure C-4 Chemical Experiment, Solver Parameters

"ISWT 1"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.89	0.91	0.78	0.82	0.80		Value		
3		$\mu$ (Y1)		760.46	43.71	154.84	-12.05	-73.42	-24.7		849.92		
4		$\sigma$ (Y2)		19.26	6.31	6.28	0	0	0		30.53		
5													
6		Constraints											
7				$\sigma$	>=	1.46					=ISWT!F12		
8				$\sigma$	<=	37.06					=ISWT!F13		
9				x1, x2	>=	-1.41							
10				x1, x2	<=	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.00		

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Chemical Experiment.xls]ISWT 1				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		0.89	0.00
10	\$F\$2	Settings x2		0.91	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value		30.53	0.00
16	\$K\$4	$\sigma$ (Y2) Value		30.53	0.00

	X	Y	Z	AA	AB	AC	AD
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Chemical Experiment.xls]ISWT 1						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Max)						
9		Cell	Name	Original Value	Final Value		
10		\$K\$3	$\mu$ (Y1) Value	760.46	849.92		
11							
12							
13	Adjustable Cells						
14		Cell	Name	Original Value	Final Value		
15		\$E\$2	Settings x1	0.00	0.89		
16		\$F\$2	Settings x2	0.00	0.91		
17							
18							
19	Constraints						
20		Cell	Name	Cell Value	Formula	Status	Slack
21		\$K\$4	$\sigma$ (Y2) Value	30.53	\$K\$4<=\$F\$8	Not Binding	6.5272737
22		\$K\$4	$\sigma$ (Y2) Value	30.53	\$K\$4>=\$F\$7	Not Binding	29.08
23		\$E\$2	Settings x1	0.89	\$E\$2<=\$F\$10	Not Binding	0.5283577
24		\$F\$2	Settings x2	0.91	\$F\$2<=\$F\$10	Not Binding	0.5084931
25		\$E\$2	Settings x1	0.89	\$E\$2>=\$F\$9	Not Binding	2.30
26		\$F\$2	Settings x2	0.91	\$F\$2>=\$F\$9	Not Binding	2.32

Figure C-5 Chemical Experiment, Scenario 1, Iteration 1



"ISWT 2"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			0.88	0.91	0.78	0.82	0.80		Value		
3		$\mu$ (Y1)		760.46	43.71	154.84	-12.05	-73.42	-24.7		849.92		
4		$\sigma$ (Y2)		19.26	6.31	6.28	0	0	0		30.52		
5													
6		Constraints											
7				$\sigma$	>=	1.46					=ISWT!F12		
8				$\sigma$	<=	30.52					=ISWT 1!F8-'ISWT 1!AD21-0.01		
9				x1, x2	>=	-1.41							
10				x1, x2	<=	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									0.01		
15													
16	Step 10:	Given that the mean equals 849.92 and the standard deviation equals 30.52,											
17		please indicate your preference for decreasing the mean by 0.01 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			1						
25				step size=			0.0101				=(G23^G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			12.56				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Chemical Experiment.xls]ISWT 2				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.88	0.00
10	\$F\$2	Settings x2		0.91	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$4	$\sigma$ (Y2) Value		30.52	0.01
16	\$K\$4	$\sigma$ (Y2) Value		30.52	0.00

Figure C-6 Chemical Experiment, Scenario 1, Iteration 2

"ISWT 3"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-1.41	0.35	2.00	0.13	-0.50		Value		
3		$\mu$ (Y1)		760.46	43.71	154.84	-12.05	-73.42	-24.7		732.55		
4		$\sigma$ (Y2)		19.26	6.31	6.28	0	0	0		12.56		
5													
6		Constraints											
7				$\sigma$	>=	1.46					=ISWT!F12		
8				$\sigma$	<=	12.56					=ISWT 2!G27		
9				x1, x2	>=	-1.41							
10				x1, x2	<=	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									21.94		
15													
16	Step 10:	Given that the mean equals 732.55 and the standard deviation equals 12.56,											
17		please indicate your preference for decreasing the mean by 21.94 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									-2.00		
22													
23	Step 11:	Update $\epsilon$		c=			0.618						
24				iteration=			2						
25				step size=			0.0152				=(G23*G24)/(SQRT((G21*ABS(K4))^2))		
26													
27				$\epsilon$ =			16.80				=F8+((G25*G21*ABS(K4))*(ISWT!F12-F8))		
28													

	O	P	Q	R	S
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Chemical Experiment.xls]ISWT 3</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-1.41	-69.39
10	\$F\$2	Settings x2		0.35	0.00
11					
12	Constraints				
13				<b>Final</b>	<b>Lagrange</b>
14	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
15	\$K\$4	$\sigma$ (Y2) Value		12.56	21.94
16	\$K\$4	$\sigma$ (Y2) Value		12.56	0.00

Figure C-7 Chemical Experiment, Scenario 1, Iteration 3

"ISWT 4"

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Step 7:			Constant	x1	x2	x1sq	x2sq	x1x2		Response		
2		Settings			-1.30	0.92	1.70	0.84	-1.20		Value		
3		$\mu$ (Y1)		760.46	43.71	154.84	-12.05	-73.42	-24.7		792.85		
4		$\sigma$ (Y2)		19.26	6.31	6.28	0	0	0		16.80		
5													
6		Constraints											
7				$\sigma$	>=	1.46				=ISWT!F12			
8				$\sigma$	<=	16.80				=ISWT 3!G27			
9				x1, x2	>=	-1.41							
10				x1, x2	<=	1.41							
11													
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"									No		
13													
14	Step 9:	Lagrange multiplier=									8.31		
15													
16	Step 10:	Given that the mean equals 792.85 and the standard deviation equals 16.8,											
17		please indicate your preference for decreasing the mean by 8.31 units for one unit decrease in the standard deviation,											
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,											
19		and -2 is a strong preference to make the opposite trade-off.											
20													
21		W=									0.00		

	O	P	Q	R	S
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Chemical Experiment.xls]ISWT 4				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		-1.30	0.00
10	\$F\$2	Settings x2		0.92	0.00
11					
12	Constraints				
13				Final	Lagrange
14	Cell	Name		Value	Multiplier
15	\$K\$4	$\sigma$ (Y2) Value		16.80	8.31
16	\$K\$4	$\sigma$ (Y2) Value		16.80	0.00

Figure C-8 Chemical Experiment, Scenario 1, Iteration 4

## **APPENDIX D: PRINTING PRESS PROBLEM**

Printing Process (Lin and Tu, 1995)

X1	X2	X3	Y1	Y2	Y3	Y (avg)	Y (std)
-1	-1	-1	34	10	28	24.0	12.5
0	-1	-1	115	116	130	120.3	8.4
1	-1	-1	192	186	263	213.7	42.8
-1	0	-1	82	88	88	86.0	3.5
0	0	-1	44	178	188	136.7	80.4
1	0	-1	322	350	350	340.7	16.2
-1	1	-1	141	110	86	112.3	27.6
0	1	-1	259	251	259	256.3	4.6
1	1	-1	290	280	245	271.7	23.6
-1	-1	0	81	81	81	81.0	0.0
0	-1	0	90	122	93	101.7	17.7
1	-1	0	319	376	376	357.0	32.9
-1	0	0	180	180	154	171.3	15.0
0	0	0	372	372	372	372.0	0.0
1	0	0	541	568	396	501.7	92.5
-1	1	0	288	192	312	264.0	63.5
0	1	0	432	336	513	427.0	88.6
1	1	0	713	725	754	730.7	21.1
-1	-1	1	364	99	199	220.7	133.8
0	-1	1	232	221	266	239.7	23.5
1	-1	1	408	415	443	422.0	18.5
-1	0	1	182	233	182	199.0	29.5
0	0	1	507	515	434	485.3	44.6
1	0	1	846	535	640	673.7	158.2
-1	1	1	236	126	168	176.7	55.5
0	1	1	660	440	403	501.0	138.9
1	1	1	878	991	1161	1010.0	142.5

X1=speed  
 X2=pressure  
 X3=distance  
 Y=quality of printing process (3 replicates)

Figure D-1 Printing Press, Experimental Design

Fitted Responses:

$$\mu = 327.6 + 117x_1 + 109.4x_2 + 131.5x_3 + 32x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3$$

$$\sigma = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3$$

Figure D-2 Printing Press, Fitted Responses

“ISWT”

	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2	Settings		-1.00	1.00	-1.00	1.00	1.00	1.00	-1.00	1.00	-1.00		Value
3	$\mu$ (Y1)	327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		74.90
4	$\sigma$ (Y2)	34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		12.50
5													
6	Constraints												
7		x1, x2, x3	>=		-1.00								
8		x1, x2, x3	<=		1.00								
9													
10		Lower Bound			74.90								
11		Upper Bound			911.10								
12													
13		Stated Goal: >=			500.00								
14					<=	500.00							

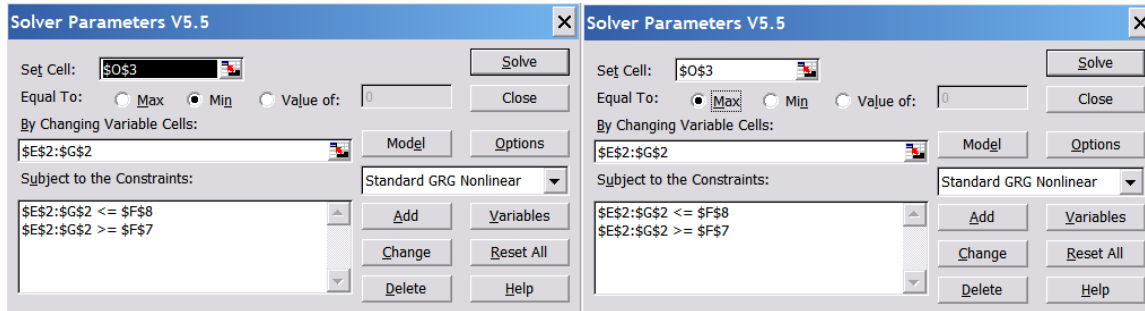


Figure D-3 Printing Press, Scenario 1, Extreme Values

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.12	-0.26	1.00	0.01	0.07	0.12	-0.26	-0.03		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		500.00
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.10
5															
6		Constraints													
7				$\mu$	$\geq$	500.00									=ISWT!F13
8				$\mu$	$\leq$	911.10									=ISWT!F11
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 500 and the standard deviation equals 45.1, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00062								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			754.06								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

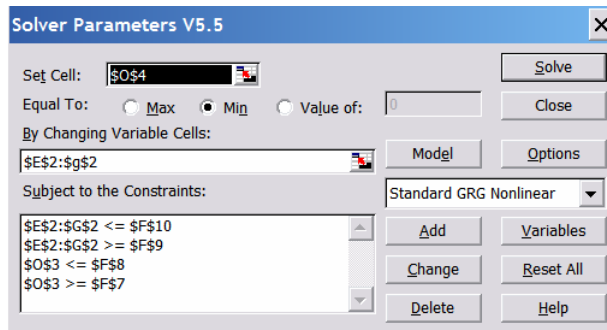


Figure D-4 Printing Press, Scenario 1, Solver Parameters

"ISWT 1"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.12	-0.26	1.00	0.01	0.07	0.12	-0.26	-0.03		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		500.00
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.10
5															
6		Constraints													
7				$\mu$	>=	500.00									=ISWT!F13
8				$\mu$	<=	911.10									=ISWT!F11
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 500 and the standard deviation equals 45.1,													
17		please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00062								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			754.06								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 1				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-7.99
10	\$F\$2	Settings x2		0.12	0.00
11	\$G\$2	Settings x3		-0.26	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		500.00	0.00
17	\$O\$3	$\mu$ (Y1) Value		500.00	0.12

Figure D-5 Printing Press, Scenario 1, Iteration 1



“ISWT 2”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.27	1.00	1.00	0.07	1.00	0.27	0.27		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		754.06
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		86.33
5															
6		Constraints													
7				$\mu$	>=	754.06				=ISWT!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.24
15															
16	Step 10:	Given that the mean equals 754.06 and the standard deviation equals 86.33,													
17		please indicate your preference for increasing the standard deviation by 0.24 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			2								
25				step size=			0.00025								=-(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			694.08								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 2				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-50.76
10	\$F\$2	Settings x2		1.00	-10.51
11	\$G\$2	Settings x3		0.27	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		754.06	0.00
17	\$O\$3	$\mu$ (Y1) Value		754.06	0.24

Figure D-6 Printing Press, Scenario 1, Iteration 2

"ISWT 3"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.02	1.00	1.00	0.00	1.00	0.02	0.02		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		694.08
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		73.17
5															
6		Constraints													
7				$\mu$	>=	694.08				=ISWT 2!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													<input type="text" value="No"/>
13															
14	Step 9:	Lagrange multiplier=													<input type="text" value="0.20"/>
15															
16	Step 10:	Given that the mean equals 694.08 and the standard deviation equals 73.17, please indicate your preference for increasing the standard deviation by 0.2 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													<input type="text" value="1.00"/>
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			3								
25				step size=			0.00034								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			745.30								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 3</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-32.86
10	\$F\$2	Settings x2		1.00	-5.14
11	\$G\$2	Settings x3		0.02	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		694.08	0.00
17	\$O\$3	$\mu$ (Y1) Value		694.08	0.20

Figure D-7 Printing Press, Scenario 1, Iteration 3

"ISWT 4"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.23	1.00	1.00	0.05	1.00	0.23	0.23		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		745.30
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		84.23
5															
6		Constraints													
7				$\mu$	>=	745.30				=ISWT 3!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.24
15															
16	Step 10:	Given that the mean equals 745.3 and the standard deviation equals 84.23,													
17		please indicate your preference for increasing the standard deviation by 0.24 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				4							
25				step size=				0.0002							=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =				721.12							=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 4</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-47.85
10	\$F\$2	Settings x2		1.00	-9.59
11	\$G\$2	Settings x3		0.23	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		745.30	0.00
17	\$O\$3	$\mu$ (Y1) Value		745.30	0.24

Figure D-8 Printing Press, Scenario 1, Iteration 4

"ISWT 5"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.13	1.00	1.00	0.02	1.00	0.13	0.13		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		721.12
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		78.75
5															
6		Constraints													
7				$\mu$	>=	721.12				=ISWT 4!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.22
15															
16	Step 10:	Given that the mean equals 721.12 and the standard deviation equals 78.75,													
17		please indicate your preference for increasing the standard deviation by 0.22 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				5							
25				step size=				0.00013							=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =				738.25							=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 5				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-40.36
10	\$F\$2	Settings x2		1.00	-7.30
11	\$G\$2	Settings x3		0.13	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		721.12	0.00
17	\$O\$3	$\mu$ (Y1) Value		721.12	0.22

Figure D-9 Printing Press, Scenario 1, Iteration 5

"ISWT 6"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.20	1.00	1.00	0.04	1.00	0.20	0.20		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		738.25
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		82.58
5															
6		Constraints													
7				$\mu$	>=	738.25				=ISWT 5!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.23
15															
16	Step 10:	Given that the mean equals 738.25 and the standard deviation equals 82.58,													
17		please indicate your preference for increasing the standard deviation by 0.23 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			6								
25				step size=			7.5E-05								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			747.88								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 6</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-45.58
10	\$F\$2	Settings x2		1.00	-8.89
11	\$G\$2	Settings x3		0.20	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		738.25	0.00
17	\$O\$3	$\mu$ (Y1) Value		738.25	0.23

Figure D-10 Printing Press, Scenario 1, Iteration 6

"ISWT 7"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.24	1.00	1.00	0.06	1.00	0.24	0.24		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		747.88
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		84.84
5															
6		Constraints													
7				$\mu$	>=	747.88				=ISWT 6!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													<input type="text" value="No"/>
13															
14	Step 9:	Lagrange multiplier=													<input type="text" value="0.24"/>
15															
16	Step 10:	Given that the mean equals 747.88 and the standard deviation equals 84.84,													
17		please indicate your preference for increasing the standard deviation by 0.24 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													<input type="text" value="0.00"/>

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 7</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-48.69
10	\$F\$2	Settings x2		1.00	-9.86
11	\$G\$2	Settings x3		0.24	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		747.88	0.00
17	\$O\$3	$\mu$ (Y1) Value		747.88	0.24

Figure D-11 Printing Press, Scenario 1, Iteration 7

"ISWT 1b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			-0.20	-1.00	-0.42	0.04	1.00	0.18	0.20	0.08	0.42		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		139.38
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		14.76
5															
6		Constraints													
7				$\mu$	>=	74.90									=ISWT!F10
8				$\mu$	<=	911.10									=ISWT!F11
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.00

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 1b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.20	0.00
10	\$F\$2	Settings x2		-1.00	10.46
11	\$G\$2	Settings x3		-0.42	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		139.38	0.00
17	\$O\$3	$\mu$ (Y1) Value		139.38	0.00

	AB	AC	AD	AE	AF	AG	AH
4	<b>Microsoft Excel 11.0 Answer Report</b>						
5	<b>Worksheet: [Printing Press.xls]ISWT 1b</b>						
6	<b>Result: Solver found a solution. All constraints and optimality conditions are satisfied.</b>						
7	<b>Engine: Standard GRG Nonlinear</b>						
8							
9	Target Cell (Min)						
10	<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>			
11	\$O\$4	$\sigma$ (Y2) Value	34.90	14.76			
12							
13							
14	Adjustable Cells						
15	<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>			
16	\$E\$2	Settings x1	0.00	-0.20			
17	\$F\$2	Settings x2	0.00	-1.00			
18	\$G\$2	Settings x3	0.00	-0.42			
19							
20							
21	Constraints						
22	<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>	
23	\$O\$3	$\mu$ (Y1) Value	139.38	\$O\$3<=\$F\$8	Not Binding	771.72008	
24	\$O\$3	$\mu$ (Y1) Value	139.38	\$O\$3>=\$F\$7	Not Binding	64.48	
25	\$E\$2	Settings x1	-0.20	\$E\$2<=\$F\$10	Not Binding	1.197752	
26	\$F\$2	Settings x2	-1.00	\$F\$2<=\$F\$10	Not Binding	2	
27	\$G\$2	Settings x3	-0.42	\$G\$2<=\$F\$10	Not Binding	1.4193888	
28	\$E\$2	Settings x1	-0.20	\$E\$2>=\$F\$9	Not Binding	0.80	
29	\$F\$2	Settings x2	-1.00	\$F\$2>=\$F\$9	Binding	0.00	
30	\$G\$2	Settings x3	-0.42	\$G\$2>=\$F\$9	Not Binding	0.58	

Figure D-12 Printing Press, Scenario 2, Iteration 1

"ISWT 2b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			-0.09	-1.00	-0.39	0.01	1.00	0.16	0.09	0.04	0.39		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		149.38
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		14.83
5															
6		Constraints													
7				$\mu$	>=	149.38									=ISWT 1b!F7+ISWT 1b!AH24+10
8				$\mu$	<=	911.10									=ISWT!F11
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													<input type="text" value="No"/>
13															
14	Step 9:	Lagrange multiplier=													<input type="text" value="0.01"/>
15															
16	Step 10:	Given that the mean equals 149.38 and the standard deviation equals 14.83, please indicate your preference for increasing the standard deviation by 0.01 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													<input type="text" value="2.00"/>
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00207								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			<input type="text" value="620.12"/>								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 2b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.09	0.00
10	\$F\$2	Settings x2		-1.00	9.88
11	\$G\$2	Settings x3		-0.39	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		149.38	0.00
17	\$O\$3	$\mu$ (Y1) Value		149.38	0.01

Figure D-13 Printing Press, Scenario 2, Iteration 2



"ISWT 3b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	-0.27	1.00	1.00	0.07	1.00	-0.27	-0.27		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		620.12
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		60.50
5															
6		Constraints													
7				$\mu$	>=	620.12				=ISWT 2b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.15
15															
16	Step 10:	Given that the mean equals 620.12 and the standard deviation equals 60.5,													
17		please indicate your preference for increasing the standard deviation by 0.15 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			2								
25				step size=			0.00031								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			508.99								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 3b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-16.16
10	\$F\$2	Settings x2		1.00	-0.97
11	\$G\$2	Settings x3		-0.27	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		620.12	0.00
17	\$O\$3	$\mu$ (Y1) Value		620.12	0.15

Figure D-14 Printing Press, Scenario 2, Iteration 3

"ISWT 4b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.20	-0.27	1.00	0.04	0.07	0.20	-0.27	-0.05		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		508.99
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		46.18
5															
6		Constraints													
7				$\mu$	>=	508.99				=ISWT 3b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 508.99 and the standard deviation equals 46.18, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			3								
25				step size=			0.00046								=-(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			414.08								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 4b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-8.09
10	\$F\$2	Settings x2		0.20	0.00
11	\$G\$2	Settings x3		-0.27	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		508.99	0.00
17	\$O\$3	$\mu$ (Y1) Value		508.99	0.12

Figure D-15 Printing Press, Scenario 2, Iteration 4

"ISWT 5b"

	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2	Settings		1.00	-0.52	-0.14	1.00	0.27	0.02	-0.52	-0.14	0.07		Value
3	$\mu$ (Y1)	327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		414.08
4	$\sigma$ (Y2)	34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		34.97
5													
6	Constraints												
7		$\mu$	>=	414.08					=ISWT 4b!G27				
8		$\mu$	<=	911.10					=ISWT!F11				
9		x1, x2, x3	>=	-1.00									
10		x1, x2, x3	<=	1.00									
11													
12	Ask DM, "Are you satisfied with the above solution?"												No
13													
14	Lagrange multiplier=												0.12
15													
16	Given that the mean equals 414.08 and the standard deviation equals 34.97,												
17	please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean,												
18	on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,												
19	and -2 is a strong preference to make the opposite trade-off.												
20													
21	W=												2.00
22													
23	Update $\epsilon$	c=			0.618								
24		iteration=			4								
25		step size=			0.00018								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26													
27		$\epsilon$ =			486.58								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28													

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 5b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-7.65
10	\$F\$2	Settings x2		-0.52	0.00
11	\$G\$2	Settings x3		-0.14	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		414.08	0.00
17	\$O\$3	$\mu$ (Y1) Value		414.08	0.12

Figure D-16 Printing Press, Scenario 2, Iteration 5

"ISWT 6b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.01	-0.24	1.00	0.00	0.06	0.01	-0.24	0.00		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		486.58
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		43.49
5															
6		Constraints													
7				$\mu$	$\geq$	486.58				=ISWT 5b!G27					
8				$\mu$	$\leq$	911.10				=ISWT!F11					
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 486.58 and the standard deviation equals 43.49, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			5								
25				step size=			0.00019								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			524.85								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 6b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-7.86
10	\$F\$2	Settings x2		0.01	0.00
11	\$G\$2	Settings x3		-0.24	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		486.58	0.00
17	\$O\$3	$\mu$ (Y1) Value		486.58	0.12

Figure D-17 Printing Press, Scenario 2, Iteration 6

"ISWT 7b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.34	-0.30	1.00	0.11	0.09	0.34	-0.30	-0.10		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		524.85
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		48.10
5															
6		Constraints													
7				$\mu$	$\geq$	524.85				=ISWT 6b!G27					
8				$\mu$	$\leq$	911.10				=ISWT!F11					
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 524.85 and the standard deviation equals 48.1, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			6								
25				step size=			0.00011								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			503.33								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 7b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-8.35
10	\$F\$2	Settings x2		0.34	0.00
11	\$G\$2	Settings x3		-0.30	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		524.85	0.00
17	\$O\$3	$\mu$ (Y1) Value		524.85	0.12

Figure D-18 Printing Press, Scenario 2, Iteration 7

"ISWT 8b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.15	-0.26	1.00	0.02	0.07	0.15	-0.26	-0.04		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		503.33
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.50
5															
6		Constraints													
7				$\mu$	>=	503.33				=ISWT 7b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 503.33 and the standard deviation equals 45.5, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			7								
25				step size=			6.8E-05								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			489.29								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 8b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-8.02
10	\$F\$2	Settings x2		0.15	0.00
11	\$G\$2	Settings x3		-0.26	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		503.33	0.00
17	\$O\$3	$\mu$ (Y1) Value		503.33	0.12

Figure D-19 Printing Press, Scenario 2, Iteration 8

"ISWT 9b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.03	-0.24	1.00	0.00	0.06	0.03	-0.24	-0.01		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		489.29
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		43.82
5															
6		Constraints													
7				$\mu$	>=	489.29				=ISWT 8b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 489.29 and the standard deviation equals 43.82, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				8							
25				step size=				4.3E-05							=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =				498.27							=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 9b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-7.88
10	\$F\$2	Settings x2		0.03	0.00
11	\$G\$2	Settings x3		-0.24	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		489.29	0.00
17	\$O\$3	$\mu$ (Y1) Value		489.29	0.12

Figure D-20 Printing Press, Scenario 2, Iteration 9

"ISWT 10b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.10	-0.26	1.00	0.01	0.07	0.10	-0.26	-0.03		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		498.27
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		44.89
5															
6		Constraints													
7				$\mu$	>=	498.27				=ISWT 9b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 498.27 and the standard deviation equals 44.89, please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean, on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off, and -2 is a strong preference to make the opposite trade-off.													
17															
18															
19															
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				9							
25				step size=				2.6E-05							=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =				503.70							=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 10b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-7.97
10	\$F\$2	Settings x2		0.10	0.00
11	\$G\$2	Settings x3		-0.26	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		498.27	0.00
17	\$O\$3	$\mu$ (Y1) Value		498.27	0.12

Figure D-21 Printing Press, Scenario 2, Iteration 10



"ISWT 11b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.15	-0.27	1.00	0.02	0.07	0.15	-0.27	-0.04		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		503.70
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.54
5															
6		Constraints													
7				$\mu$	>=	503.70				=ISWT 10b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 503.7 and the standard deviation equals 45.54,													
17		please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			10								
25				step size=			1.6E-05								=(G23^G24)/(SQRT((G21*ABS(O3))^2))
26															
27				$\epsilon$ =			500.38								=F7+((G25*G21*ABS(O3))*(ISWT!F11-F7))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 11b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	-8.03
10	\$F\$2	Settings x2		0.15	0.00
11	\$G\$2	Settings x3		-0.27	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$3	$\mu$ (Y1) Value		503.70	0.00
17	\$O\$3	$\mu$ (Y1) Value		503.70	0.12

Figure D-22 Printing Press, Scenario 2, Iteration 11

"ISWT 12b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.12	-0.26	1.00	0.01	0.07	0.12	-0.26	-0.03		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		500.38
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.14
5															
6		Constraints													
7				$\mu$	>=	500.38				=ISWT 11b!G27					
8				$\mu$	<=	911.10				=ISWT!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													0.12
15															
16	Step 10:	Given that the mean equals 500.38 and the standard deviation equals 45.14,													
17		please indicate your preference for increasing the standard deviation by 0.12 units for one unit increase in the mean,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													0.00

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 12b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	-7.99
10	\$F\$2	Settings x2		0.12	0.00
11	\$G\$2	Settings x3		-0.26	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$3	$\mu$ (Y1) Value		500.38	0.00
17	\$O\$3	$\mu$ (Y1) Value		500.38	0.12

Figure D-23 Printing Press, Scenario 2, Iteration 12

“ISWT c”

	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2	Settings		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Value
3	$\mu$ (Y1)	327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		911.10
4	$\sigma$ (Y2)	34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		137.50
5													
6	Constraints												
7		x1, x2, x3	>=		-1.00								
8		x1, x2, x3	<=		1.00								
9													
10		Lower Bound			12.50								
11		Upper Bound			137.50								
12													
13		Stated Goal:	>=		60.00	75.00	90.00						
14			<=		60.00	75.00	90.00						

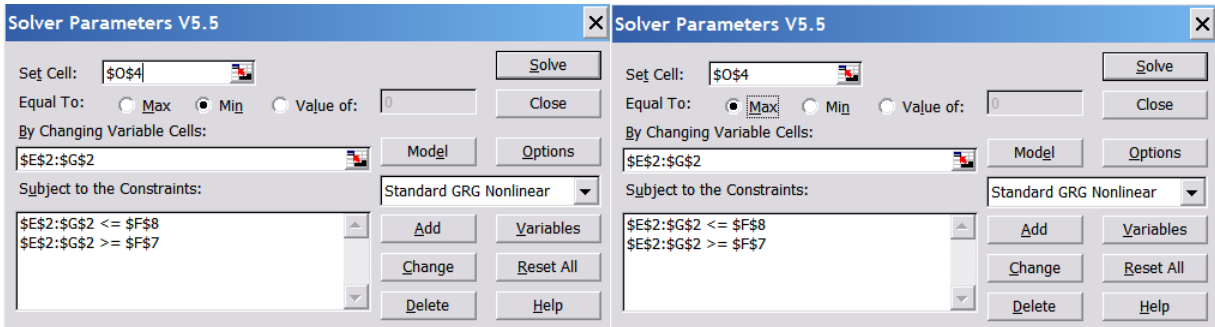


Figure D-24 Printing Press, Scenario 3, Extreme Values

"ISWT 1c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	-0.28	1.00	1.00	0.08	1.00	-0.28	-0.28		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		616.70
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		60.00
5															
6		Constraints													
7				$\sigma$	$\geq$	12.50									=ISWT c!F10
8				$\sigma$	$\leq$	60.00									=ISWT c!F14
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													6.86
15															
16	Step 10:	Given that the mean equals 616.7 and the standard deviation equals 60,													
17		please indicate your preference for decreasing the mean by 6.86 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00515								=-(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			30.65								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

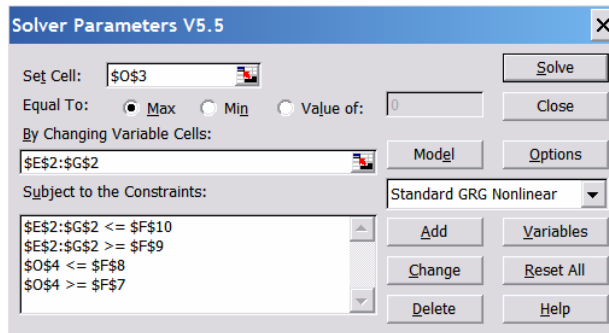


Figure D-25 Printing Press, Scenario 3, Solver Parameters

"ISWT 1c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	-0.28	1.00	1.00	0.08	1.00	-0.28	-0.28		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		616.70
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		60.00
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	60.00				=ISWT c!F14					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													6.86
15															
16	Step 10:	Given that the mean equals 616.7 and the standard deviation equals 60,													
17		please indicate your preference for decreasing the mean by 6.86 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				1							
25				step size=				0.00515							=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =				30.65							=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 1c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	106.32
10	\$F\$2	Settings x2		1.00	5.67
11	\$G\$2	Settings x3		-0.28	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		60.00	6.86
17	\$O\$4	$\sigma$ (Y2) Value		60.00	0.00

Figure D-26 Printing Press, Scenario 3, Iteration 1

"ISWT 2c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	-0.75	-0.09	1.00	0.56	0.01	-0.75	-0.09	0.07		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		376.75
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		30.65
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	30.65				=ISWT 1c!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													8.66
15															
16	Step 10:	Given that the mean equals 376.75 and the standard deviation equals 30.65,													
17		please indicate your preference for decreasing the mean by 8.66 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			2								
25				step size=			0.00623								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			23.71								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 2c</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	66.43
10	\$F\$2	Settings x2		-0.75	0.00
11	\$G\$2	Settings x3		-0.09	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		30.65	8.66
17	\$O\$4	$\sigma$ (Y2) Value		30.65	0.00

Figure D-27 Printing Press, Scenario 3, Iteration 2

"ISWT 3c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	-1.00	-0.15	1.00	1.00	0.02	-1.00	-0.15	0.15		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		314.39
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		23.71
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	23.71				=ISWT 2c!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													11.23
15															
16	Step 10:	Given that the mean equals 314.39 and the standard deviation equals 23.71,													
17		please indicate your preference for decreasing the mean by 11.23 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				3							
25				step size=				0.00995							=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =				21.07							=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 3c</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	35.36
10	\$F\$2	Settings x2		-1.00	-50.53
11	\$G\$2	Settings x3		-0.15	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		23.71	11.23
17	\$O\$4	$\sigma$ (Y2) Value		23.71	0.00

Figure D-28 Printing Press, Scenario 3, Iteration 3

"ISWT 4c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			0.88	-1.00	-0.25	0.77	1.00	0.06	-0.88	-0.22	0.25		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		278.13
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		21.07
5															
6		Constraints													
7				$\sigma$	$\geq$	12.50									=ISWT c!F10
8				$\sigma$	$\leq$	21.07									=ISWT 3c!G27
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													14.98
15															
16	Step 10:	Given that the mean equals 278.13 and the standard deviation equals 21.07,													
17		please indicate your preference for decreasing the mean by 14.98 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			4								
25				step size=			0.00692								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			22.32								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 4c				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		0.88	0.00
10	\$F\$2	Settings x2		-1.00	-115.76
11	\$G\$2	Settings x3		-0.25	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		21.07	14.98
17	\$O\$4	$\sigma$ (Y2) Value		21.07	0.00

Figure D-29 Printing Press, Scenario 3, Iteration 4



"ISWT 5c"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			0.98	-1.00	-0.23	0.97	1.00	0.06	-0.98	-0.23	0.23		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		296.46
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		22.32
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	22.32				=ISWT 4c!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													14.38
15															
16	Step 10:	Given that the mean equals 296.46 and the standard deviation equals 22.32,													
17		please indicate your preference for decreasing the mean by 14.38 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													0.00

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 5c</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.98	0.00
10	\$F\$2	Settings x2		-1.00	-109.86
11	\$G\$2	Settings x3		-0.23	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		22.32	14.38
17	\$O\$4	$\sigma$ (Y2) Value		22.32	0.00

Figure D-30 Printing Press, Scenario 3, Iteration 5

"ISWT 1d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.05	1.00	1.00	0.00	1.00	0.05	0.05		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		703.23
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		75.00
5															
6		Constraints													
7				$\sigma$	$\geq$	12.50									=ISWT c!F10
8				$\sigma$	$\leq$	75.00									=ISWT c!G14
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													4.92
15															
16	Step 10:	Given that the mean equals 703.23 and the standard deviation equals 75,													
17		please indicate your preference for decreasing the mean by 4.92 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00412								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			36.38								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

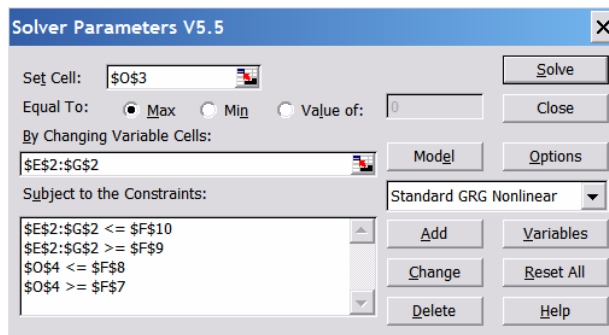


Figure D-31 Printing Press, Scenario 4, Solver Parameters

"ISWT 1d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.05	1.00	1.00	0.00	1.00	0.05	0.05		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		703.23
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		75.00
5															
6		Constraints													
7				$\sigma$	$\geq$	12.50									=ISWT c!F10
8				$\sigma$	$\leq$	75.00									=ISWT c!G14
9				x1, x2, x3	$\geq$	-1.00									
10				x1, x2, x3	$\leq$	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													4.92
15															
16	Step 10:	Given that the mean equals 703.23 and the standard deviation equals 75,													
17		please indicate your preference for decreasing the mean by 4.92 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00412								=-(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			36.38								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 1d				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	173.84
10	\$F\$2	Settings x2		1.00	28.72
11	\$G\$2	Settings x3		0.05	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		75.00	4.92
17	\$O\$4	$\sigma$ (Y2) Value		75.00	0.00

Figure D-32 Printing Press, Scenario 4, Iteration 1

"ISWT 2d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	-0.44	-0.15	1.00	0.19	0.02	-0.44	-0.15	0.07		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		426.12
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		36.38
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	36.38				=ISWT 1d!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													8.57
15															
16	Step 10:	Given that the mean equals 426.12 and the standard deviation equals 36.38,													
17		please indicate your preference for decreasing the mean by 8.57 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			2								
25				step size=			0.0105								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			45.49								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 2d</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	65.56
10	\$F\$2	Settings x2		-0.44	0.00
11	\$G\$2	Settings x3		-0.15	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		36.38	8.57
17	\$O\$4	$\sigma$ (Y2) Value		36.38	0.00

Figure D-33 Printing Press, Scenario 4, Iteration 2

"ISWT 3d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.15	-0.26	1.00	0.02	0.07	0.15	-0.26	-0.04		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		503.30
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		45.49
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	45.49				=ISWT 2d!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													8.33
15															
16	Step 10:	Given that the mean equals 503.3 and the standard deviation equals 45.49,													
17		please indicate your preference for decreasing the mean by 8.33 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			3								
25				step size=			0.00519								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			53.28								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 3d				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	66.80
10	\$F\$2	Settings x2		0.15	0.00
11	\$G\$2	Settings x3		-0.26	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		45.49	8.33
17	\$O\$4	$\sigma$ (Y2) Value		45.49	0.00

Figure D-34 Printing Press, Scenario 4, Iteration 3

"ISWT 4d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.73	-0.35	1.00	0.54	0.12	0.73	-0.35	-0.26		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		566.60
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		53.28
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	53.28				=ISWT 3d!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													7.87
15															
16	Step 10:	Given that the mean equals 566.6 and the standard deviation equals 53.28,													
17		please indicate your preference for decreasing the mean by 7.87 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				4							
25				step size=				0.00274							=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =				47.33							=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 4d				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	76.08
10	\$F\$2	Settings x2		0.73	0.00
11	\$G\$2	Settings x3		-0.35	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		53.28	7.87
17	\$O\$4	$\sigma$ (Y2) Value		53.28	0.00

Figure D-35 Printing Press, Scenario 4, Iteration 4

"ISWT 5d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.28	-0.29	1.00	0.08	0.08	0.28	-0.29	-0.08		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		518.54
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		47.33
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	47.33				=ISWT 4d!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													8.25
15															
16	Step 10:	Given that the mean equals 518.54 and the standard deviation equals 47.33,													
17		please indicate your preference for decreasing the mean by 8.25 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			5								
25				step size=			0.00095								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			50.47								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 5d				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	67.90
10	\$F\$2	Settings x2		0.28	0.00
11	\$G\$2	Settings x3		-0.29	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		47.33	8.25
17	\$O\$4	$\sigma$ (Y2) Value		47.33	0.00

Figure D-36 Printing Press, Scenario 4, Iteration 5

"ISWT 6d"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	0.52	-0.32	1.00	0.27	0.10	0.52	-0.32	-0.17		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		544.19
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		50.47
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	50.47				=ISWT 5d!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													8.08
15															
16	Step 10:	Given that the mean equals 544.19 and the standard deviation equals 50.47,													
17		please indicate your preference for decreasing the mean by 8.08 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													0.00

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 6d</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	71.14
10	\$F\$2	Settings x2		0.52	0.00
11	\$G\$2	Settings x3		-0.32	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		50.47	8.08
17	\$O\$4	$\sigma$ (Y2) Value		50.47	0.00

Figure D-37 Printing Press, Scenario 4, Iteration 6



“ISWT 1e”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.33	1.00	1.00	0.11	1.00	0.33	0.33		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		768.73
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		90.00
5															
6		Constraints													
7				$\sigma$	>=	12.50									=ISWT c!F10
8				$\sigma$	<=	90.00									=ISWT c!H14
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													3.90
15															
16	Step 10:	Given that the mean equals 768.73 and the standard deviation equals 90,													
17		please indicate your preference for decreasing the mean by 3.9 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													0.00

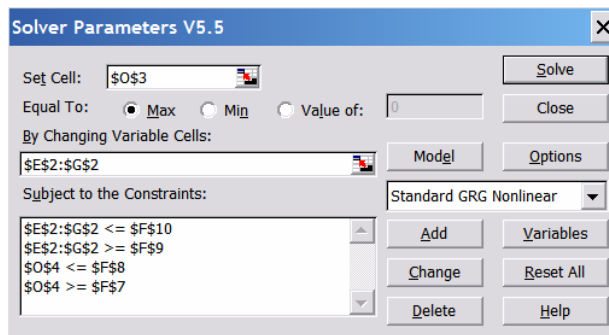


Figure D-38 Printing Press, Scenario 5, Solver Parameters

"ISWT 1e"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.33	1.00	1.00	0.11	1.00	0.33	0.33		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		768.73
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		90.00
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	90.00				=ISWT c!H14					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													<input type="text" value="No"/>
13															
14	Step 9:	Lagrange multiplier=													<input type="text" value="3.90"/>
15															
16	Step 10:	Given that the mean equals 768.73 and the standard deviation equals 90,													
17		please indicate your preference for decreasing the mean by 3.9 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													<input type="text" value="0.00"/>

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press.xls]ISWT 1e</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	217.76
10	\$F\$2	Settings x2		1.00	47.42
11	\$G\$2	Settings x3		0.33	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		90.00	3.90
17	\$O\$4	$\sigma$ (Y2) Value		90.00	0.00

Figure D-39 Printing Press, Scenario 5, Iteration 1

“ISWT 1f”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		911.10
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		137.50
5															
6		Constraints													
7				$\sigma$	>=	12.50									=ISWT c!F10
8				$\sigma$	<=	137.50									=ISWT c!F11
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													2.35
15															
16	Step 10:	Given that the mean equals 911.1 and the standard deviation equals 137.5,													
17		please indicate your preference for decreasing the mean by 2.35 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			1								
25				step size=			0.00449								=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =			214.75								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

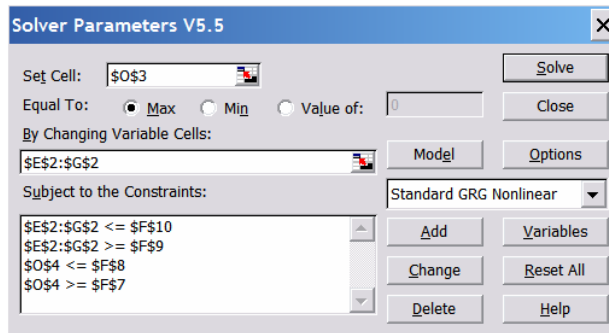


Figure D-40 Printing Press, Scenario 6, Solver Parameters

"ISWT 1f"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		911.10
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		137.50
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	137.50				=ISWT c!F11					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													2.35
15															
16	Step 10:	Given that the mean equals 911.1 and the standard deviation equals 137.5,													
17		please indicate your preference for decreasing the mean by 2.35 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													-1.00
22															
23	Step 11:	Update $\epsilon$		c=				0.618							
24				iteration=				1							
25				step size=				0.00449							=(G23^G24)/(SQRT((G21*ABS(O4))^2))
26															
27				$\epsilon$ =				214.75							=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))
28															

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press.xls]ISWT 1f				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		1.00	305.77
10	\$F\$2	Settings x2		1.00	93.25
11	\$G\$2	Settings x3		1.00	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value		137.50	2.35
17	\$O\$4	$\sigma$ (Y2) Value		137.50	0.00

Figure D-41 Printing Press, Scenario 6, Iteration 1

"ISWT 2f"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	-0.75	-0.09	1.00	0.56	0.01	-0.75	-0.09	0.07		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		376.75
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		30.65
5															
6		Constraints													
7				$\sigma$	>=	12.50									=ISWT c!F10
8				$\sigma$	<=	214.75									=ISWT 1f!G27
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									

Figure D-42 Printing Press, Scenario 6, Iteration 2

"ISWT 3f"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		911.10
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		137.50
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	137.50				=ISWT 1!F8					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													2.35
15															
16	Step 10:	Given that the mean equals 911.1 and the standard deviation equals 137.5,													
17		please indicate your preference for decreasing the mean by 2.35 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													2.00
22															
23	Step 11:	Update $\epsilon$		c=			0.618								
24				iteration=			2								
25				step size=		0.00139									=((G23^G24)/(SQRT((G21*ABS(O4))^2)))
26															
27				$\epsilon$ =			89.76								=F8+((G25*G21*ABS(O4))*(ISWT c!F10-F8))

	S	T	U	V	W
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Printing Press rev.xls]ISWT 3f				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	1.00	305.77	
10	\$F\$2	Settings x2	1.00	93.25	
11	\$G\$2	Settings x3	1.00	0.00	
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name	Value	Value	Multiplier
16	\$O\$4	$\sigma$ (Y2) Value	137.50	2.35	
17	\$O\$4	$\sigma$ (Y2) Value	137.50	0.00	

Figure D-43 Printing Press, Scenario 6, Iteration 3

"ISWT 4f"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3		Response
2		Settings			1.00	1.00	0.32	1.00	1.00	0.11	1.00	0.32	0.32		Value
3		$\mu$ (Y1)		327.60	177.00	109.40	131.50	32.00	-22.40	-29.10	66.00	75.50	43.60		767.80
4		$\sigma$ (Y2)		34.90	11.50	15.30	29.20	4.20	-1.30	16.80	7.70	5.10	14.10		89.76
5															
6		Constraints													
7				$\sigma$	>=	12.50				=ISWT c!F10					
8				$\sigma$	<=	89.76				=ISWT 3!G27					
9				x1, x2, x3	>=	-1.00									
10				x1, x2, x3	<=	1.00									
11															
12	Step 8:	Ask DM, "Are you satisfied with the above solution?"													No
13															
14	Step 9:	Lagrange multiplier=													3.91
15															
16	Step 10:	Given that the mean equals 767.8 and the standard deviation equals 89.76,													
17		please indicate your preference for decreasing the mean by 3.91 units for one unit decrease in the standard deviation,													
18		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
19		and -2 is a strong preference to make the opposite trade-off.													
20															
21		W=													0.00

	S	T	U	V	W
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Printing Press rev.xls]ISWT 4f</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		1.00	217.15
10	\$F\$2	Settings x2		1.00	47.14
11	\$G\$2	Settings x3		0.32	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$O\$4	$\sigma$ (Y2) Value		89.76	3.91
17	\$O\$4	$\sigma$ (Y2) Value		89.76	0.00

Figure D-44 Printing Press, Scenario 6, Iteration 4

## **APPENDIX E: PROPELLANT MIXTURE PROBLEM**



Propellant Experiment (Del Castillo and Montgomery, 1993)

X1	X2	X3	Y1	Y2	Y3
1	0	0	32.5	4.1	32
1	0	0	37.9	3.7	25
0.5	0.5	0	44.0	6.8	20
0.5	0	0.5	63.2	4.7	18
0	1	0	54.5	8.9	18
0	1	0	32.5	9.2	21
0	0.5	0.5	94.0	4.5	17
0	0	1	64.0	14.0	14
0	0	1	78.5	13.0	16
0.667	0.167	0.167	67.1	3.5	20
0.167	0.667	0.167	73.0	5.2	22
0.167	0.167	0.667	87.5	7.0	17
0.333	0.333	0.333	112.5	4.6	19
0.333	0.333	0.333	98.5	3.5	20
0.333	0.333	0.333	103.6	3.0	18

X1=fuel (.3, .5)  
 X2=oxidizer (.2, .4)  
 X3=binder (.2, .4)  
 Y1=burning rate  
 Y2=variability of burning rate  
 Y3=manufacturability index

Figure E-1 Propellant Mixture, Experimental Design

Fitted Responses:

$$Y1 = 35.4945x_1 + 42.7755x_2 + 70.3612x_3 + 16.0204x_1x_2 + 36.3347x_1x_3 + 136.8204x_2x_3 + 854.9818x_1x_2$$

$$Y2 = 3.8815x_1 + 9.0387x_2 + 13.6339x_3 - 0.1904x_1x_2 - 16.6195x_1x_3 - 27.6761x_2x_3$$

$$Y3 = 23.1333x_1 + 19.7333x_2 + 14.7333x_3$$

Figure E-2 Propellant Mixture, Fitted Responses

“ISWT”

	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2	Settings		0.21	0.34	0.44	0.05	0.12	0.20	0.07	0.09	0.15	0.03		Value
3	Y1		0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98	106.65
4	Y2		0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00	4.18
5	Y3		0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.24
6														
7	Constraints													
8		x1		>=	0.00									
9		x1		<=	1.00									
10		x2		>=	0.00									
11		x2		<=	1.00									
12		x3		>=	0.00									
13		x3		<=	1.00									
14		1.00	x1+x2+x3	>	0.00									
15			x1+x2+x3	<=	1.00									
16														
17					Y1	Y2	Y3							
18		Lower Bound			0.00	0.00	0.00							
19		Upper Bound			106.65	13.63	23.13							
20														
21		Case <=			4.50	20.00								

The figure displays six Solver Parameters V5.5 dialog boxes arranged in a 3x2 grid. Each dialog box represents a different optimization scenario for the 'Propellant Mixture, Extreme Values' problem. The 'Set Cell' (target) is \$P\$3, \$P\$4, or \$P\$5. The 'Equal To' options are Max, Min, or Value of: 0. The 'By Changing Variable Cells' is consistently \$E\$2:\$G\$2. The 'Subject to the Constraints' list varies between dialog boxes, including constraints like \$C\$14 <= \$F\$15, \$C\$14 >= \$F\$14, \$E\$2:\$G\$2 <= \$F\$9, and \$E\$2:\$G\$2 >= \$F\$8. The dialog boxes are titled 'Solver Parameters V5.5' and include buttons for Solve, Close, Model, Options, Add, Variables, Change, Reset All, Delete, and Help.

Figure E-3 Propellant Mixture, Extreme Values

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.21	0.34	0.44	0.05	0.12	0.20	0.07	0.09	0.15	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.65
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.18
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.24
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.50		=ISWT!G21								
10				Y3	>=	0.00										
11				Y3	<=	20.00		=ISWT!H21								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													0.00
21			Lagrange multiplier Y3=													0.00

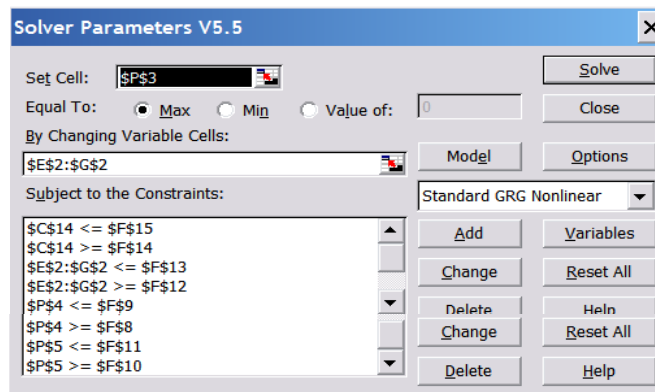


Figure E-4 Propellant Mixture, Scenario1, Solver Parameters

"ISWT 1"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.21	0.34	0.44	0.05	0.12	0.20	0.07	0.09	0.15	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.65
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.18
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.24
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.50		=ISWT!G21								
10				Y3	>=	0.00										
11				Y3	<=	20.00		=ISWT!H21								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													0.00
21			Lagrange multiplier Y3=													0.00

	T	U	V	W	X
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Propellant Experiment.xls]ISWT 1</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.21	0.00
10	\$F\$2	Settings x2		0.34	0.00
11	\$G\$2	Settings x3		0.44	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$C\$14	Constraints		1.00	187.54
17	\$C\$14	Constraints		1.00	0.00
18	\$P\$4	Y2 Value		4.18	0.00
19	\$P\$4	Y2 Value		4.18	0.00
20	\$P\$5	Y3 Value		18.24	0.00
21	\$P\$5	Y3 Value		18.24	0.00

	AC	AD	AE	AF	AG	AH	AI																																																																														
3	Microsoft Excel 11.0 Answer Report																																																																																				
4	Worksheet: [Propellant Experiment.xls]ISWT 1																																																																																				
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.																																																																																				
6	Engine: Standard GRG Nonlinear																																																																																				
7																																																																																					
8	Target Cell (Max)																																																																																				
9	<table border="1"> <thead> <tr> <th>Cell</th> <th>Name</th> <th>Original Value</th> <th>Final Value</th> </tr> </thead> <tbody> <tr> <td>SP\$3</td> <td>Y1 Value</td> <td>0.00</td> <td>106.65</td> </tr> </tbody> </table>							Cell	Name	Original Value	Final Value	SP\$3	Y1 Value	0.00	106.65																																																																						
Cell	Name	Original Value	Final Value																																																																																		
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Cell	Name	Cell Value	Formula	Status	Slack																																																																																
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SC\$14	Constraints	1.00	SC\$14>=FF\$14	Binding	0.00																																																																																
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SP\$5	Y3 Value	18.24	SP\$5<=FF\$11	Not Binding	1.7633866																																																																																
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Figure E-5 Propellant Mixture, Scenario 1, Iteration 1

“ISWT 2”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2		Settings			0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3		Y1		0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4		Y2		0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5		Y3		0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7		Constraints														
8				Y2	>=	0.00										
9				Y2	<=	4.17		=ISWT!G21-ISWT 1!AI24-0.01								
10				Y3	>=	0.00										
11				Y3	<=	18.23		=ISWT!H21-ISWT 1!AI26-0.01								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"														No
19																
20	Step 9:	Lagrange multiplier Y2=														5.02
21		Lagrange multiplier Y3=														2.63
22																
23	Step 10:	Given that Y1 equals 106.61 and Y2 equals 4.17,														
24		please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,														
25		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,														
26		and -2 is a strong preference to make the opposite trade-off.														
27																
28		W (Y2)=														-1.00
29																
30		Given that Y1 equals 106.61 and Y3 equals 18.23,														
31		please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,														
32		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,														
33		and -2 is a strong preference to make the opposite trade-off.														
34																
35		W (Y3)=														2.00
36																
37																
38	Step 11:	Update $\epsilon$		c=			0.618									
39				iteration=			1									
40				step size=			0.0168		=(G38^G39)/(SQRT((G28*ABS(P4))^2+(G35*ABS(P5))^2))							
41																
42				$\epsilon$ (Y2)=			4.46		=F9+((G40*G28*ABS(P4))*(ISWT!G18-F9))							
43				$\epsilon$ (Y3)=			7.04		=F11+((G40*G35*ABS(P5))*(ISWT!H18-F11))							
44																

	T	U	V	W	X
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Propellant Experiment.xls]ISWT 2				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		0.20	0.00
10	\$F\$2	Settings x2		0.35	0.00
11	\$G\$2	Settings x3		0.44	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$C\$14	Constraints		1.00	0.00
17	\$C\$14	Constraints		1.00	0.00
18	\$P\$4	Y2 Value		4.17	5.02
19	\$P\$4	Y2 Value		4.17	0.00
20	\$P\$5	Y3 Value		18.23	2.63
21	\$P\$5	Y3 Value		18.23	0.00

Figure E-6 Propellant Mixture, Scenario 1, Iteration 2

“ISWT 3”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2		Settings			0.00	0.11	0.33	0.00	0.01	0.11	0.00	0.00	0.04	0.00		Value
3		Y1		0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		32.85
4		Y2		0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.46
5		Y3		0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		7.04
6																
7		Constraints														
8				Y2	>=	0.00										
9				Y2	<=	4.46		=ISWT 2!G42								
10				Y3	>=	0.00										
11				Y3	<=	7.04		=ISWT 2!G43								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				0.44 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										

Figure E-7 Propellant Mixture, Scenario 1, Iteration 3

“ISWT 4”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.17										=ISWT!G21-ISWT 1!AI24-0.01
10				Y3	>=	0.00										
11				Y3	<=	18.23										=ISWT!H21-ISWT 1!AI26-0.01
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													5.02
21			Lagrange multiplier Y3=													2.63
22																
23	Step 10:		Given that Y1 equals 106.61 and Y2 equals 4.17,													
24			please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,													
25			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
26			and -2 is a strong preference to make the opposite trade-off.													
27																
28			W (Y2)=													0.00
29																
30			Given that Y1 equals 106.61 and Y3 equals 18.23,													
31			please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,													
32			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
33			and -2 is a strong preference to make the opposite trade-off.													
34																
35			W (Y3)=													2.00
36																
37																
38	Step 11:		Update $\epsilon$	c=			0.618									
39				iteration=			1									
40				step size=			0.017									= $(G38^G39)/(SQRT((G28*ABS(P4))^2+(G35*ABS(P5))^2))$
41																
42				$\epsilon$ (Y2)=			4.17									=F9+((G40*G28*ABS(P4))*(ISWT!G18-F9))
43				$\epsilon$ (Y3)=			6.96									=F11+((G40*G35*ABS(P5))*(ISWT!H18-F11))
44																

Figure E-8 Propellant Mixture, Scenario 1, Iteration 4



"ISWT 5"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2		Settings			0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3		Y1		0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4		Y2		0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5		Y3		0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7		Constraints														
8				Y2	>=	0.00										
9				Y2	<=	4.17		=ISWT 4!G42								
10				Y3	>=	0.00										
11				Y3	<=	18.23		=ISWT 4!F11								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"														No
19																
20	Step 9:	Lagrange multiplier Y2=														5.02
21		Lagrange multiplier Y3=														2.63
22																
23	Step 10:	Given that Y1 equals 106.61 and Y2 equals 4.17,														
24		please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,														
25		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,														
26		and -2 is a strong preference to make the opposite trade-off.														
27																
28		W (Y2)=														0.00
29																
30		Given that Y1 equals 106.61 and Y3 equals 18.23,														
31		please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,														
32		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,														
33		and -2 is a strong preference to make the opposite trade-off.														
34																
35		W (Y3)=														0.00

	T	U	V	W	X
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Propellant Experiment.xls]ISWT 5				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		0.20	0.00
10	\$F\$2	Settings x2		0.35	0.00
11	\$G\$2	Settings x3		0.44	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$C\$14	Constraints		1.00	0.00
17	\$C\$14	Constraints		1.00	0.00
18	\$P\$4	Y2 Value		4.17	5.02
19	\$P\$4	Y2 Value		4.17	0.00
20	\$P\$5	Y3 Value		18.23	2.63
21	\$P\$5	Y3 Value		18.23	0.00

Figure E-9 Propellant Mixture, Scenario 1, Iteration 5

“ISWT 1b”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2		Settings			0.21	0.34	0.44	0.05	0.12	0.20	0.07	0.09	0.15	0.03		Value
3		Y1		0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.65
4		Y2		0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.18
5		Y3		0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.24
6																
7		Constraints														
8				Y2	>=	0.00										
9				Y2	<=	13.63		=ISWT!G19								
10				Y3	>=	0.00										
11				Y3	<=	23.13		=ISWT!H19								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"						No								
19																
20	Step 9:	Lagrange multiplier Y2=						0.00								
21		Lagrange multiplier Y3=						0.00								

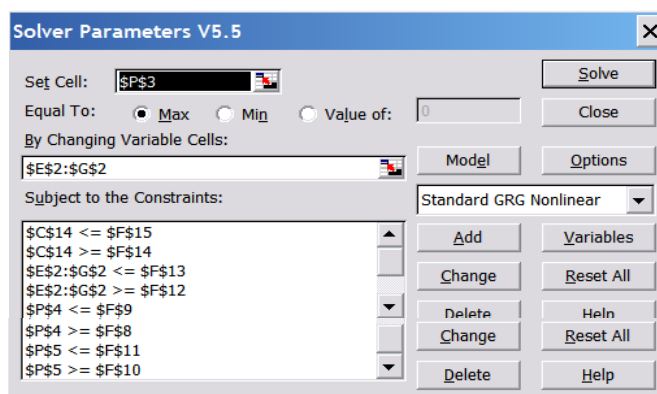


Figure E-10 Propellant Mixture, Scenario 2, Solver Parameters

"ISWT 1b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.21	0.34	0.44	0.05	0.12	0.20	0.07	0.09	0.15	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.65
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.18
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.24
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	13.63		=ISWT!G19								
10				Y3	>=	0.00										
11				Y3	<=	23.13		=ISWT!H19								
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													0.00
21			Lagrange multiplier Y3=													0.00

	T	U	V	W	X
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Propellant Experiment.xls]ISWT 1b</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		0.21	0.00
10	\$F\$2	Settings x2		0.34	0.00
11	\$G\$2	Settings x3		0.44	0.00
12					
13	Constraints				
14				<b>Final</b>	<b>Lagrange</b>
15	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
16	\$C\$14	Constraints		1.00	187.54
17	\$C\$14	Constraints		1.00	0.00
18	\$P\$4	Y2 Value		4.18	0.00
19	\$P\$4	Y2 Value		4.18	0.00
20	\$P\$5	Y3 Value		18.24	0.00
21	\$P\$5	Y3 Value		18.24	0.00

	AC	AD	AE	AF	AG	AH	AI																																																																														
3	Microsoft Excel 11.0 Answer Report																																																																																				
4	Worksheet: [Propellant Experiment.xls]ISWT 1b																																																																																				
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.																																																																																				
6	Engine: Standard GRG Nonlinear																																																																																				
7																																																																																					
8	Target Cell (Max)																																																																																				
9	<table border="1"> <thead> <tr> <th>Cell</th> <th>Name</th> <th>Original Value</th> <th>Final Value</th> </tr> </thead> <tbody> <tr> <td>\$P\$3</td> <td>Y1 Value</td> <td>0.00</td> <td>106.65</td> </tr> </tbody> </table>							Cell	Name	Original Value	Final Value	\$P\$3	Y1 Value	0.00	106.65																																																																						
Cell	Name	Original Value	Final Value																																																																																		
\$P\$3	Y1 Value	0.00	106.65																																																																																		
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Cell	Name	Original Value	Final Value																																																																																		
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\$F\$2	Settings x2	0.00	0.34																																																																																		
\$G\$2	Settings x3	0.00	0.44																																																																																		
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Cell	Name	Cell Value	Formula	Status	Slack																																																																																
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\$P\$4	Y2 Value	4.18	\$P\$4<=\$F\$9	Not Binding	9.4544124																																																																																
\$P\$4	Y2 Value	4.18	\$P\$4>=\$F\$8	Not Binding	4.18																																																																																
\$P\$5	Y3 Value	18.24	\$P\$5<=\$F\$11	Not Binding	4.8966866																																																																																
\$P\$5	Y3 Value	18.24	\$P\$5>=\$F\$10	Not Binding	18.24																																																																																
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\$F\$2	Settings x2	0.34	\$F\$2<=\$F\$13	Not Binding	0.6563287																																																																																
\$G\$2	Settings x3	0.44	\$G\$2<=\$F\$13	Not Binding	0.5561634																																																																																
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Figure E-11 Propellant Mixture, Scenario 2, Iteration 1

"ISWT 2b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.17										=ISWT!G19-ISWT 1b!A124-0.01
10				Y3	>=	0.00										
11				Y3	<=	18.23										=ISWT!H19-ISWT 1b!A126-0.01
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													5.02
21			Lagrange multiplier Y3=													2.63
22																
23	Step 10:		Given that Y1 equals 106.61 and Y2 equals 4.17,													
24			please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,													
25			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
26			and -2 is a strong preference to make the opposite trade-off.													
27																
28			W (Y2)=													-1.00
29																
30			Given that Y1 equals 106.61 and Y3 equals 18.23,													
31			please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,													
32			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
33			and -2 is a strong preference to make the opposite trade-off.													
34																
35			W (Y3)=													2.00
36																
37																
38	Step 11:		Update $\epsilon$	c=			0.618									
39				iteration=			1									
40				step size=			0.0168									=((G38^G39)/(SQRT((G28*ABS(P4))^2+(G35*ABS(P5))^2)))
41																
42				$\epsilon$ (Y2)=			4.46									=F9+((G40*G28*ABS(P4))*(ISWT!G18-F9))
43				$\epsilon$ (Y3)=			7.04									=F11+((G40*G35*ABS(P5))*(ISWT!H18-F11))
44																

	T	U	V	W	X
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Propellant Experiment.xls]ISWT 2b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name	Value	Value	Gradient
9	\$E\$2	Settings x1	0.20	0.00	
10	\$F\$2	Settings x2	0.35	0.00	
11	\$G\$2	Settings x3	0.44	0.00	
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name	Value	Value	Multiplier
16	\$C\$14	Constraints	1.00	0.00	
17	\$C\$14	Constraints	1.00	0.00	
18	\$P\$4	Y2 Value	4.17	5.02	
19	\$P\$4	Y2 Value	4.17	0.00	
20	\$P\$5	Y3 Value	18.23	2.63	
21	\$P\$5	Y3 Value	18.23	0.00	

Figure E-12 Propellant Mixture, Scenario 2, Iteration 2

“ISWT 3b”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:		Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3			Response
2		Settings			0.00	0.11	0.33	0.00	0.01	0.11	0.00	0.00	0.04	0.00		Value
3		Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98			32.85
4		Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00			4.46
5		Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00			7.04
6																
7		Constraints														
8			Y2	>=	0.00											
9			Y2	<=	4.46					=ISWT 2b!G42						
10			Y3	>=	0.00											
11			Y3	<=	7.04					=ISWT 2b!G43						
12			x1, x2, x3	>=	0.00											
13			x1, x2, x3	<=	1.00											
14			0.44 x1+x2+x3	>=	1.00											
15			x1+x2+x3	<=	1.00											

Figure E-13 Propellant Mixture, Scenario 2, Iteration 3

"ISWT 4b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.17										=ISWT!G19-ISWT 1b!A124-0.01
10				Y3	>=	0.00										
11				Y3	<=	18.23										=ISWT!H19-ISWT 1b!A126-0.01
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													5.02
21			Lagrange multiplier Y3=													2.63
22																
23	Step 10:		Given that Y1 equals 106.61 and Y2 equals 4.17,													
24			please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,													
25			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
26			and -2 is a strong preference to make the opposite trade-off.													
27																
28			W (Y2)=													0.00
29																
30			Given that Y1 equals 106.61 and Y3 equals 18.23,													
31			please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,													
32			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
33			and -2 is a strong preference to make the opposite trade-off.													
34																
35			W (Y3)=													2.00
36																
37																
38	Step 11:		Update $\epsilon$	c=			0.618									
39				iteration=			1									
40				step size=			0.017									=(G38^G39)/(SQRT((G28*ABS(P4))^2+(G35*ABS(P5))^2))
41																
42				$\epsilon$ (Y2)=			4.17									=F9+((G40*G28*ABS(P4))*(ISWT!G18-F9))
43				$\epsilon$ (Y3)=			6.96									=F11+((G40*G35*ABS(P5))*(ISWT!H18-F11))
44																

Figure E-14 Propellant Mixture, Scenario 2, Iteration 4

"ISWT 5b"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Step 7:			Constant	x1	x2	x3	x1sq	x2sq	x3sq	x1x2	x1x3	x2x3	x1x2x3		Response
2			Settings		0.20	0.35	0.44	0.04	0.13	0.19	0.07	0.09	0.16	0.03		Value
3			Y1	0.00	35.49	42.78	70.36	0.00	0.00	0.00	16.02	36.33	136.82	854.98		106.61
4			Y2	0.00	3.88	9.04	13.63	0.00	0.00	0.00	-0.19	-16.62	-27.68	0.00		4.17
5			Y3	0.00	23.13	19.73	14.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00		18.23
6																
7			Constraints													
8				Y2	>=	0.00										
9				Y2	<=	4.17										=ISWT!G19-ISWT 1b!A124-0.01
10				Y3	>=	0.00										
11				Y3	<=	18.23										=ISWT!H19-ISWT 1b!A126-0.01
12				x1, x2, x3	>=	0.00										
13				x1, x2, x3	<=	1.00										
14				1.00 x1+x2+x3	>=	1.00										
15				x1+x2+x3	<=	1.00										
16																
17																
18	Step 8:		Ask DM, "Are you satisfied with the above solution?"													No
19																
20	Step 9:		Lagrange multiplier Y2=													5.02
21			Lagrange multiplier Y3=													2.63
22																
23	Step 10:		Given that Y1 equals 106.61 and Y2 equals 4.17,													
24			please indicate your preference for decreasing Y1 by 5.02 units for one unit decrease in Y2,													
25			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
26			and -2 is a strong preference to make the opposite trade-off.													
27																
28			W (Y2)=													0.00
29																
30			Given that Y1 equals 106.61 and Y3 equals 18.23,													
31			please indicate your preference for decreasing Y1 by 2.63 units for one unit decrease in Y3,													
32			on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,													
33			and -2 is a strong preference to make the opposite trade-off.													
34																
35			W (Y3)=													0.00

	T	U	V	W	X
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Propellant Experiment.xls]ISWT 5b				
5					
6	Adjustable Cells				
7				Final	Reduced
8	Cell	Name		Value	Gradient
9	\$E\$2	Settings x1		0.20	0.00
10	\$F\$2	Settings x2		0.35	0.00
11	\$G\$2	Settings x3		0.44	0.00
12					
13	Constraints				
14				Final	Lagrange
15	Cell	Name		Value	Multiplier
16	\$C\$14	Constraints		1.00	0.00
17	\$C\$14	Constraints		1.00	0.00
18	\$P\$4	Y2 Value		4.17	5.02
19	\$P\$4	Y2 Value		4.17	0.00
20	\$P\$5	Y3 Value		18.23	2.63
21	\$P\$5	Y3 Value		18.23	0.00

Figure E-15 Propellant Mixture, Scenario 2, Iteration 5



## **APPENDIX F: FOAMING PROPERTIES OF WHEY PROTEIN PROBLEM**

Foaming Properties of Whey Protein (Khuri and Conlon, 1981)

X1	X2	X3	X4	X5	Y1	Y2	Y3	Y4
-1	-1	-1	-1	1	1082	4.5	80.6	81.4
1	-1	-1	-1	-1	824	7.5	67.9	69.6
-1	1	-1	-1	-1	953	8.3	83.1	105.0
1	1	-1	-1	1	759	17.0	38.1	81.2
-1	-1	1	-1	-1	1163	6.7	79.7	80.8
1	-1	1	-1	1	839	9.5	74.7	76.3
-1	1	1	-1	1	1343	12.0	71.2	103.0
1	1	1	-1	-1	736	36.0	36.8	76.9
-1	-1	-1	1	-1	1027	4.0	81.7	87.2
1	-1	-1	1	1	836	5.0	66.8	74.0
-1	1	-1	1	1	1272	12.5	73.0	98.5
1	1	-1	1	-1	825	20.0	40.5	94.1
-1	-1	1	1	1	1363	15.0	74.9	95.9
1	-1	1	1	-1	855	7.5	74.2	76.8
-1	1	1	1	-1	1284	18.5	63.5	100.0
1	1	1	1	1	851	12.0	42.8	104.0
-2	0	0	0	0	1283	12.0	80.9	100.0
2	0	0	0	0	651	8.5	42.4	50.5
0	-2	0	0	0	1217	4.5	73.4	71.2
0	2	0	0	0	982	10.5	45.0	101.0
0	0	-2	0	0	884	9.0	66.0	85.8
0	0	2	0	0	1147	9.0	71.7	103.0
0	0	0	-2	0	1081	9.0	77.5	104.0
0	0	0	2	0	1036	10.0	76.3	89.4
0	0	0	0	-2	1213	16.0	67.4	105.0
0	0	0	0	2	1103	8.5	86.5	113.0
0	0	0	0	0	1171	11.0	77.4	102.0
0	0	0	0	0	1179	9.0	74.6	104.0
0	0	0	0	0	1183	9.0	79.8	107.0
0	0	0	0	0	1120	10.0	78.3	104.0
0	0	0	0	0	1180	9.5	74.8	101.0
0	0	0	0	0	1195	11.0	80.9	103.0

X1=heating temperature  
X2=PH level  
X3=redox potential  
X4=sodium oxalate  
X5=soldium lauryl sulfate  
Y1=maximum overrun  
Y2=time at first drop  
Y3=undenatured protein  
Y4=soluble protein

Figure F-1 Whey Protein, Experimental Design

Fitted Responses:

$$Y1 = 1176.98 - 176.08x_1 - 18.17x_2 + 57.58x_3 + 21.83x_4 + 19.08x_5 - 56.97x_1^2 - 23.84x_2^2 - 44.84x_3^2 - 34.09x_4^2 - 9.22x_5^2 \\ - 25.00x_1x_2 - 48.88x_1x_3 - 12.25x_1x_4 - 36.75x_1x_5 - 2.88x_2x_3 + 16.75x_2x_4 + 11.00x_2x_5 - 4.38x_3x_4 + 2.38x_3x_5 - 1.00x_4x_5$$

$$Y2 = 9.44 + 1.08x_1 + 3.69x_2 + 1.60x_3 - 0.21x_4 - 1.50x_5 + 0.56x_1^2 - 0.13x_2^2 + 0.24x_3^2 + 0.37x_4^2 + 1.06x_5^2 \\ + 2.15x_1x_2 - 0.46x_1x_3 - 2.75x_1x_4 - 2.13x_1x_5 + 0.19x_2x_3 - 0.85x_2x_4 - 2.35x_2x_5 - 0.96x_3x_4 - 1.21x_3x_5 + 0.63x_4x_5$$

$$Y3 = 77.79 - 10.12x_1 - 8.68x_2 - 0.10x_3 - 0.71x_4 + 1.37x_5 - 4.16x_1^2 - 4.77x_2^2 - 2.36x_3^2 - 0.34x_4^2 - 0.33x_5^2 \\ - 6.21x_1x_2 + 2.77x_1x_3 + 1.77x_1x_4 + 0.71x_1x_5 - 1.68x_2x_3 - 0.26x_2x_4 + 0.48x_2x_5 + 0.04x_3x_4 + 1.51x_3x_5 + 0.03x_4x_5$$

$$Y4 = 103.81 - 8.25x_1 + 7.51x_2 + 2.38x_3 + 1.13x_4 + 1.66x_5 - 7.37x_1^2 - 4.66x_2^2 - 2.58x_3^2 - 2.01x_4^2 + 1.07x_5^2 \\ - 0.11x_1x_2 + 0.47x_1x_3 + 2.09x_1x_4 + 0.77x_1x_5 - 0.78x_2x_3 + 0.29x_2x_4 - 0.16x_2x_5 + 1.44x_3x_4 + 4.09x_3x_5 + 0.29x_4x_5$$

Figure F-2 Whey Protein, Fitted Responses

“ISWT”

	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
1		Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response		
2	Settings		-0.25	0.64	2.00	1.06	2.00	0.06	0.41	4.00	1.12	4.00	-0.16	-0.49	-0.26	-0.49	1.28	0.68	1.28	2.12	4.00	2.12	Value		
3	Y1	1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1186.45		
4	Y2	9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	9.88		
5	Y3	77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	66.44		
6	Y4	103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	127.15		
7																									
8	Constraints																								
9		x1, x2, x3, x4, x5 >=																					-2		
10		x1, x2, x3, x4, x5 <=																						2	
11																									
12				Y1	Y2	Y3	Y4																		
13		Lower Bound		0.00	0.00	0.00	0.00																		
14		Upper Bound		1640.54	85.04	97.79	127.15																		
15																									
16		Solved Lower Bound		-44.86	-22.44	-42.45	-13.41																		

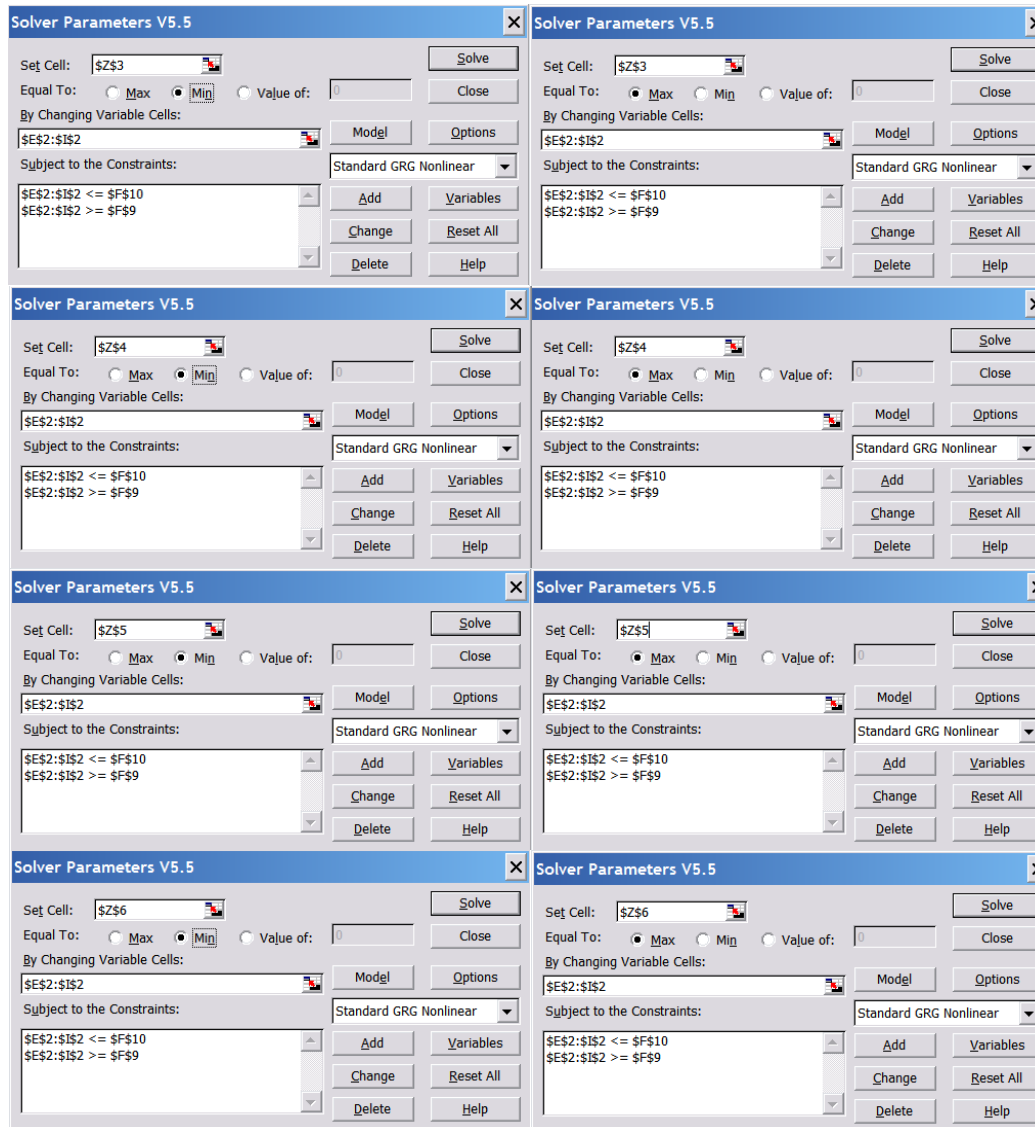


Figure F-3 Why Protein, Extreme Values

“ISWT 1”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:			Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response	
2		Settings			-2.00	1.33	1.70	0.87	2.00	4.00	1.77	2.89	0.75	4.00	-2.66	-3.40	-1.74	-4.00	2.26	1.16	2.66	1.48	3.40	1.74	Value	
3		Y1		1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1640.54	
4		Y2		9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	16.84	
5		Y3		77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	58.56	
6		Y4		103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	102.89	
7		Constraints																								
8			Y2	≥		0.00			=ISWT!G13																	
9			Y2	≤		85.04			=ISWT!G14																	
10			Y3	≥		0.00			=ISWT!H13																	
11			Y3	≤		97.79			=ISWT!H14																	
12			Y4	≥		0.00			=ISWT!I13																	
13			Y4	≤		127.15			=ISWT!I14																	
14			x1, x2, x3, x4, x5	≥		-2.00																				
15			x1, x2, x3, x4, x5	≤		2.00																				
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"							No																	
19																										
20	Step 9:	Lagrange multiplier Y2=							0.00																	
21		Lagrange multiplier Y3=							0.00																	
22		Lagrange multiplier Y4=							0.00																	

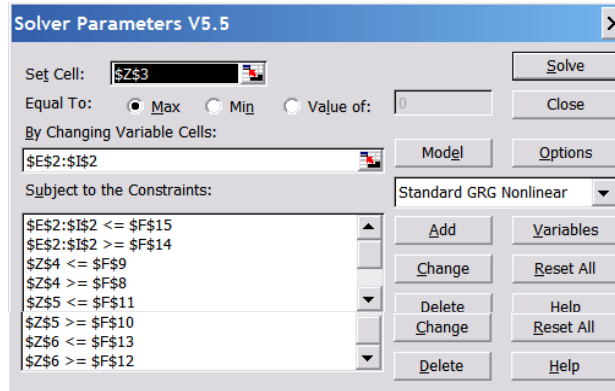


Figure F-4 Whey Protein, Solver Parameters

"ISWT 1"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:			Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response	
2		Settings			-2.00	1.33	1.70	0.87	2.00	4.00	1.77	2.89	0.75	4.00	-2.66	-3.40	-1.74	-4.00	2.26	1.16	2.66	1.48	3.40	1.74	Value	
3		Y1		1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1640.54	
4		Y2		9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	16.84	
5		Y3		77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	58.56	
6		Y4		103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	102.89	
7		Constraints																								
8			Y2	≥	0.00				=ISWT!G13																	
9			Y2	≤	85.04				=ISWT!G14																	
10			Y3	≥	0.00				=ISWT!H13																	
11			Y3	≤	97.79				=ISWT!H14																	
12			Y4	≥	0.00				=ISWT!I13																	
13			Y4	≤	127.15				=ISWT!I14																	
14			x1, x2, x3, x4, x5	≥	-2.00																					
15			x1, x2, x3, x4, x5	≤	2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"																							No	
19																										
20	Step 9:	Lagrange multiplier Y2=																							0.00	
21		Lagrange multiplier Y3=																								0.00
22		Lagrange multiplier Y4=																								0.00

	AE	AC	AD	AE	AF
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Whey Protein.xls]ISWT 1</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-2.00	-148.63
10	\$F\$2	Settings x2		1.33	0.00
11	\$G\$2	Settings x3		1.70	0.00
12	\$H\$2	Settings x4		0.87	0.00
13	\$I\$2	Settings x5		2.00	73.52
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		16.84	0.00
19	\$Z\$4	Y2 Value		16.84	0.00
20	\$Z\$5	Y3 Value		58.56	0.00
21	\$Z\$5	Y3 Value		58.56	0.00
22	\$Z\$6	Y4 Value		102.89	0.00
23	\$Z\$6	Y4 Value		102.89	0.00

	AK	AL	AM	AN	AO	AP	AQ
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Whey Protein.xls]ISWT 1						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Max)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$Z\$3	Y1 Value	1176.98	1640.54		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-2.00		
16		\$F\$2	Settings x2	0.00	1.33		
17		\$G\$2	Settings x3	0.00	1.70		
18		\$H\$2	Settings x4	0.00	0.87		
19		\$I\$2	Settings x5	0.00	2.00		
20							
21							
22	Constraints						
23		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
24		\$Z\$4	Y2 Value	16.84	\$Z\$4<=\$F\$9	Not Binding	68.200098
25		\$Z\$4	Y2 Value	16.84	\$Z\$4>=\$F\$8	Not Binding	16.84
26		\$Z\$5	Y3 Value	58.56	\$Z\$5<=\$F\$11	Not Binding	39.233374
27		\$Z\$5	Y3 Value	58.56	\$Z\$5>=\$F\$10	Not Binding	58.56
28		\$Z\$6	Y4 Value	102.89	\$Z\$6<=\$F\$13	Not Binding	24.263596
29		\$Z\$6	Y4 Value	102.89	\$Z\$6>=\$F\$12	Not Binding	102.89
30		\$E\$2	Settings x1	-2.00	\$E\$2<=\$F\$15	Not Binding	4
31		\$F\$2	Settings x2	1.33	\$F\$2<=\$F\$15	Not Binding	0.6687688
32		\$G\$2	Settings x3	1.70	\$G\$2<=\$F\$15	Not Binding	0.2999095
33		\$H\$2	Settings x4	0.87	\$H\$2<=\$F\$15	Not Binding	1.1319782
34		\$I\$2	Settings x5	2.00	\$I\$2<=\$F\$15	Binding	0
35		\$E\$2	Settings x1	-2.00	\$E\$2>=\$F\$14	Binding	0.00
36		\$F\$2	Settings x2	1.33	\$F\$2>=\$F\$14	Not Binding	3.33
37		\$G\$2	Settings x3	1.70	\$G\$2>=\$F\$14	Not Binding	3.70
38		\$H\$2	Settings x4	0.87	\$H\$2>=\$F\$14	Not Binding	2.87
39		\$I\$2	Settings x5	2.00	\$I\$2>=\$F\$14	Not Binding	4.00
40		\$F\$2	Settings x2	0.34	\$F\$2>=\$F\$12	Not Binding	0.34
41		\$G\$2	Settings x3	0.44	\$G\$2>=\$F\$12	Not Binding	0.44

Figure F-5 Whey Protein, Scenario 1, Iteration 1



“ISWT 2”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:		Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5			Response
2		Settings		-2.00	1.35	1.70	0.87	2.00	4.00	1.82	2.89	0.75	4.00	-2.70	-3.40	-1.74	-4.00	2.29	1.17	2.70	1.48	3.40	1.74			Value
3		Y1	1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00			1640.54
4		Y2	9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63			16.74
5		Y3	77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03			58.36
6		Y4	103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29			102.79
7		Constraints																								
8		Y2	≥		0.00				=ISWT!G13																	
9		Y2	≤		16.74				=ISWT!G14-ISWT 1!AQ24-0.1																	
10		Y3	≥		0.00				=ISWT!H13																	
11		Y3	≤		58.36				=ISWT!H14-ISWT 1!AQ26-0.2																	
12		Y4	≥		0.00				=ISWT!I13																	
13		Y4	≤		102.79				=ISWT!I14-ISWT 1!AQ28-0.1																	
14		x1, x2, x3, x4, x5	≥		-2.00																					
15		x1, x2, x3, x4, x5	≤		2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"																								No
19																										
20	Step 9:	Lagrange multiplier Y2=																								0.09
21		Lagrange multiplier Y3=																								0.02
22		Lagrange multiplier Y4=																								0.01
23																										
24	Step 10:	Given that Y1 equals 1640.54 and Y2 equals 16.74,																								
25		please indicate your preference for decreasing Y1 by 0.09 units for one unit increase in Y2,																								
26		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
27		and -2 is a strong preference to make the opposite trade-off.																								
28																										
29		W (Y2)=																								2.00
30																										
31		Given that Y1 equals 1640.54 and Y3 equals 58.36,																								
32		please indicate your preference for decreasing Y1 by 0.02 units for one unit increase in Y3,																								
33		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
34		and -2 is a strong preference to make the opposite trade-off.																								
35																										
36		W (Y3)=																								-2.00
37																										
38		Given that Y1 equals 1640.54 and Y4 equals 102.79,																								
39		please indicate your preference for decreasing Y1 by 0.01 units for one unit increase in Y4,																								
40		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
41		and -2 is a strong preference to make the opposite trade-off.																								
42																										
43		W (Y4)=																								1.00
44																										
45																										
46	Step 11:	Update ε	c=					0.618																		
47			iteration=					1																		
48			step size=					0.0039																		= (G46^G47)/(SQRT((G29*ABS(Z4))^2+(G36*ABS(Z5))^2+(G43*ABS(Z6))^2))
49																										
50			ε (Y2)=					25.62																		=F9+((G48*G29*ABS(Z4))*(ISWT!G14-F9))
51			ε (Y3)=					40.48																		=F11+((G48*G36*ABS(Z5))*(ISWT!H14-F11))
52			ε (Y4)=					112.52																		=F13+((G48*G43*ABS(Z6))*(ISWT!I14-F13))
53																										

	AE	AC	AD	AE	AF	
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>					
4	<b>Worksheet: [Whey Protein.xls]ISWT 2</b>					
5						
6	Adjustable Cells					
7					<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>	
9	\$E\$2	Settings x1		-2.00	-149.00	
10	\$F\$2	Settings x2		1.35	0.00	
11	\$G\$2	Settings x3		1.70	0.00	
12	\$H\$2	Settings x4		0.87	0.00	
13	\$I\$2	Settings x5		2.00	73.33	
14						
15	Constraints					
16					<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>	
18	\$Z\$4	Y2 Value		16.74	0.09	
19	\$Z\$4	Y2 Value		16.74	0.00	
20	\$Z\$5	Y3 Value		58.36	0.02	
21	\$Z\$5	Y3 Value		58.36	0.00	
22	\$Z\$6	Y4 Value		102.79	0.01	
23	\$Z\$6	Y4 Value		102.79	0.00	

Figure F-6 Whey Protein, Scenario 1, Iteration 2

“ISWT 3”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:			Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response	
2		Settings			-2.00	2.00	2.00	1.62	2.00	4.00	4.00	4.00	2.63	4.00	-4.00	-4.00	-3.25	-4.00	4.00	3.25	4.00	3.25	4.00	3.25	Value	
3		Y1		1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1613.32	
4		Y2		9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	15.79	
5		Y3		77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	40.48	
6		Y4		103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	93.64	
7		Constraints																								
8			Y2	≥	0.00				=ISWT!G13																	
9			Y2	≤	25.62				=ISWT!G50																	
10			Y3	≥	0.00				=ISWT!H13																	
11			Y3	≤	40.48				=ISWT!G51																	
12			Y4	≥	0.00				=ISWT!I13																	
13			Y4	≤	112.52				=ISWT!G52																	
14			x1, x2, x3, x4, x5	≥	-2.00																					
15			x1, x2, x3, x4, x5	≤	2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"																							No	
19																										
20	Step 9:	Lagrange multiplier Y2=																							0.00	
21		Lagrange multiplier Y3=																								7.25
22		Lagrange multiplier Y4=																								0.00

	AE	AC	AD	AE	AF
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Whey Protein.xls]ISWT 3</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-2.00	-217.76
10	\$F\$2	Settings x2		2.00	111.55
11	\$G\$2	Settings x3		2.00	79.18
12	\$H\$2	Settings x4		1.62	0.00
13	\$I\$2	Settings x5		2.00	61.56
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		15.79	0.00
19	\$Z\$4	Y2 Value		15.79	0.00
20	\$Z\$5	Y3 Value		40.48	7.25
21	\$Z\$5	Y3 Value		40.48	0.00
22	\$Z\$6	Y4 Value		93.64	0.00
23	\$Z\$6	Y4 Value		93.64	0.00

	AK	AL	AM	AN	AO	AP	AQ
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Whey Protein.xls]ISWT 3						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Max)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$Z\$3	Y1 Value	1176.98	1613.32		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-2.00		
16		\$F\$2	Settings x2	0.00	2.00		
17		\$G\$2	Settings x3	0.00	2.00		
18		\$H\$2	Settings x4	0.00	1.62		
19		\$I\$2	Settings x5	0.00	2.00		
20							
21							
22	Constraints						
23		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
24		\$Z\$4	Y2 Value	15.79	\$Z\$4<=\$F\$9	Not Binding	9.8339014
25		\$Z\$4	Y2 Value	15.79	\$Z\$4>=\$F\$8	Not Binding	15.79
26		\$Z\$5	Y3 Value	40.48	\$Z\$5<=\$F\$11	Binding	0
27		\$Z\$5	Y3 Value	40.48	\$Z\$5>=\$F\$10	Not Binding	40.48
28		\$Z\$6	Y4 Value	93.64	\$Z\$6<=\$F\$13	Not Binding	18.874306
29		\$Z\$6	Y4 Value	93.64	\$Z\$6>=\$F\$12	Not Binding	93.64
30		\$E\$2	Settings x1	-2.00	\$E\$2<=\$F\$15	Not Binding	4
31		\$F\$2	Settings x2	2.00	\$F\$2<=\$F\$15	Binding	0
32		\$G\$2	Settings x3	2.00	\$G\$2<=\$F\$15	Binding	0
33		\$H\$2	Settings x4	1.62	\$H\$2<=\$F\$15	Not Binding	0.3773502
34		\$I\$2	Settings x5	2.00	\$I\$2<=\$F\$15	Binding	0
35		\$E\$2	Settings x1	-2.00	\$E\$2>=\$F\$14	Binding	0.00
36		\$F\$2	Settings x2	2.00	\$F\$2>=\$F\$14	Not Binding	4.00
37		\$G\$2	Settings x3	2.00	\$G\$2>=\$F\$14	Not Binding	4.00
38		\$H\$2	Settings x4	1.62	\$H\$2>=\$F\$14	Not Binding	3.62
39		\$I\$2	Settings x5	2.00	\$I\$2>=\$F\$14	Not Binding	4.00
40		\$F\$2	Settings x2	0.34	\$F\$2>=\$F\$12	Not Binding	0.34
41		\$G\$2	Settings x3	0.44	\$G\$2>=\$F\$12	Not Binding	0.44

Figure F-7 Whey Protein, Scenario 1, Iteration 3

"ISWT 4"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:			Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response	
2		Settings			-0.04	2.00	-0.52	2.00	0.76	0.00	4.00	0.27	4.00	0.58	-0.08	0.02	-0.08	-0.03	-1.04	4.00	1.52	-1.04	-0.40	1.52	Value	
3		Y1		1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1018.57	
4		Y2		9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	11.39	
5		Y3		77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	40.48	
6		Y4		103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	93.54	
7		Constraints																								
8			Y2	≥	0.00				=ISWT!G13																	
9			Y2	≤	15.69				=ISWT 2!G50-ISWT 3!AQ24-0.1																	
10			Y3	≥	0.00				=ISWT!H13																	
11			Y3	≤	40.48				=ISWT 2!G51																	
12			Y4	≥	0.00				=ISWT!I13																	
13			Y4	≤	93.54				=ISWT 2!G52-ISWT 3!AQ28-0.1																	
14			x1, x2, x3, x4, x5	≥	-2.00																					
15			x1, x2, x3, x4, x5	≤	2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"																							No	
19																										
20	Step 9:	Lagrange multiplier Y2=																							0.00	
21		Lagrange multiplier Y3=																								11.03
22		Lagrange multiplier Y4=																								9.73

	AE	AC	AD	AE	AF
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Whey Protein.xls]ISWT 4</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.04	0.00
10	\$F\$2	Settings x2		2.00	326.09
11	\$G\$2	Settings x3		-0.52	0.00
12	\$H\$2	Settings x4		2.00	17.90
13	\$I\$2	Settings x5		0.76	0.00
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		11.39	0.00
19	\$Z\$4	Y2 Value		11.39	0.00
20	\$Z\$5	Y3 Value		40.48	11.03
21	\$Z\$5	Y3 Value		40.48	0.00
22	\$Z\$6	Y4 Value		93.54	9.73
23	\$Z\$6	Y4 Value		93.54	0.00

	AK	AL	AM	AN	AO	AP	AQ
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Whey Protein.xls]ISWT 4						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Max)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$Z\$3	Y1 Value	1176.98	1018.57		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-0.04		
16		\$F\$2	Settings x2	0.00	2.00		
17		\$G\$2	Settings x3	0.00	-0.52		
18		\$H\$2	Settings x4	0.00	2.00		
19		\$I\$2	Settings x5	0.00	0.76		
20							
21							
22	Constraints						
23		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
24		\$Z\$4	Y2 Value	11.39	\$Z\$4<=\$F\$9	Not Binding	4.2997579
25		\$Z\$4	Y2 Value	11.39	\$Z\$4>=\$F\$8	Not Binding	11.39
26		\$Z\$5	Y3 Value	40.48	\$Z\$5<=\$F\$11	Binding	0
27		\$Z\$5	Y3 Value	40.48	\$Z\$5>=\$F\$10	Not Binding	40.48
28		\$Z\$6	Y4 Value	93.54	\$Z\$6<=\$F\$13	Binding	0
29		\$Z\$6	Y4 Value	93.54	\$Z\$6>=\$F\$12	Not Binding	93.54
30		\$E\$2	Settings x1	-0.04	\$E\$2<=\$F\$15	Not Binding	2.0406307
31		\$F\$2	Settings x2	2.00	\$F\$2<=\$F\$15	Binding	0
32		\$G\$2	Settings x3	-0.52	\$G\$2<=\$F\$15	Not Binding	2.5216736
33		\$H\$2	Settings x4	2.00	\$H\$2<=\$F\$15	Binding	0
34		\$I\$2	Settings x5	0.76	\$I\$2<=\$F\$15	Not Binding	1.2382329
35		\$E\$2	Settings x1	-0.04	\$E\$2>=\$F\$14	Not Binding	1.96
36		\$F\$2	Settings x2	2.00	\$F\$2>=\$F\$14	Not Binding	4.00
37		\$G\$2	Settings x3	-0.52	\$G\$2>=\$F\$14	Not Binding	1.48
38		\$H\$2	Settings x4	2.00	\$H\$2>=\$F\$14	Not Binding	4.00
39		\$I\$2	Settings x5	0.76	\$I\$2>=\$F\$14	Not Binding	2.76
40		\$F\$2	Settings x2	0.34	\$F\$2>=\$F\$12	Not Binding	0.34
41		\$G\$2	Settings x3	0.44	\$G\$2>=\$F\$12	Not Binding	0.44

Figure F-8 Whey Protein, Scenario 1, Iteration 4

“ISWT 5”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:		Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5			Response
2		Settings		-0.04	2.00	-0.53	2.00	0.80	0.00	4.00	0.28	4.00	0.64	-0.08	0.02	-0.08	-0.03	-1.05	4.00	1.60	-1.05	-0.42	1.60			Value
3		Y1	1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00			1018.56
4		Y2	9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63			11.29
5		Y3	77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03			40.48
6		Y4	103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29			93.54
7		Constraints																								
8		Y2	≥	0.00					=ISWT!G13																	
9		Y2	≤	11.29					=ISWT 4!F9-ISWT 4!AQ24-0.1																	
10		Y3	≥	0.00					=ISWT!H13																	
11		Y3	≤	40.48					=ISWT 4!F11																	
12		Y4	≥	0.00					=ISWT!I13																	
13		Y4	≤	93.54					=ISWT 4!F13																	
14		x1, x2, x3, x4, x5	≥	-2.00																						
15		x1, x2, x3, x4, x5	≤	2.00																						
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"							No																	
19																										
20	Step 9:	Lagrange multiplier Y2=							0.35																	
21		Lagrange multiplier Y3=							11.11																	
22		Lagrange multiplier Y4=							9.52																	
23																										
24	Step 10:	Given that Y1 equals 1018.56 and Y2 equals 11.29,																								
25		please indicate your preference for decreasing Y1 by 0.35 units for one unit increase in Y2,																								
26		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
27		and -2 is a strong preference to make the opposite trade-off.																								
28																										
29		W (Y2)=							1.00																	
30																										
31		Given that Y1 equals 1018.56 and Y3 equals 40.48,																								
32		please indicate your preference for decreasing Y1 by 11.11 units for one unit increase in Y3,																								
33		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
34		and -2 is a strong preference to make the opposite trade-off.																								
35																										
36		W (Y3)=							0.00																	
37																										
38		Given that Y1 equals 1018.56 and Y4 equals 93.54,																								
39		please indicate your preference for decreasing Y1 by 9.52 units for one unit increase in Y4,																								
40		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
41		and -2 is a strong preference to make the opposite trade-off.																								
42																										
43		W (Y4)=							2.00																	
44																										
45																										
46	Step 11:	Update ε	c=						0.618																	
47			iteration=						2																	
48			step size=						0.002																	
49																										
50			ε (Y2)=						12.99																	
51			ε (Y3)=						40.48																	
52			ε (Y4)=						106.36																	
53																										

	AE	AC	AD	AE	AF
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Whey Protein.xls]ISWT 5				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-0.04	0.00
10	\$F\$2	Settings x2		2.00	326.50
11	\$G\$2	Settings x3		-0.53	0.00
12	\$H\$2	Settings x4		2.00	16.24
13	\$I\$2	Settings x5		0.80	0.00
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		11.29	0.35
19	\$Z\$4	Y2 Value		11.29	0.00
20	\$Z\$5	Y3 Value		40.48	11.11
21	\$Z\$5	Y3 Value		40.48	0.00
22	\$Z\$6	Y4 Value		93.54	9.52
23	\$Z\$6	Y4 Value		93.54	0.00

Figure F-9 Whey Protein, Scenario 1, Iteration 5



"ISWT 6"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:			Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5	Response	
2		Settings			-1.53	2.00	2.00	1.61	1.68	2.36	4.00	4.00	2.58	2.81	-3.07	-3.07	-2.47	-2.57	4.00	3.21	3.35	3.21	3.35	2.69	Value	
3		Y1		1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00	1492.04	
4		Y2		9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63	12.99	
5		Y3		77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03	40.48	
6		Y4		103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29	100.49	
7		Constraints																								
8			Y2	≥	0.00			=ISWT!G13																		
9			Y2	≤	12.99			=ISWT 5!G50																		
10			Y3	≥	0.00			=ISWT!H13																		
11			Y3	≤	40.48			=ISWT 5!G51																		
12			Y4	≥	0.00			=ISWT!I13																		
13			Y4	≤	106.36			=ISWT 5!G52																		
14			x1, x2, x3, x4, x5	≥	-2.00																					
15			x1, x2, x3, x4, x5	≤	2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"						No																		
19																										
20	Step 9:	Lagrange multiplier Y2=						42.66																		
21		Lagrange multiplier Y3=						32.25																		
22		Lagrange multiplier Y4=						0.00																		

	AE	AC	AD	AE	AF
3	<b>Microsoft Excel 11.0 Sensitivity Report</b>				
4	<b>Worksheet: [Whey Protein.xls]ISWT 6</b>				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-1.53	0.00
10	\$F\$2	Settings x2		2.00	863.70
11	\$G\$2	Settings x3		2.00	410.65
12	\$H\$2	Settings x4		1.61	0.00
13	\$I\$2	Settings x5		1.68	0.00
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		12.99	42.66
19	\$Z\$4	Y2 Value		12.99	0.00
20	\$Z\$5	Y3 Value		40.48	32.25
21	\$Z\$5	Y3 Value		40.48	0.00
22	\$Z\$6	Y4 Value		100.49	0.00
23	\$Z\$6	Y4 Value		100.49	0.00

	AK	AL	AM	AN	AO	AP	AQ
3	Microsoft Excel 11.0 Answer Report						
4	Worksheet: [Whey Protein.xls]ISWT 6						
5	Result: Solver found a solution. All constraints and optimality conditions are satisfied.						
6	Engine: Standard GRG Nonlinear						
7							
8	Target Cell (Max)						
9		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
10		\$Z\$3	Y1 Value	1176.98	1492.04		
11							
12							
13	Adjustable Cells						
14		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
15		\$E\$2	Settings x1	0.00	-1.53		
16		\$F\$2	Settings x2	0.00	2.00		
17		\$G\$2	Settings x3	0.00	2.00		
18		\$H\$2	Settings x4	0.00	1.61		
19		\$I\$2	Settings x5	0.00	1.68		
20							
21							
22	Constraints						
23		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
24		\$Z\$4	Y2 Value	12.99	\$Z\$4<=\$F\$9	Binding	0
25		\$Z\$4	Y2 Value	12.99	\$Z\$4>=\$F\$8	Not Binding	12.99
26		\$Z\$5	Y3 Value	40.48	\$Z\$5<=\$F\$11	Binding	0
27		\$Z\$5	Y3 Value	40.48	\$Z\$5>=\$F\$10	Not Binding	40.48
28		\$Z\$6	Y4 Value	100.49	\$Z\$6<=\$F\$13	Not Binding	5.8705947
29		\$Z\$6	Y4 Value	100.49	\$Z\$6>=\$F\$12	Not Binding	100.49
30		\$E\$2	Settings x1	-1.53	\$E\$2<=\$F\$15	Not Binding	3.5349638
31		\$F\$2	Settings x2	2.00	\$F\$2<=\$F\$15	Binding	0
32		\$G\$2	Settings x3	2.00	\$G\$2<=\$F\$15	Binding	0
33		\$H\$2	Settings x4	1.61	\$H\$2<=\$F\$15	Not Binding	0.392831
34		\$I\$2	Settings x5	1.68	\$I\$2<=\$F\$15	Not Binding	0.3246185
35		\$E\$2	Settings x1	-1.53	\$E\$2>=\$F\$14	Not Binding	0.47
36		\$F\$2	Settings x2	2.00	\$F\$2>=\$F\$14	Not Binding	4.00
37		\$G\$2	Settings x3	2.00	\$G\$2>=\$F\$14	Not Binding	4.00
38		\$H\$2	Settings x4	1.61	\$H\$2>=\$F\$14	Not Binding	3.61
39		\$I\$2	Settings x5	1.68	\$I\$2>=\$F\$14	Not Binding	3.68
40		\$F\$2	Settings x2	0.34	\$F\$2>=\$F\$12	Not Binding	0.34
41		\$G\$2	Settings x3	0.44	\$G\$2>=\$F\$12	Not Binding	0.44

Figure F-10 Whey Protein, Scenario 1, Iteration 6

“ISWT 7”

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Step 7:		Constant	x1	x2	x3	x4	x5	x1sq	x2sq	x3sq	x4sq	x5sq	x1x2	x1x3	x1x4	x1x5	x2x3	x2x4	x2x5	x3x4	x3x5	x4x5			Response
2		Settings		-1.54	2.00	2.00	1.60	1.67	2.36	4.00	4.00	2.57	2.78	-3.07	-3.07	-2.46	-2.56	4.00	3.20	3.34	3.20	3.34	2.67			Value
3		Y1	1176.98	-176.00	-18.17	57.58	21.83	19.08	-56.97	-23.84	-44.84	-34.09	-9.22	-25.00	-48.88	-12.25	-36.75	-2.88	16.75	11.00	-4.38	2.38	-1.00			1492.04
4		Y2	9.44	1.08	3.69	1.60	-0.21	-1.50	0.56	-0.13	0.24	0.37	1.06	2.15	-0.46	-2.75	-2.13	0.19	-0.85	-2.35	-0.96	-1.21	0.63			12.99
5		Y3	77.79	-10.12	-8.68	-0.10	-0.71	1.37	-4.16	-4.77	-2.36	-0.34	-0.33	-6.21	2.77	1.77	0.71	-1.68	-0.26	0.48	0.04	1.51	0.03			40.48
6		Y4	103.81	-8.25	7.51	2.38	1.13	1.66	-7.37	-4.66	-2.58	-2.01	1.07	-0.11	0.47	2.09	0.77	-0.78	0.29	-0.16	1.44	4.09	0.29			100.39
7		Constraints																								
8			Y2	≥	0.00				=ISWT!G13																	
9			Y2	≤	12.99				=ISWT 5!G50																	
10			Y3	≥	0.00				=ISWT!H13																	
11			Y3	≤	40.48				=ISWT 5!G51																	
12			Y4	≥	0.00				=ISWT!I13																	
13			Y4	≤	100.39				=ISWT 6!F13-ISWT 6!AQ28-0.1																	
14			x1, x2, x3, x4, x5	≥	-2.00																					
15			x1, x2, x3, x4, x5	≤	2.00																					
16																										
17																										
18	Step 8:	Ask DM, "Are you satisfied with the above solution?"																								No
19																										
20	Step 9:	Lagrange multiplier Y2=																								43.08
21		Lagrange multiplier Y3=																								32.29
22		Lagrange multiplier Y4=																								0.08
23																										
24	Step 10:	Given that Y1 equals 1492.04 and Y2 equals 12.99,																								
25		please indicate your preference for decreasing Y1 by 43.08 units for one unit increase in Y2,																								
26		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
27		and -2 is a strong preference to make the opposite trade-off.																								
28																										
29		W (Y2)=																								0.00
30																										
31		Given that Y1 equals 1492.04 and Y3 equals 40.48,																								
32		please indicate your preference for decreasing Y1 by 32.29 units for one unit increase in Y3,																								
33		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
34		and -2 is a strong preference to make the opposite trade-off.																								
35																										
36		W (Y3)=																								0.00
37																										
38		Given that Y1 equals 1492.04 and Y4 equals 100.39,																								
39		please indicate your preference for decreasing Y1 by 0.08 units for one unit increase in Y4,																								
40		on a 5-point scale where 2 is a strong preference for making the trade-off, 0 is a preference to make no trade-off,																								
41		and -2 is a strong preference to make the opposite trade-off.																								
42																										
43		W (Y4)=																								0.00

	AE	AC	AD	AE	AF
3	Microsoft Excel 11.0 Sensitivity Report				
4	Worksheet: [Whey Protein.xls]ISWT 7				
5					
6	Adjustable Cells				
7				<b>Final</b>	<b>Reduced</b>
8	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Gradient</b>
9	\$E\$2	Settings x1		-1.54	0.00
10	\$F\$2	Settings x2		2.00	866.54
11	\$G\$2	Settings x3		2.00	411.23
12	\$H\$2	Settings x4		1.60	0.00
13	\$I\$2	Settings x5		1.67	0.00
14					
15	Constraints				
16				<b>Final</b>	<b>Lagrange</b>
17	<b>Cell</b>	<b>Name</b>		<b>Value</b>	<b>Multiplier</b>
18	\$Z\$4	Y2 Value		12.99	43.08
19	\$Z\$4	Y2 Value		12.99	0.00
20	\$Z\$5	Y3 Value		40.48	32.29
21	\$Z\$5	Y3 Value		40.48	0.00
22	\$Z\$6	Y4 Value		100.39	0.08
23	\$Z\$6	Y4 Value		100.39	0.00

Figure F-11 Whey Protein, Scenario 1, Iteration 7

## **APPENDIX G: AUTOMOTIVE PROBLEM**

## Usability Example

Use the automotive test problem that Dvorak used in his dissertation to test the Distant Minimization technique. The study could be duplicated using the real decision-makers to evaluate the process. In the Engine Research Laboratory at the University of Central Florida, a study was conducted to determine the exhaust configuration of a restrictor-plate Winston Cup race engine that would deliver the optimum performance of the racecar. There were two response functions: the horsepower of the race car in the draft (Y1) and the horsepower of the race car out of the draft (Y2). The independent variables are oil temperature, engine speed, and exhaust header length. A fourth independent variable in the Y2 function is pressure. The goal of the study was to optimize the restrictor-plate so that the engine performed well both as the leader and as a follower in the draft. The regression results obtained by Dvorak will be used for consistency.

Goals:

- Test process in a “real-world” environment
  - Ask the DM to select the primary objective
  - Ask the DM to select bounds on the secondary objective
    - If inside the extreme values, use the DM’s bounds
- Gain feedback on the usability of the process from a decision-maker’s point of view

Questions for decision-maker:

- Where possible, ask for a rating using a five point scale
- Please explain why you rated ... that way
- Other comments

Question	Rating	Comments
Overall		
Ease of Use		
Level of interaction		
Solutions reflect preferences		
Use of trade-offs		
Number of iterations		
Satisfaction with final solution		
Confidence in the solution		

- Feedback on wording of trade-off question
- 5 point scale vs. 20 point scale (-2 to 2 vs. -10 to 10)
- How does this compare with other methods previously used such as Dvorak’s Distant Minimization Technique?
- What could make this process easier for the decision-maker?
- Would having an initial set of solutions (e.g., SWT) first and then conducting trade-offs be easier for the DM?

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