A STUDY ON THE PLASTICITY AND FRACTURE OF AISI 4340 STEEL UNDER DIFFERENT LOADING CONDITIONS CONSIDERING HEAT TREATMENT AND MICROMECHANICS

by

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ABSTRACT

Accurate predictions of material strength under different loading conditions with large plastic deformation and ductile fracture are of great importance. This dissertation aims to develop an essential understanding of ductile fracture of AISI 4340 steel alloy using both empirical and micromechanics based models. For this purpose, 29 specimens of different geometries with different heat-treatments were designed to investigate the effects of stress states. These specimens are: (a) 13 round bars with different notches (axial symmetric tension); (b) 13 flat grooved specimens with different grooves; (c) 3 small round cylinders. Mechanical tests up to fracture were conducted on these specimens to characterize the influence of hydrostatic stress and Lode angle on material plasticity and fracture. Scanning electron microscopy (SEM) observations were performed on both original and fractured specimens to investigate different micromechanical revelations and features. The plasticity model with pressure and Lode angle effects (PPL model) and the modified Mohr-Coulomb (MMC) fracture criterion were used to predict plastic flow and fracture initiation behaviors under different loading conditions in finite element simulations. A model optimization method using ISIGHT was set up to achieve simulation results that were well correlated with experimental data. The effects of heat-treatment on material strength and ductility of AISI 4340 steel were also discussed. This work was further carried onto the microvoids based metal plasticity theory. The well-known Gurson-Tvergaard-Needleman (GTN) model was calibrated for AISI 4340 steel. It is found that the GTN model is not sufficient in simulating test data for the 16 Rockwell hardness plane strain specimens. Therefore, The GTN model is modified to include the Lode angle dependence on matrix material plasticity. It is also found that using a
fixed or constant microvoid volume fraction at failure ($f_t$) for all loading conditions is inadequate. Following a similar derivation of the MMC fracture model, the microvoid volume fraction at failure ($f_t$) becomes a function of both stress triaxiality and Lode angle. This new criterion is named (GTN-MMC). The proposed plasticity and fracture models were implemented into ABAQUS through a user-defined material subroutine (VUMAT) for FE simulations. Good correlations were achieved between experimental results and numerical simulations.
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CHAPTER 1:  
INTRODUCTION

1.1 Background

Many engineering designs have inherent flaws that could affect the reliability and performance intended. Therefore, insufficient safety measurements and poor understanding of fracture mechanics could result in catastrophic accidents. The use of “factors of safety” is a standard method in engineering design to account for inherent flaws and defects of materials. However, this does not give any physical understanding of the problem and may still lead to failure. Enormous disasters of the twentieth century such as the Boston molasses failure, Comet disaster, and the Aloha airline Boeing fuselage failure has caused the engineering community to sit and reevaluate the conventional design approaches. The analysis of these failures emphasized the role of a solid understanding and implementation of fracture mechanics into design stages (especially when considering the effects of inherent flaws).

Predictions of different fracture phenomena are essential to validate the proposed plasticity and fracture models. Many efforts have been taken over the past few decades to model the correct plasticity and fracture behaviors. The importance of this subject arises in many fields were specific safety measures should be taken. For example, car manufacturers are always looking for optimal designs and solutions that are safe, reliable and save money. These designs should always address the fracture and failure phenomena. Various simulation packages are gaining much popularity due to recent evolutions in computing power. Successful simulations of any system under extreme loading events require high fidelity models. Ductile fracture in steels is believed to be a multistep
process that occurs due to void microscopic activity ranging from growth, nucleation, shear, and coalescence of microvoids (Garrison and Moody 1987). A solid physical understanding of different ductile fracture models and criteria are essential to enable the advancement of simulations in this area. The present dissertation is concerned with empirical and micromechanical based aspects of ductile fracture of AISI 4340 steel.

1.2  **AISI 4340 Steel Alloy and Experiments**

A clear understanding of mechanical properties and behaviors of materials are crucial in predicting failure. Due to the diversity of different materials, finite element method techniques (FEM) are gaining much publicity in many engineering applications. The use of (FEM) tools and software allows unnecessary test repetitiveness and thus saving time and money. To be able to use such tools, a comprehensive elasticity, plasticity, and fracture models become essential. Many models have been proposed and extensively investigated by the mechanic’s community over the past four decades; several macroscopic and microscopic models have been developed to describe the ductile fracture behaviors of metals. These models can be categorized into three groups: physical based models, phenomenological and empirical models. (Cockcroft and Latham 1968, McClintock 1968, Rice and Tracey 1969, Gurson 1975, Gurson 1977, Wilkins, Streit et al. 1980, Johnson and Cook 1985, Wierzbicki, Bao et al. 2005, Nahshon and Hutchinson 2008, Xue 2008, LauNielsen and Tvergaard 2009, Dunand and Mohr 2011, Bai and Wierzbicki 2015). This dissertation aims to investigate the applicability of the plasticity, pressure and Lode angle (PPL) model (An empirically based model developed by (Bai 2008) ) coupled with the modified Mohr-Coulomb (MMC) criterion in predicting fracture for AISI 4340 steel considering heat-treatment
effects. Nevertheless, revisit and evaluate the applicability of the Gurson-Tvergaard-Needleman (GTN) model (physics-based model) in predicting the correct mechanical behaviors under different loading conditions. This reevaluation of the (GTN) model has resulted in some suggested modifications to include the Lode angle effects which are not considered in the original derivation. Nevertheless, a new failure criterion known as (GTN-MMC) is developed.

AISI 4340 steel is a low alloy steel that contains nickel, chromium, and molybdenum. It is very famous for its high toughness and capability of developing high strength after proper heat-treatment while still retaining good fatigue strength (Sirin, Sirin et al. 2013). This alloy has many applications such as commercial and military aircraft landing gears, power transmissions gears and shafts; nevertheless, it is also used in designing many war projectiles (ASTM-STEEL, HÅKAN 2011). The melting point of this tough material is roughly about 1427°C which makes good use in high-temperature environments (ASTM-STEEL). It is also worth noting that this material has good ductility, especially in the unheated phase.

pressure) is a parameter to control the size of the yield surface. On the other hand, the Lode angle usually controls the shape of the yield surface. The recent empirical model proposed by (Bai and Wierzbicki 2008) (PPL) considers these two parameters (stress triaxiality, Lode angle parameter) for both plasticity and ductile fracture. This model is used along with the modified Mohr-Coulomb fracture criterion under the assumption of proportional loading to predict different fracture initiation modes. Over the past few years, the modified Mohr-Coulomb has gained much publicity due to its ability to capture the correct fracture mode (Bai and Wierzbicki 2009, Jia, Ghazali et al. 2017).

1.3 Heat-treatments

Heat treated AISI 4340 steel has been investigated and studied extensively by many different researchers. (Tanaka and Spretnak 1973, Bell and Loh 1982, Hickey and Anctil 1985, O'Brien and D 1991, Lee and Su 1999, Odeshi and Bassim 2009, Foster, Chen et al. 2011, Podder, Mondal et al. 2012, Pang, Li et al. 2013, Sirin, Sirin et al. 2013). A recent study on this subject was performed by Sirin and his team (Sirin, Sirin et al. 2013). They investigated the influence of ion nitride on AISI 4340 steel mechanical and fatigue properties by using both unheated and heat-treated specimens by austenitizing. The ion nitriding process is done by applying some DC volts between a container wall that is positively charged and an insulated center post supporting the work to be nitrided that is negatively charged. Once the job starts a glow of charge of the iodized gas accelerates and diffuses towards the specimen’s core. The study concluded that initial microstructure would influence the material response to nitriding. This observation was evident when comparing the heat-treated specimens with those that are not. The obtained case depths
deposited in the heat-treated samples were a little more substantial and increased as time and temperature increased.

Podder (Podder, Mondal et al. 2012) investigated the effects of heat-treatment on flow formability and mechanical properties of AISI 4340 steel. A linkage between mechanical properties and the microstructure type was established. It was noted that spheroidized, annealed and tempered then annealed respectively exhibited an increase in strength for both preforms and flow formable tubes. On the other hand, a reduction in ductility is observed for all preforms. These observations confirm the existence of a direct linkage between microstructure and mechanical properties. The martensite microstructure, typically the hardest steel type microstructure, exhibited the highest strength and lowest ductility. In contrary, the spheroidized microstructure, typically the softest of steel microstructure, clearly demonstrates the lowest strength and highest ductility.

To understand the microstructural features of AISI 4340 steel under quenched and tempered conditions, Lee and Su (Lee and Su 1999) studied the effects of seven different tempering temperatures as well as quenching using a period of 2 and 48 hours respectively. They observed that while holding the tempering time constant, the higher the tempering temperature, the less strength and hardness the material reveals. The only exception to this observation was at a tempering temperature of 300 °C and tempering period of 2 hours. An increase in hardness was observed. The loss of strength is assumed to be due to the formation of interlath carbide films and the retained interlath austenite.

The effects of high strain rate on fracture for steel 4340 under different tempering conditions were investigated by Odeshi and Bassim (Odeshi and Bassim 2009). It was observed and noted
that thermal softening as a result of adiabatic heating controls the deformation, fracture behavior and void growth and nucleation. It is worth noting that the dimples observed under the microscope are narrower for specimens tempered at lower temperatures. Thus, higher tempering temperatures typically enhance void growth rather than dislocation activities.

1.4 Dissertation Overview

This work is partitioned into seven chapters. CHAPTER 2: is a brief overview of different fracture models and their implementations. CHAPTER 3: will discuss the experimental methods and design of the specimens used. CHAPTER 4: illustrates the framework modeling of the Bai-Wierzbicki, Linear Drucker-Prager models and discussion of their results. CHAPTER 5: discusses the potential improvements that can be performed on the GTN model. CHAPTER 6: illustrates some SEM images of fractured specimens at different magnifications. CHAPTER 7: concludes the research and suggests various topics for future work.
CHAPTER 2:
LITERATURE REVIEW

Several coupled and uncoupled fracture models exist in the literature. (Tresca 1864, Cockcroft and Latham 1968, McClintock 1968, Rice JR 1969, Gurson 1977, Wilkins, Streit et al. 1980, Johnson and Cook 1985, Thomason 1985, Børvik, Hopperstad et al. 2001, Bai 2008, Xue and Wierzbicki 2008, Erice and Gálvez 2014). The uncoupled models do not incorporate any damage accumulation parameters in their formulations, while the coupled models do. The uncoupled models are usually accommodated with independent failure criteria to indicate fracture initiation and propagation. Many uncoupled models are gaining much publicity due to their ease of calibration and implementations into various finite element packages. This chapter aims to give a general overview of different ductile plasticity and fracture theories and models. The models that are discussed are categorized into three groups: physics-based, phenomenological and empirical models. Differences between these models are explained and detailed. Finally, different theories of void behaviors are listed and discussed.

2.1 Empirical Based Models

Empirical modeling refers to models that are created by observation and experiments. These models do not necessarily have any physical sound. However, they have found many application implementations due to their ease of calibration and their accurate predictions. This section will discuss Johnson-Cook and Wilkins models.
2.1.1 Johnson-Cook Model (J-C)

This model is purely empirical which has many advantages over other models. The J-C model incorporates the effects of stress triaxiality (normalized hydrostatic pressure), strain rate and could also include temperature dependency into its formulation (Johnson and Cook 1985). The J-C model is read as follows:

\[
\bar{\sigma} = [A + B(\varepsilon_{pl}^n)] \left[ 1 + C \ln \left( \frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_0} \right) \right] (1 - \hat{\theta}^m) \tag{2-1}
\]

where A, B, n, C, \(\dot{\varepsilon}_0\) and m are material parameters, and (\(\varepsilon_{pl}, \hat{\theta}^m\)) are equivalent plastic strain and non-dimensional temperature respectively.

\[
\hat{\theta}^m = \frac{\theta - \theta_r}{\theta_{melt} - \theta_r} \tag{2-2}
\]

where \(\theta, \theta_r, \theta_{melt}\) are the current, reference and melting temperatures respectively.

Neglecting temperature effects, the J-C fracture model (Johnson and Cook 1985) can also be defined in the strain domain as follows:

\[
\dot{\varepsilon}_f = c_0 + c_1 \exp \left( -c_2 \frac{\sigma_m}{\bar{\sigma}} \right) \tag{2-3}
\]

Where \(c_0, c_1\) and \(c_2\) are parameters that need calibration, and \(\sigma_m, \bar{\sigma}\) are the mean and equivalent stresses respectively.

2.1.2 Wilkins Model

This model was developed by (Wilkins, Streit et al. 1980) as a cumulative strain-damage criterion to predict fracture initiation and propagation. Fracture is activated once damage reaches a critical value \(D_C\) over a critical distance.
\[ D_c = \int_0^{\bar{\varepsilon}_f} \frac{1}{(1 - a\sigma_m)^\lambda} (2 - A)^\mu \ d\bar{\varepsilon} \]  

(2-4)

\[ A = \max \left( \frac{s_2}{s_3}, \frac{s_2}{s_1} \right) \]  

(2-5)

where the A parameter is within \( 0 \leq A \leq 1 \) and \( (s_1, s_2, s_3) \) are three deviatoric principal stresses. \( a \) is the coefficient for mean stress. \( \lambda \) and \( \mu \) are constants that need calibration.

Assuming proportional loading and integrating Eq. (2-4), the fracture locus is expressed as:

\[ \bar{\varepsilon}_f = \frac{D_c (1 - a\sigma_m)^\lambda}{(2 - A)^\mu} \]  

(2-6)

2.2 Phenomenological Based Models

Phenomenological based models are models that define the empirical relationship between different phenomena. These models have some physical association embedded, but the main thing is to explain why and how specific variables interact and relate to one another. This section will discuss maximum shear stress, pressure modified maximum shear stress, and Cockcroft-Latham models.

2.2.1 Maximum Shear Stress Model

Maximum shear stress theory assumes failure whenever the maximum shear stress in the designed element reaches a critical value \( \tau_s \) which equals the maximum shear stress in uniaxial tension. The (Tresca 1864) criterion is one of the most famous criteria used in many civil engineering applications. It asserts that \( \tau_{max} = \tau_y \), where \( \tau_{max} \) is defined as follows:
\[ \tau_{\text{max}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]  

(2-7)

where \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) are the maximum and minimum principal stresses respectively.

The maximum shear stress is then compared to the yield failure as follows:

\[ \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \leq S_{\text{yld}} \]  

(2-8)

The factor of safety (FOS) is defined as:

\[ \text{FOS} = \frac{S_{\text{yld}}}{\tau_{\text{max}}} \]  

(2-9)

2.2.2 Pressure Modified Maximum Shear Stress Theory

The traditional maximum shear stress theory has no hydrostatic pressure dependence on the ductile fracture. However, many materials illustrate such reliance. The pressure modified maximum shear stress reads as follows:

\[ (\tau_{\text{max}} + c_1 \sigma_m)_f = c_2 \]  

(2-10)

where \( \tau_{\text{max}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \) and \((c_1, c_2)\) are material constants. \( \sigma_m \) is the mean stress or negative pressure \((-p)\).

2.2.3 Cockcroft-Latham Model

The fracture criterion developed by (Cockcroft and Latham 1968) is an increment integral damage parameter value. This model is most suitable in the range of small and negative hydrostatic stresses. This criterion has only one material constant that needs calibration which makes it relatively preferable when sufficient data is absent. The criterion reads as follows:
\[ D_C = \int_0^{\bar{\varepsilon}_f} \left< \sigma_{\text{max}} \right> \, d\bar{\varepsilon}_{\text{pl}} \]  

(2-11)

where \( \sigma_{\text{max}} \) is the maximum principal stress and \( \bar{\varepsilon}_f \) is the equivalent strain at fracture. \( D_C \) is the damage material constant to indicate the limitation of the ductile model.

### 2.3 Physics-Based Model (Micro-Mechanical Models)

Unlike empirical and phenomenological models, physics-based models explain the microscopic and macroscopic based phenomena in mathematical terms to resemble the ongoing physical observations.

#### 2.3.1 McClintock Model

(McClintock 1968) analyzed the expansion (growth) and linkage (coalescence) of cylindrically shaped voids. His main finding is that stress triaxiality is the primary contributor parameter in governing the fracture behavior. This model focused on the growth of a single void in an infinite elastic-plastic matrix to determine the critical stain for coalescence. The model reads as follows:

\[
\frac{dD}{d\bar{\varepsilon}} = \frac{1}{\ln F^f_{zb}} \left[ \frac{\sqrt{3}}{2(1-n)} \sinh \left( \frac{\sqrt{3}}{2} \frac{(1-n) \sigma_a + \sigma_b}{\sigma} \right) + 3 \frac{\sigma_a - \sigma_b}{4} \right] 
\]

(2-12)

Where \( \sigma_a, \sigma_b \) and \( F^f_{zb} \) are the two principal stresses and the relative void growth factor to fracture. \( D \) is the damage accumulation indicator.
2.3.2 Rice-Tracey Model

(Rice and Tracey 1969) studied the influence of stress triaxiality on the growth of an isolated spherical void within an infinite plastic medium. Their model read as:

\[
\hat{\varepsilon}_f = c_1 \exp \left( -c_2 \frac{\sigma_m}{\bar{\sigma}} \right)
\]  \hspace{1cm} (2-13)

where \( c_1, c_2 \) are constants that calibration to best describe the fracture locus, and \( \sigma_m, \bar{\sigma} \) are the mean and equivalent stresses respectively.

This model is at most taken as a fracture criterion since it does not consider the effects of void growth on the material. The Rice and Tracey model has been used in some engineering applications and was found to give an adequate description of the ductile fracture process (Sun, Siegele et al. 1989, Zhang 1994).

2.3.3 Thomason’s Model

Thomason’s criterion is criterion that assumes a rigid non-hardening plastic solid containing a regular distribution of spherical microvoids. Unlike many porous models, this model considers the effects of stress triaxiality on void behaviors. This is done by assuming that the material fails once his criterion reaches the upper-bound theorem. The void growth rates and geometry changes are calculated using (Rice and Tracey 1969) void growth model. This model reads as follows (Thomason 1985):

\[
\left( \frac{\sigma_n}{Y} \right)_{lim} = \frac{0.4}{(a + a)^2} + \frac{1.67}{\left[ \frac{b}{b + d} + \frac{c}{c + e} \right]^{1/2}} \hspace{1cm} (2-14)
\]
Where a, b, c, d, and e are geometry dimensions of the void unit-cell system. \( \left( \frac{\sigma_n}{Y} \right)_{\text{lim}} \) is the incipient limit-load failure of the inter-void matrix based on the current geometry.

\[
\left( \frac{\sigma_n}{Y} \right)_{\text{crit}} \left[ 1 - \left( \frac{3\sqrt{\pi} V_f}{4} \right)^{2/3} \left( \frac{b}{b_0} \right) \left( \frac{c}{c_0} \right) e^{\epsilon_i} \right] = \frac{\sigma_m}{Y} + \frac{3 + \nu}{3 \sqrt{\nu^2 + 3}}
\]

(2-15)

\( \left( \frac{\sigma_n}{Y} \right)_{\text{crit}} \) is the critical condition for incipient ductile fracture based on the upper-bound theorem.

Failure is assumed once the following is satisfied:

\[
\left( \frac{\sigma_n}{Y} \right)_{\text{lim}} = \left( \frac{\sigma_n}{Y} \right)_{\text{crit}}
\]

(2-16)

2.3.4 Brown and Embury Model

(Brown and Embury 1973) proposed a coalescence criterion that is based on internal necking observed between two adjacent voids. It is assumed that void coalescence initiates once the radius of the elongated voids is equal to some critical distance between two neighboring void centers.

2.3.5 Bouaziz and Maire Criterion

The (Bouaziz, Maire et al. 2008) criterion is developed with the aid of X-ray tomography observations. It is assumed that the void density (N) is a function of hydrostatic stress (\( \eta \)) and equivalent plastic strain as follows:

\[
N = A \left( \frac{\varepsilon_{eq}^p}{\varepsilon_N} \right) \exp \left( \frac{\varepsilon_{eq}^p}{\varepsilon_N} \right)
\]

\( \varepsilon_N = \varepsilon_{N0} \exp(-\eta) \)

(2-17)

(2-18)

Where \( A \) and \( \varepsilon_{N0} \) are parameters that onsets the nucleation criterion that needs calibration.
2.3.6 Gurson Type Models

It is often assumed that hydrostatic stress (stress triaxiality) has little or no effect on yield. However, studies have shown that the existence of microvoids and impurities will cause macroscopic dilatation and thus hydrostatic stress should be considered. (Gurson 1977) developed a yield criterion that links the preexisting of void behavior along with matrix material. The adapted model is based on a unit-cell of a single void, where the ratio of this void to the cell is defined as $f$. Unlike the Rice-Tracy model, Gurson’s model considers not only void expansion and change of shape, but also the degradation of the materials carrying capacity. However, it was still argued by many that the model is insufficient to model many realistic materials. Gurson assumed that the material fails once its porosity reaches unity. This assumption is too big for most materials. (Brown and Embury 1973) found that once nucleation cavities are formed along the tensile loading, the two neighboring voids start to coalesce if their lengths are comparable to their spacing (i.e., coalescences happens faster than what is assumed by Gurson). Therefore, (Chu and Needleman 1980, Tvergaard 1981, Tvergaard 1982, LauNielsen and Tvergaard 2009) modified Gurson’s work and included some constant parameters that control the plastic flow potential. They also added a new parameter known as critical volume fraction to control the coalescence effects. Nevertheless, a new void nucleation mechanism was developed in addition to void growth. This model is often referred to as GTN model and is further explained in CHAPTER 5.

Gurson assumed that voids remain spherical regardless of the stress state. However, this is not the case for low hydrostatic stress, where voids tend to elongate which could have a significant effect on the prediction of ductile fracture (Benzerga, Leblond et al. 2016). These kinds of
extensions to the Gurson model were first carried out by (Gologanu, Leblond et al. 1993, Castañeda and Zaidman 1994). The reader is highly referred to the recent work of (Benzerka, Leblond et al. 2016) for a comprehensive review of the different Gurson type models and many suggested improvements.

2.4 Microvoid Behaviors

It has been identified that under high positive stress triaxiality voids grow, nucleate, and coalesce as illustrated in Figure 2-1. This section aims to shed some light on the former mechanisms as well as a brief discussion of initial assumptions of void shapes and their effects.
Figure 2-1: Evolution of the damage process in a ductile material that is under tensile loading showing different stages of void behavior (Abbassi, Pantalé et al. 2010)
2.4.1 Void Nucleation

X-ray tomography reviles that in heterogeneous nucleation, the most encountered kind, voids nucleate either by decohesion of the matrix-particle or by fracture of the particle as seen Figure 2-2 (Puttick 1959, Goods and Brown 1979). Tvergaard argues that nucleation appears to be controlled by either plastic strain or sometimes by the maximum normal stress when the particle-matrix reaches a critical value.

![Figure 2-2: Damage in aluminum 6061 matrix reinforce with Al₂O₃ (a) decohesion; (b) particle fracture (Kanetake, Nomura et al. 1995)](image)

By using radiographs on predesigned materials that contain controlled voids drilled by laser, (Weck 2007) proved that the void nucleation and coalescence strains could be determined. He also established that the volume void fraction of particles has an inverse relation to both nucleation and coalescence strains respectively. However, void nucleation is still a challenging subject to model as it involves many assumptions about the shape and nature of the nucleation, especially when dealing with real materials. (Tvergaard 1990) argued that these simple shape assumptions are a reasonable approximation at early stages. However, near-final failure stages these shape
approximations are very poor. Landron has illustrated that when using higher X-ray resolutions voids that appear to nucleate are nothing more than small voids expanding. (Landron 2011) uncovered that the initial microstructure of the material will affect void nucleation. She observed that dual phased steels experienced higher nucleation rates when compared to single phase steels.

Void nucleation criterions that are frequently used are the (Argon and Im 1975), (Beremin 1981) and (Chu and Needleman 1980) criterions. The Argon criterion is a phenomenological critical stress criterion that is defined as:

\[ \sigma_{eq} + \sigma_H = \sigma_C \]  
(2-19)

where \( \sigma_{eq}, \sigma_H \) and \( \sigma_C \) are the equivalent, hydrostatic and critical stresses respectively.

The (Beremin 1981) criterion is also a stress-based criterion. This criterion has the advantage of having a stress concentration factor \( (k_s) \) that considers the shape of the particle and is expressed as follows:

\[ \sigma_{I_{max}} + k_s (\sigma_{eq} - \sigma_0) = \sigma_C \]  
(2-20)

where \( \sigma_{I_{max}}, \sigma_0 \) are the maximum principal stress in the loading direction and the equivalent stress within the matrix respectively.

The (Chu and Needleman 1980) criterion is a strain based criterion. It proposes that void nucleation appears once a critical strain is reached. This criterion is used in the well-known Gurson- Tvergaard-Needleman (GTN) model.
It is believed that void nucleation is a continuous process due to inhomogeneity and local stress concentrations within the material. Voids are found to nucleate at faster rates when secondary phase particles, impurities, and large inclusions are present. It has also been observed that voids favor decohesion at the soft matrix, while the hard matrix material usually leads to particle fracture (Babout, Brechet et al. 2004).

2.4.2 Void Growth

It has been established that by (McClintock 1968) that hydrostatic stress, also known as stress triaxiality is the crucial parameter in controlling void dilatation. In most cases, a direct relation between stress triaxiality and void growth rates can be established. Recent studies by (Weck 2007, Maire, Bouaziz et al. 2008, Landron 2011) using X-ray tomography have also concluded the importance of the stress triaxiality in enhancing void growth.

Voids are highly dispersed within a matrix which makes them hard to study and analyze. Therefore, (Landron 2011) considered the average diameter of twenty of the largest voids within a population as a representation of the growth of a single void. She concluded that the initial microstructures of the specimen would affect the void growth rates. She observed that dual phased steel specimens with higher yield strength experienced faster growth rates.

Many computational studies were performed (unit cell modeling) to understand the mechanism of void dilations (Hill 1984, Becker, Needleman et al. 1988, Koplik and Needleman 1988, Kuna and Sun 1996). Direct relations between initial void sizes and hydrostatic stresses have been established. It was also concluded that large voids were observed at high-stress triaxiality.
2.4.3 Void Coalescence

Adjacent voids tend to coalesce after the growth has reached its critical state $f_c$. According to (Koplik and Needleman 1988) this critical value varies slowly with hydrostatic stress and matrix strain hardening. However, initial void volume fraction has a strong influence on this critical value. The phenomena of voids joining are usually seen near the fracture of the specimen. The mechanism of coalescence has not been studied extensively, due to its speed of occurrence and difficulty of distinguishing. It is believed that coalescence occurs at three stages, namely internal necking (Thompson 1987), shear localization (Weck 2007) and necklace coalescence (Benzerga 2000). The internal necking can be defined as the breakage of the ligament between 2 voids, and it is the most encountered type. (Weck 2007) has illustrated that localization is the primary phenomenon that is usually companied with void coalescence. Coalescence has many factors that influence its behaviors. These factors include; initial void volume fraction, void spacing, and ligament length, void shapes, and void distributions effects. Many studies on these different effects have been carried on throughout the last decade. These studies resulted in different model criteria to describe such behaviors. Experiments involving different notch sizes performed by (Weck 2007) illustrated that a drastic decrease in coalescence is observed with an increase in stress triaxiality. Therefore, Weck proposed a modified version of the Brown and Embury model that takes into account void ligament length in modeling the coalescence behavior under different stress states.

2.4.4 Void Closure and Healing

Under compressive loadings, voids tend to close as illustrated in Figure 2-3. (Ståhlberg 1986) demonstrated that small voids are more accessible to remove than large ones. It was observed that
void volume fraction may increase at the early stages due to enormous stresses, but will finally decrease resulting in void closure. The principal recognized parameters that influence the closure process are temperature, pressure, compressive stresses and healing time (Afshan 2013). It was observed by (Wang, Thomson et al. 1996) that the closure rate needs a specified level of stress triaxiality to achieve the closure. It was also observed that high temperatures enhance the process enormously. The concept of void closure and healing is applicable in metal joining (Wei, Han et al. 2004, Wang, Huang et al. 2007, Afshan 2013).

Figure 2-3: Void closure steps forming a crack (Zhang, Cui et al. 2009)

2.4.5 Void Size and Shape Effects

It has been established by many scientists that void size dramatically influences the growth rate (Fond, Lobbrecht et al. 1996, Fleck 1997, Zhang, Bai et al. 1999, Gao, Wang et al. 2005). In general, larger voids tend to have a faster growth rate than small voids. The intrinsic material length, when considered, accounts for the void size effect (Liu, Qiu et al. 2003). (Wen, Huang et al. 2005) extended the Gurson model based on the Taylor dislocation model to include void size
effects. It was noted that a direct relation could be established between void size and initial void volume fraction, where void size effects could be significant at some stage.
CHAPTER 3: EXPERIMENTAL METHODS AND SPECIMEN DESIGN

The material being investigated in this paper is AISI 4340 steel Alloy. The mechanical properties of this material were captured using MTS machine. Tests were carried out at room temperature under different loading conditions. All deformations were measured and recorded by a Digital Image Correlation (DIC) system.

3.1 Material Specification and Heat-treatment

AISI 4340 steel is a medium carbon, low alloy steel known for its toughness and strength in relatively large sections (ASTM-STEEL). It is also one of the existing types of nickel-chromium-molybdenum steel materials. This material has excellent shock and impact resistance as well as wear and abrasion resistance in the hardened condition (ASTM-STEEL). The properties of this material offer good ductility in the annealed condition, allowing it to be bent or formed. This material can also be machined by many conventional methods. For its chemical composition, please refer to Table 3-1.

Table 3-1: Chemical composition in %wt. of Steel 4340 being tested

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.25</td>
<td>0.73</td>
<td>0.004</td>
<td>0.0008</td>
<td>0.63</td>
<td>0.25</td>
<td>1.76</td>
<td>0.13</td>
</tr>
</tbody>
</table>
The heat-treatment of this material is performed by, Braddock Metallurgical Inc.-Daytona Beach, which is an accredited heat-treatment facility in Florida.

The heat-treatment process for the 32HRC was summarized as follows:

a) Parts were preheated at 1000 °F (±10) for 60 minutes in furnace #643 then loaded into a hardening furnace hot.

b) Parts were hardened in a furnace #630 at 1525 °F (±25) for 60 minutes then oil quenched.

c) Parts were tempered in a furnace #632 at 1100 °F (±15) for 120 minutes, and air cooled to room temperature.

d) All parts were processed in a mercury-free environment

The heat-treatment process for the 39HRC was summarized as follows:

a) Parts were preheated at 1200 °F (±25) for 120 minutes

b) Parts were hardened at 1525 °F (±25) for 60 minutes then oil quenched.

c) Parts were tempered in a furnace at 950 °F (±15) for 120 minutes and air cooled to room temperature.

Results of heat-treatment indicate that the hardness achieved correspond to 32HRC and 39HRC respectively. All heat-treatment was conducted within the requirements of AMS 2750E.
3.2 Representation of Stress State

The state of stress of isotropic material is commonly defined in Cartesian coordinates \((\sigma_1, \sigma_2, \sigma_3)\). The plasticity, pressure and Lode angle model (PPL) model convert stress from a Cartesian representation to a spherical representation. The three stress invariants \((p, q, r)\) can be defined as follows:

\[ p = -\sigma_m = \frac{1}{3} \text{tr}(\sigma) = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \]  
\[ (3-1) \]

\[ q = \bar{\sigma} = \sqrt{\frac{3}{2}} J_2 = \sqrt{\frac{3}{2} [S] : [S]} = \frac{1}{\sqrt{2}} \left[ ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2) \right] \]  
\[ (3-2) \]

\[ r = \left( \frac{9}{2} [S] : [S] : [S] \right)^{1/3} = \left[ \frac{27}{2} \text{det}(\sigma) \right]^{1/3} \]  
\[ = \left[ \frac{27}{2} (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m) \right]^{1/3} \]  
\[ (3-3) \]

\[ [S] = [\sigma] + p[I] \]  
\[ (3-4) \]

The parameters \([S]\) and \([I]\) respectively denote the deviatoric stress tensor and the identity tensor.

It is assumed that the principal stresses are represented by \(\sigma_1 \geq \sigma_2 \geq \sigma_3\). For convenience stress triaxiality \((\eta)\) is defined as:

\[ \eta = \frac{-p}{q} = \frac{\sigma_m}{\sigma} \]  
\[ (3-5) \]
The $\eta$ parameter has been widely used in many ductile fracture formulations (McClintock 1968, Rice and Tracey 1969, Hancock and Mackenzie 1976, Bao and Wierzbicki 2004). However, the incorporation of Lode parameter is neglected in many of the previous models. The geometrical meaning of Lode angle $\theta$ is illustrated in Figure 3-1.

**Figure 3-1**: Principal stresses in spherical coordinate representation (Bai and Wierzbicki 2008)

The Lode parameter is related to the third deviatoric stress invariant $\xi$ through the following (Malvern 1969) equations:

$$\xi = \left(\frac{r}{q}\right)^3 = \cos(3\theta) \quad (3-6)$$

Furthermore, the Lode angle parameter $\tilde{\theta}$ is normalized and is expressed as:
\[
\bar{\theta} = 1 - \frac{6\theta}{\pi} = 1 - \frac{2}{\pi} \cos^{-1}(\xi)
\] (3-7)

The state of stress of the isotropic material is usually defined in Cartesian coordinate \((\sigma_1, \sigma_2, \sigma_3)\). The Cartesian coordinates can be simply transformed into either cylindrical or spherical coordinate by appropriate mapping methods. From Figure 3-1, the vector \(\overrightarrow{OO'}\) is perpendicular to the \(\pi\) plane that is located at the origin \(O\), and every point on it represents a hydrostatic stress state. Point \((P)\) represents an arbitrary point that is located on the \(O'\) plane. To describe the state of stress of point \((P)\) in spherical coordinates the vector \(\overrightarrow{OP}\) must be determined. By vector addition analogy, \(\overrightarrow{OP}\) is described by the addition of the two vectors \(\overrightarrow{OO'}\) and \(\overrightarrow{O'P}\) (Bai and Wierzbicki 2008).

3.3 Design of Specimen Geometries

This study investigated 29 specimens of different shapes and at different tempering conditions. There were 13 round bars (axial symmetric tension), 13 flat grooved (plane stain) samples and three small round cylinders (upsetting test). Six round bars and six flat grooved specimens were heat treated with 32 Rockwell Hardness (32 HRC). Four other round bars and four flat grooved specimens were heat treated with 39 Rockwell Hardness (39 HRC). The hardness for the initially untreated specimens was 16 Rockwell Hardness (16 HRC). The round bars are named as follows: smooth bar (SB), large notch (LN) sharp notch (SN) and a small compressive cylinder (SCC). The flat grooved were named as follows; large flat grooved (LFG), medium flat grooved (MFG) and small flat grooved (SFG). Some of these specimens are illustrated in Figure 3-2 and Figure 3-3. The design of different shapes enables to study the effects of varying stress states.
These specimens are summarized in Table 3-2 and Table 3-3. Detail drawings are shown in Figure 3-4 through Figure 3-6.

Table 3-2: Round bar specimen’s names and dimensions (mm)

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Symbol</th>
<th>Notch Radius</th>
<th>Minimum Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC Smooth Bar</td>
<td>16 SB</td>
<td>N/A</td>
<td>5</td>
</tr>
<tr>
<td>32 HRC Heat-treated Smooth Bar</td>
<td>32 SB</td>
<td>N/A</td>
<td>5</td>
</tr>
<tr>
<td>39 HRC Heat-treated Smooth Bar</td>
<td>39 SB</td>
<td>N/A</td>
<td>5</td>
</tr>
<tr>
<td>16 HRC Small Compressive Cylinder</td>
<td>16 SCC</td>
<td>N/A</td>
<td>7</td>
</tr>
<tr>
<td>16 HRC Large Notched Bar</td>
<td>16 LN</td>
<td>7.5</td>
<td>5</td>
</tr>
<tr>
<td>32 HRC Heat-treated Large Notched Bar</td>
<td>32 LN</td>
<td>7.5</td>
<td>5</td>
</tr>
<tr>
<td>16 HRC Sharp Notched Bar</td>
<td>16 SN</td>
<td>3.5</td>
<td>5</td>
</tr>
<tr>
<td>32 HRC Heat-treated Sharp Notched Bar</td>
<td>32 SN</td>
<td>3.5</td>
<td>5</td>
</tr>
<tr>
<td>39 HRC Heat-treated Sharp Notched Bar</td>
<td>39 SN</td>
<td>3.5</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 3-3: Flat grooved specimen’s names and dimensions (mm)

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Symbol</th>
<th>Notch Radius</th>
<th>Minimum Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC Large Flat Grooved</td>
<td>16 LFG</td>
<td>40.625</td>
<td>1</td>
</tr>
<tr>
<td>32 HRC Heat-treated Large Flat Grooved</td>
<td>32 LFG</td>
<td>40.625</td>
<td>1</td>
</tr>
<tr>
<td>39 HRC Heat-treated Large Flat Grooved</td>
<td>39 LFG</td>
<td>40.625</td>
<td>1</td>
</tr>
<tr>
<td>16 HRC Medium Flat Grooved</td>
<td>16 MFG</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>32 HRC Heat Treated Medium Flat Grooved</td>
<td>32 MFG</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>16 HRC Small Flat Grooved</td>
<td>16 SFG</td>
<td>1.575</td>
<td>1</td>
</tr>
<tr>
<td>32 HRC Heat-treated Small Flat Grooved</td>
<td>32 SFG</td>
<td>1.575</td>
<td>1</td>
</tr>
<tr>
<td>39 HRC Heat-treated Small Flat Grooved</td>
<td>39 SFG</td>
<td>1.575</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 3-2: Round bars and flat grooved specimens in the unheated case (16 HRC)

Figure 3-3: samples of other round bar specimens with different heat-treatment conditions
Figure 3-4: Drawings of the small compressive cylinder

Figure 3-5: Drawings of round bars (SB, LN, and SN)
3.4 Theoretical Analysis of Stress States

In theory, each specimen with different geometry shall experience different stress state \((\eta, \bar{\theta})\). Fracture strain can be estimated using the area reduction method of the fractured specimen. Lode angle parameter \((\bar{\theta})\) and stress triaxiality \((\eta)\) can be determined using analytical solutions. Please refer to Ref. (Bai and Wierzbicki 2008, Algarni, Bai et al. 2015) for more details.

The smooth bar specimen (SB) will be used to calibrate the necessary mechanical properties including the strain hardening curve. The process of analyzing SB tensile test data provided by the MTS machine and the DIC software are as follows: First, load-displacement curves are recorded and analyzed. The curves are then converted to the engineering stress-strain curves \((\sigma_E, \varepsilon_E)\). The engineering stress-strain curves are then converted to true stress-strain curves up to the necking.
initiation point. Once necking is observed, a Swift power function is introduced to describe strain hardening beyond necking. Several FEA simulations are run to calibrate the parameters of the swift power function until an excellent correlation between FEA and experimental load-displacement results is achieved. Once FEA and experimental data match, an approximate true stress-strain is established. Please refer to equations (3-8) through (3-10).

\[ \sigma_t = \sigma_E (1 + \varepsilon_E) \]  \hspace{2cm} (3-8)

\[ \varepsilon_t = \ln (1 + \varepsilon_E) \]  \hspace{2cm} (3-9)

\[ \sigma (\overline{\varepsilon}_{pl}) = K (\varepsilon_0 + \overline{\varepsilon}_{pl})^n \]  \hspace{2cm} (3-10)

where \( \sigma_E, \varepsilon_E, \sigma_t, \varepsilon_t, \varepsilon_0, \overline{\varepsilon}_{pl}, K, \) and \( n \) are engineering stress-strain, true stress-strain, first yield strain, strength index, and strain hardening exponent, consecutively.

The cylindrical compression specimens enable the determination of whether the shape of the yield surface is symmetric or asymmetric. It has been proven that both the yield and fracture loci are not necessarily symmetric (Bai and Wierzbicki 2008, Algarni, Bai et al. 2015, Bai and Wierzbicki 2015, Bai Massachusetts Institute of Technology; 2008). Please refer to Figure 3-4 through Figure 3-6 for the detailed design of the specimen’s geometry.

The notched specimens are designed to investigate the effects of stress triaxiality (\( \eta \)) on material’s behavior. The deeper the notch is, the higher the stress triaxiality (\( \eta \)). From analytical solutions, the round bar specimens (axial symmetric tension) will exhibit a Lode angle parameter of unity \( \bar{\theta} = 1 \), while the plane strain specimens (Flat Grooved) and the small compressive cylinder will exhibit Lode angle parameters of \( \bar{\theta} = 0, -1 \) respectively. Thus, a comprehensive
study of the effects of stress triaxiality ($\eta$) and Lode angle parameter dependence on material’s plasticity behavior is achieved. Please refer to Figure 3-5 for detailed drawings of the round specimens.

The plane strain specimens have the same range of stress triaxialities ($\eta$) as the round bar specimens, but the Lode angle parameter is $\bar{\theta} = 0$. See Figure 3-6 for specimen drawings. This fact, in association with previous tests, will adequately determine the shape of the fracture locus. Analytical solutions for plane strain specimens are illustrated below (Bai 2008, Algarni 2015).

\[
\bar{\varepsilon}_f = \frac{2}{\sqrt{3}} \ln \left( \frac{t_o}{t} \right) \tag{3-11}
\]

\[
\eta = \frac{\sigma_{\text{m}}}{\sigma} = \frac{\sqrt{3}}{3} \left[ 1 + 2 \ln \left( 1 + \frac{a}{2R} \right) \right] \tag{3-12}
\]

\[
\sigma_{xx} = \frac{2}{\sqrt{3}} \bar{\sigma} \ln \left( \frac{a^2 + 2aR - x^2}{2aR} \right) \tag{3-13}
\]

\[
\sigma_{yy} = \frac{2}{\sqrt{3}} \bar{\sigma} \left[ \frac{1}{2} + \ln \left( \frac{a^2 + 2aR - x^2}{2aR} \right) \right] \tag{3-14}
\]

\[
\sigma_{zz} = \frac{2}{\sqrt{3}} \bar{\sigma} \left[ 1 + \ln \left( \frac{a^2 + 2aR - x^2}{2aR} \right) \right] \tag{3-15}
\]
3.5 Experimental Setup

Twenty-nine samples of different shapes and different heat-treatment were fabricated from an AISI 4340 steel alloy square bar material. The testing took place at room temperature with quasi-static loading conditions. All tests were carried out using an MTS testing machine that can deliver up to 100 (KN). The selected loading rate was 0.15 (mm/s). The displacement of the specimens was recorded by a camera and analyzed by DIC software. The period selected for all experiments was one frame per second. The DIC requires a pattern spray on the specimen to enable proper displacement depiction and detection. For this purpose, all specimens were painted using an airbrush. An alternative method to measure a specimen’s displacement is by using an extensometer. This method was not used here. Please refer to Figure 3-7 for test configuration setup and to Figure 3-8 and Figure 3-9 for samples of fractured specimens.
Figure 3-7: Test configuration setup
Figure 3-8: Samples of round bar fractured specimens (cup and cone fractured surface)

Figure 3-9: Samples of flat grooved fractured specimens (slant fractured surface)
CHAPTER 4:
PLASTICITY AND FRACTURE MODELS CALIBRATION AND NUMERICAL VERIFICATION

In this section, the Drucker and Prager and the Bai and Wierzbicki (PPL) models are presented. The PPL and the Drucker and Prager models are both pressure dependent models. However, the PPL model also incorporates the effects of the Lode parameter in a merit method that considers different stress loading conditions.

4.1 The Drucker and Prager Plasticity Model

The Drucker and Prager (DP) model (Drucker and Prager 1952) is a pressure dependence model that has been extensively used to describe pressure sensitive materials such as rocks and concrete. This model and many of its extensions are built in many software packages including the ABAQUS software.

The linear Drucker-Prager model (Simulia 2017) reads as follows:

\[ F = t - p \tan \beta - d = 0 \]  \hspace{1cm} (4-1)

\[ t = \frac{1}{2} \ q \left[ 1 + \frac{1}{K} - \left( 1 - \frac{1}{K} \right) \left( \frac{r}{q} \right)^3 \right] \]  \hspace{1cm} (4-2)

K is defined as the ratio of the yield stress in triaxial tension to that of triaxial compression, and K is suggested to be in the range of \( 0.778 \leq K \leq 1.0 \) to ensure the convexity of the yield surface. In the case of uniaxial tension, the relationship between parameter \( d \) and yield stress is,
\[ d = \left( 1 + \frac{1}{3} \tan \beta \right) \sigma_t \]  

(4-3)

The angle \( \beta \) is often referred to as the friction angle and \( d \) as the cohesion of the material.

The plastic potential function \( G \) in the DP model is defined in accordance with the following equation:

\[ G = t - p \tan \psi \]  

(4-4)

It is worth noting that the original (DP) model is retrieved by \( \beta = \psi \) and \( K = 1 \).

### 4.2 The PPL Model and Failure Criterion

#### 4.2.1 Modification to the Original Bai and Wierzbicki Plasticity Model

The calibration of AISI 4340 steel was obtained by invoking the Bai and Wierzbicki (PPL) plasticity model (Bai and Wierzbicki 2008, Algarni, Bai et al. 2015). This model grasps the effects of plastic strain, pressure dependence, and Lode angle effects, on plasticity and fracture. The model can be expressed as:

\[
\sigma_{yld}(\varepsilon_{pl}, \eta, \theta) = \bar{\sigma}(\bar{\varepsilon}_{pl}) \left[ 1 - c_\eta (\bar{\eta} - \eta_o) \right] \left[ c_s^\theta + (c_\theta^{ax} - c_s^\theta) \left( \frac{m + 1}{m} \right) \left( \gamma - \gamma^{m+1} \right) \right] \]  

(4-5)

(Algarni 2015) recently introduced a new correction factor \( \left( \frac{m+1}{m} \right) \) to make the model more user-friendly. Additionally, please note that the stress triaxiality term \( (\eta) \) in the original model is modified to be \( (\bar{\eta}) \), also known as modified stress triaxiality ratio, defined as:
\[
\tilde{\eta} = \frac{\sigma_m}{\sigma_o}
\] (4-6)

where \( \sigma_o = \overline{\sigma}(\overline{\varepsilon}_{pl}) \) is the yield stress at the reference loading of \( \eta_o \). This modification ensures the stability of the model at extreme loading conditions (i.e., when the \( q \to 0 \) in Eq. (3-5)). (Algarni 2015) claimed that the \( c^s_0 \) could be a function of the Lode angle parameter. A similar claim could be issued regarding the \( c_\eta \) parameter (APPENDIX A). However, for calibration proposes only, it is best that these two parameters \( (c^s_0, c_\eta) \) are kept as constant parameters as intended by the original model derivation (Bai and Wierzbicki 2008).

A Swift power function is introduced to describe the term \( \overline{\sigma}(\overline{\varepsilon}_{pl}) \) for metal plasticity as seen in Eq. (3-10). The \( c_\eta \) is a coefficient that controls the effects of hydrostatic pressure on yielding and needs to be calibrated. The \( c^s_0, c^{ax}_0 \) and \( m \) are coefficients that control the effects of Lode angle on material plasticity. The \( \gamma \) and \( c^{ax}_0 \) parameters are defined in Eq. (4-7) and (4-8) as follows:

\[
\gamma = 6.4641 \left[ \sec(\theta - \pi/6) - 1 \right]
\] (4-7)

\[
c^{ax}_0 = \begin{cases} c^t_0 & \text{for } \bar{\theta} \geq 0 \\ c^c_0 & \text{for } \bar{\theta} < 0 \end{cases}
\] (4-8)

The terms \( c^t_0 \) and \( c^c_0 \) are parameters denoting tension and compression respectively. It is worth noting that the parameters \( c^t_0 = 1 \) and \( \eta_o = 1/3 \) are uniquely defined if a smooth bar tension test is used as a reference test (Bai and Wierzbicki 2008).

Most yield criteria in literature lack either pressure or Lode angle dependencies. Yield criteria that include both pressure and Lode dependence are not easy to calibrate. The PPL model
consists of both of the former dependencies and adopts a simplified calibration approach. The PPL model is also very convenient for describing the state of stress at low, intermediate, and high-stress triaxialities, consecutively. The parameters of the PPL plasticity model \((c_\eta, c^t_\theta, c^c_\theta, c^s_\theta, m)\) can be adjusted to match many of the well-known yield criteria, as seen in Table 4-1. Furthermore, the PPL model predicts adequate results for specimens experiencing different stress states, which makes it viable for industrial usage (i.e., all notched round bars and flat grooved specimens are calibrated by a fixed set of PPL parameters).

Table 4-1: Achieving different yield criterions by adjusting the PPL parameters (Algarni 2017)

<table>
<thead>
<tr>
<th>Yield Criterion</th>
<th>(c_\eta)</th>
<th>(c^t_\theta)</th>
<th>(c^c_\theta)</th>
<th>(c^s_\theta)</th>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von-Mises</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tresca</td>
<td>0</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>1</td>
<td>1</td>
<td>+(\infty)</td>
</tr>
<tr>
<td>Pressure-modified Tresca</td>
<td>(\neq 0)</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>1</td>
<td>1</td>
<td>+(\infty)</td>
</tr>
</tbody>
</table>
4.2.2 The Deviatoric Associated Flow Rule of the Modified PPL Model

The conventional, associated flow rule, can be expressed as

\[ d\varepsilon_{ij}^{pl} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \]  

(4-9)

where the plastic flow is defined as:

\[ f = q - \bar{\sigma}(\bar{\varepsilon}^p) \left[ 1 - c_\eta (\bar{\eta} - \eta_0) \right] \left[ c_\theta^s + (c_\theta^a - c_\theta^s)(\gamma - \frac{\gamma^m}{m + 1})(1 + \frac{1}{m}) \right] \]  

(4-10)

Variable \( q \) is defined in Eq. (3-2) and \( d\lambda \) is the equivalent plastic strain increment.

The conventionally associated flow rule follows:

\[ \frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial q}{\partial \sigma_{ij}} + \bar{\sigma}(\bar{\varepsilon}^p) \frac{c_\eta}{\partial \sigma_{ij}} \left[ c_\theta^s + (c_\theta^a - c_\theta^s)(\gamma - \frac{\gamma^m}{m + 1})(1 + \frac{1}{m}) \right] \frac{\partial \bar{\eta}}{\partial \sigma_{ij}} \]

\[ - \bar{\sigma}(\bar{\varepsilon}^p) \left[ 1 - c_\eta (\bar{\eta} - \eta_0) \right] (c_\theta^a - c_\theta^s)(1 + \frac{1}{m})(1 - \gamma^m) \frac{\partial \gamma}{\partial \sigma_{ij}} \]  

(4-11)

Where \( \frac{\partial q}{\partial \sigma_{ij}}, \frac{\partial \bar{\eta}}{\partial \sigma_{ij}} \) and \( \frac{\partial \gamma}{\partial \sigma_{ij}} \) are expressed as:

\[ \frac{\partial q}{\partial \sigma_{ij}} = \frac{3}{2 q} s_{ij} \]  

(4-12)

\[ \frac{\partial \bar{\eta}}{\partial \sigma_{ij}} = \frac{\delta_{ij}}{3 \sigma_s} \]  

(4-13)

\[ \frac{\partial \gamma}{\partial \sigma_{ij}} = \left( \frac{3\sqrt{3}}{2 - \sqrt{3}} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \frac{1}{q \sin(3\theta)} \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta)}{2q} s_{ij} - \frac{3}{2q^2} s_{lk}s_{kj} \right) \]  

(4-14)
To satisfy the assumption of plastic incompressibility, the term \( \frac{\delta_{ij}}{3 \sigma_s} \) in Eq. (4-13) is removed. Thus, this paper uses deviatoric associativity rather than the conventional associativity as illustrated in Eq. (4-15).

\[
d\varepsilon_{ij}^{pl} = d\lambda \left\{ \frac{3}{2q} s_{ij} - \bar{\sigma}(\varepsilon_p) \left[ 1 - c_\eta (\eta - \eta_0) \right] (c_{\theta}^{ax} - c_{\theta}^{s}) \left( 1 + \frac{1}{m} \right) (1 - \gamma^m) \right. \\
\times \left. \left( \frac{3\sqrt{3}}{2 - \sqrt{3}} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \frac{1}{q} \cos(3\theta) \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta)}{2q} s_{ij} \right) \right. \\
- \left. \frac{3}{2q^2} s_{ij} s_{kj} \right\} \\
(4-15)
\]

4.2.3 Modified Mohr-Coulomb Criterion Failure Criteria

To construct the fracture locus of a material, it is vital to correctly capture the fracture initiation locations of the specimens that are under different stress states. This is obtained by invoking the modified Mohr-Coulomb criterion (MMC). The MMC is expressed as (Bai and Wierzbicki 2009):

\[
\bar{\varepsilon}_f = \left\{ \frac{A}{c_2} \left[ \frac{\tilde{s}}{c_0} + \frac{\sqrt{3}}{2 - \sqrt{3}} (c_{\theta}^{ax} - c_{\theta}^{s}) \left( \sec \left( \frac{\bar{\theta} \pi}{6} \right) - 1 \right) \right] \times \left[ \sqrt{\frac{1 + c_1^2}{3}} \cos \left( \frac{\bar{\theta} \pi}{6} \right) \right. \\
+ c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\bar{\theta} \pi}{6} \right) \right) \right\}^{-1/N} \\
(4-16)
\]
The parameters \((A, N, \tilde{C}_0^S, \tilde{C}_0^C, C_1, C_2)\) need to be calibrated. The parameters \(A\) and \(N\) are power strain hardening parameters that can be found from the reference test (smooth bar). Other parameters are then found by the inverse method (trial and error) until all simulated specimens agree with their experimental values. MATLAB code was developed to obtain the initial parameters of the MMC fracture locus (Algarni, Bai et al. 2015). It is assumed that fracture is initiated when the damage indicator (D) defined in Eq. (4-17) reaches unity.

\[
D = D \left( \bar{\varepsilon}_{pt} \right) = \int_0^{\bar{\varepsilon}_{pt}} \frac{d\bar{\varepsilon}_{pt}}{\bar{\varepsilon}_f(\eta, \theta)}
\]  \hspace{1cm} (4-17)

### 4.3 Calibration Procedure

A series of tests were conducted to determine the material parameters. The first test is the smooth bar, which is taken as the reference test. The smooth bar test uniquely identifies the power function parameters \((K, \varepsilon_0, n)\) pre-necking described in Eq. (3-10) and Table 4-2.

Figure 4-1 indicates the true pre-necking stress-strain curve represented using the solid curve, while post-necking is represented by dashed curves with diamond symbols. The post-necking points were obtained in such a way that experimental data correlated with the simulated data by the inverse method (or known as trial-and-error method). The DIC enables continuous tracking of the minimum cross-sectional area of the specimen at all times. Knowing the exact dimensions allows the utilizing Bridgman’s stress-strain solutions. This method was also used to verify the post-necking behavior of the specimen. Please refer to APPENDIX B for detailed derivations.
The round notched bars and the flat grooved specimens of different sizes enable the study of the modified stress triaxiality (\(\bar{\eta}\)), and the Lode angle effects on material plasticity in a sequential manner. The small compressive cylinder is used to study the material plasticity behaviors in the negative stress triaxiality region. A friction coefficient of 0.05 is assumed for simulations involving small compressive cylinder.

Table 4-2: List of material fundamental plasticity properties used in the FEA

<table>
<thead>
<tr>
<th>Material Hardness</th>
<th>Elastic Modulus E (GPa)</th>
<th>Poisson ratio (\nu) (-)</th>
<th>(\sigma_y) (MPa)</th>
<th>K (MPa)</th>
<th>(\varepsilon_0) (-)</th>
<th>n (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>208</td>
<td>0.29</td>
<td>535.58</td>
<td>1151</td>
<td>0.0040</td>
<td>0.140</td>
</tr>
<tr>
<td>32 HRC</td>
<td>208</td>
<td>0.29</td>
<td>946.8</td>
<td>1387</td>
<td>0.0100</td>
<td>0.088</td>
</tr>
<tr>
<td>39 HRC</td>
<td>208</td>
<td>0.29</td>
<td>1122.6</td>
<td>1638</td>
<td>0.0112</td>
<td>0.086</td>
</tr>
</tbody>
</table>
Figure 4-1: Comparison of stress-strain curves of heated (32 Rockwell) and unheated (16 Rockwell) specimens

4.3.1 Calibration of the Drucker and Prager Plasticity Model

The linear Drucker and Prager (DP) plasticity model built in ABAQUS has three constants that needed calibration (β, K, and ψ). As mentioned above, the smooth round bar provides the true stress-strain of the material. After the determination of the true stress-strain data, a parametric study of the three constants was conducted using the software ISIGHT provided by SIMULIA, until optimum solutions were achieved. The calibrated plasticity parameters using the DP model are listed in Table 4-3.
Table 4-3: List of material parameters used in the DP plasticity model

<table>
<thead>
<tr>
<th>AISI 4340 steel hardness</th>
<th>β</th>
<th>K</th>
<th>ψ</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>10°</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>32 HRC</td>
<td>5°</td>
<td>1</td>
<td>0°</td>
</tr>
<tr>
<td>39 HRC</td>
<td>5°</td>
<td>1</td>
<td>0°</td>
</tr>
</tbody>
</table>

4.3.2 Calibration of the PPL Plasticity Model

The second group of tests comprises round bars with different notch sizes. The introduction of different notch sizes enables the study of the effects of stress triaxiality (η) on material plasticity, and the determination of $c_\eta$ and $m$ variables. The third group of tests comprises flat grooved specimens of different sizes. This group enables the study of Lode angle effects on material plasticity and the determination of $c_\theta$. The final test is the small cylindrical upsetting test, which determines the $c_\psi$ variable. The calibrated plasticity parameters using the PPL model are listed in Table 4-4. It is important to note that the small compressive cylinders were not heat-treated because their strength would exceed our load cell and fixture capabilities. Therefore, the $c_\psi$ values for the heat-treated cases were assumed to be unity.

The MMC model was used for predicting the initiation of the fracture. Given knowledge of the true stress-strain curve for the smooth bar, the power hardening parameters (A and n) are fitted based on the entire true stress-strain curves. Table 4-5 shows the MMC parameters for the three types of material hardness. The stress triaxiality (η), Lode angle parameter $\bar{\theta}$, and fracture strain ($\bar{\varepsilon}_f$) were estimated based on the theoretical formulas. The MMC model was then calibrated, and results are summarized in Table 4-5.
Table 4-4: List of material parameters used in the PPL plasticity model.

<table>
<thead>
<tr>
<th>Material description</th>
<th>$c_\eta$</th>
<th>$\eta_o$</th>
<th>$c_{\theta}^C$</th>
<th>$c_{\theta}^S$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>0.2</td>
<td>$\frac{1}{3}$</td>
<td>0.75</td>
<td>0.93</td>
<td>0.75</td>
</tr>
<tr>
<td>32 HRC</td>
<td>0.2</td>
<td>$\frac{1}{3}$</td>
<td>1.0</td>
<td>1.03</td>
<td>0.75</td>
</tr>
<tr>
<td>39 HRC</td>
<td>0.05</td>
<td>$\frac{1}{3}$</td>
<td>1.0</td>
<td>0.93</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 4-5: List of material fracture parameters using MMC model

<table>
<thead>
<tr>
<th>Material description</th>
<th>A</th>
<th>n</th>
<th>C_1</th>
<th>C_2</th>
<th>$\tilde{c}_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>1151</td>
<td>0.14</td>
<td>0.157</td>
<td>647</td>
<td>1.1</td>
</tr>
<tr>
<td>32 HRC</td>
<td>1383</td>
<td>0.0806</td>
<td>0.085</td>
<td>732</td>
<td>0.99</td>
</tr>
<tr>
<td>39 HRC</td>
<td>1500</td>
<td>0.0526</td>
<td>0.055</td>
<td>777</td>
<td>0.96</td>
</tr>
</tbody>
</table>
4.4 Comparison of Experimental and Numerical Simulations Results

This section presents finite element simulation results compared side by side with experimental results. The PPL was run using Abaqus/Explicit with a VUMAT subroutine. The round specimens (SB, LN, and SN) were simulated using a quarter of the model with axisymmetric elements (CAX4R) type owing to symmetry. Conversely, the flat grooved specimens (LFG, MFG, and SFG) were constructed using one-eighth of the model with 3D solid elements (C3D8R) type owing to symmetry. The small compressive cylinder (SCC) was constructed using 3D solid elements (C3D8R) type as seen in Figure 4-2.
Figure 4-2: Samples of meshes used in Abaqus for simulations
4.4.1 Plastic Strength and Flow of AISI 4340 Steel with Different Hardness

A direct comparison of FEA and experimental results for the necking configuration observed immediately before the fracture is illustrated in Figure 4-3 through Figure 4-9. Diffuse necking for flat grooved specimens is indicated by circles.

Each fractured specimen was measured at various locations, and an average fractured diameter was then determined. Table 4-6 and Table 4-7 compare the measurements of specimens before and after tests with FE results. The FE fracture results were within a 5% of the error margin, compared to the experimental results.

Figure 4-3: Comparison of plastic deformations between test and simulations for un-heated (16 HRC) round specimens
Figure 4-4: Comparison of plastic deformations between test and simulations for heat treated (32 HRC) round specimens
Figure 4-5: Comparison of plastic deformations between test and simulations for heat treated (39 HRC) round specimens
Figure 4-6: Comparison of plastic deformations between test and simulations for un-heated (16 HRC) flat grooved specimens

Figure 4-7: Comparison of plastic deformations between test and simulations for heat treated (32 HRC) flat grooved specimens

Figure 4-8: Comparison of plastic deformations between test and simulations for heat treated (39 HRC) flat grooved specimens
Figure 4-9: Comparison of plastic deformations between test and simulations for un-heated (16 Rockwell) small compressive cylinder
Table 4-6: Dimensions of round bar specimens (axially symmetric) before and after the test.

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Initial diameter (mm)</th>
<th>Test fractured diameter (mm)</th>
<th>FEA fractured diameter (mm)</th>
<th>Actual % Area reduction</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 SB</td>
<td>5</td>
<td>3.42</td>
<td>3.25</td>
<td>31.6</td>
<td>4.97</td>
</tr>
<tr>
<td>16 LN</td>
<td>5</td>
<td>4.02</td>
<td>3.86</td>
<td>19.6</td>
<td>3.98</td>
</tr>
<tr>
<td>16 SN</td>
<td>5</td>
<td>3.65</td>
<td>3.55</td>
<td>27</td>
<td>2.74</td>
</tr>
<tr>
<td>32 SB</td>
<td>5</td>
<td>3.5</td>
<td>3.34</td>
<td>30</td>
<td>4.57</td>
</tr>
<tr>
<td>32 LN</td>
<td>5</td>
<td>3.88</td>
<td>3.67</td>
<td>22.4</td>
<td>5.41</td>
</tr>
<tr>
<td>32 SN</td>
<td>5</td>
<td>4.07</td>
<td>3.95</td>
<td>18.6</td>
<td>2.95</td>
</tr>
<tr>
<td>39 SB</td>
<td>5</td>
<td>3.54</td>
<td>3.48</td>
<td>29.2</td>
<td>1.69</td>
</tr>
<tr>
<td>39 SN</td>
<td>5</td>
<td>4.19</td>
<td>4.04</td>
<td>16.2</td>
<td>3.58</td>
</tr>
</tbody>
</table>

Table 4-7: Dimensions of flat grooved (plane strain) before and after the test.

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Initial thickness (mm)</th>
<th>Test fractured diameter (mm)</th>
<th>FEA fractured diameter (mm)</th>
<th>Actual % Area reduction</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 LFG</td>
<td>1</td>
<td>0.87</td>
<td>0.83</td>
<td>13</td>
<td>4.60</td>
</tr>
<tr>
<td>16 MFG</td>
<td>1</td>
<td>0.90</td>
<td>0.86</td>
<td>10</td>
<td>4.44</td>
</tr>
<tr>
<td>16 SFG</td>
<td>1</td>
<td>0.93</td>
<td>0.89</td>
<td>7</td>
<td>4.30</td>
</tr>
<tr>
<td>32 LFG</td>
<td>1</td>
<td>0.88</td>
<td>0.85</td>
<td>12</td>
<td>3.41</td>
</tr>
<tr>
<td>32 MFG</td>
<td>1</td>
<td>0.90</td>
<td>0.87</td>
<td>10</td>
<td>3.33</td>
</tr>
<tr>
<td>32 SFG</td>
<td>1</td>
<td>0.92</td>
<td>0.89</td>
<td>8</td>
<td>3.26</td>
</tr>
<tr>
<td>39 LFG</td>
<td>1</td>
<td>0.92</td>
<td>0.88</td>
<td>8</td>
<td>4.35</td>
</tr>
<tr>
<td>39 SFG</td>
<td>1</td>
<td>0.94</td>
<td>0.90</td>
<td>6</td>
<td>4.26</td>
</tr>
</tbody>
</table>
The results elicited by the Drucker and Prager model are in good agreement with the experimental results when compared to the J2 results, as seen in Figure 4-10 through Figure 4-12. The dashed curves and the dotted curves indicate the results corresponding to the DP and J2 simulations, respectively. The solid curves show the experimental results. Noteworthy is the fact that the cross marks in all the simulations (Figure 4-10 through Figure 4-15) indicate material failure by separation. The main advantage of the DP model is its ease of calibration. However, the DP model cannot still accurately model the 16 HRC flat grooved specimens seen in Figure 4-10. This is owed to the Lode angle dependence which is not accounted for in the current DP model. The DP model has an asymmetric yield surface. Therefore, using the tensile hardening curve data for compressive loading paths may lead to inadequate results for some materials (Figure 4-16). The goal here is to use one hardening curve to capture all complex loading conditions with acceptable accuracy (i.e., use a plasticity model that can capture the effects of different loading conditions). The modified PPL model is designed to achieve the premise.

Good agreement with the experimental results is achieved using the modified PPL model. This is well illustrated in Figure 4-13 through Figure 4-15. The dashed curves and the dotted curves indicate the results elicited by the PPL and J2 simulations, respectively. The solid curves indicate experimental results.

A proportional relation between hardness and strength can be established. It is observed that increasing hardness will increase the strength and lower the ductility. Hence, an inverse relationship between hardness and ductility is observed.
Figure 4-10: Force-displacement curves of AISI 4340 steel (16 HRC) using DP model
Figure 4-11: Force-displacement curves of AISI 4340 steel (32 HRC) using DP model

32 HRC AISI 4340 Steel

- TEST
- J2
- Drucker and Prager

Small Flat Grooved
Medium Flat Grooved
Large Flat Grooved
Sharp Notch
Large Notch
Smooth Bar
Figure 4-12: Force-displacement curves of AISI 4340 steel (39 HRC) using DP model
Figure 4-13: Force-displacement curves of AISI 4340 steel (16 HRC) using the PPL model
Figure 4-14: Force-displacement curves of AISI 4340 steel (32 HRC) using the PPL model
Figure 4-15: Force-displacement curves of AISI 4340 steel (39 HRC) using PPL model
Figure 4-16: Comparison of true stress-strain curves from experimental and FEA results for untreated AISI 4340 steel small compressive cylinder (16 HRC)
4.4.2 Fracture Surface Determination

As previously mentioned, the axial symmetric round bars in tension determine the fracture locus at $\bar{\theta} = 1$ and the flat grooved (plane strain) determines the locus at $\bar{\theta} = 0$. The small compressive cylinder can give fracture data at $\bar{\theta} = -1$ but no fracture was observed. Figure 4-17 shows the 2D fracture locus of the axial symmetric tension $\bar{\theta} = 1$ and the plane strain in tension at $\bar{\theta} = 0$, which shows the effects of stress triaxiality and Lode angle parameters. The calibrated 3D fracture loci of different tempering conditions are shown in Figure 4-18 through Figure 4-20.

It is found that the heat-treated specimens (32 HRC and 39 HRC) have some very similar fracture surfaces to that of the unheated specimens (16 HRC) with some shift downwards indicating loss of ductility due to heat-treatment effects.
Figure 4-17: Calibrated 2D fracture locus of axial symmetric tension (round bar specimens) at $\bar{\theta} = 1$ and plane strain tension (flat grooved specimens) at $\bar{\theta} = 0$
Figure 4-18: Calibrated 3D fracture locus of unheated AISI 4340 steel (16 HRC).
Figure 4-19: Calibrated 3D fracture locus of heat-treated AISI 4340 steel (32 HRC).
Figure 4-20: Calibrated 3D fracture locus of heat-treated AISI 4340 steel (39 HRC).
4.5 Results Discussion of the PPL Model Coupled With MMC Fracture Criteria

The PPL model and MMC fracture criterion were used to study the mechanical properties of AISI 4340 steel samples. Twenty-nine different specimens of different shapes and heat-treatments were tested and were used to calibrate the models. It was observed that this material exhibited both stress triaxiality and Lode angle dependence behaviors. It is also worth noting that the shape of the yield surface of this material changed with heat-treatment effects. The PPL model for the unheated case (16 HRC) was contained within the Tresca and von Mises yield surfaces. The results of the heat treated case (32 HRC) exhibited a yield surface shape change and exceeded the von Mises envelope. Further, heat-treatment (39 HRC) presented similar effects to those observed for the unheated sample (16 HRC). This is well demonstrated in Figure 4-21, where the shapes of the initial yield surfaces at the $\pi$-plane are plotted. This confirms previous studies (Tanaka and Spretnak 1973, Hickey and Anctil 1985, Podder, Mondal et al. 2012, Sirin, Sirin et al. 2013) showing that heat-treatments not only affects mechanical properties but also changes the yield surface shape.

The 3D fracture loci of both the heat-treated and untreated samples revealed similarities. Note that the compressive heat-treated cylinder test was not conducted because it was beyond our machine capabilities. Therefore, the compression behaviors for the heat treated specimens (32 and 39 HRC) were assumed to be identical to those of the untreated specimens (16 HRC) for simplicity.

It is found that the heat-treatment increased strength and sacrificed ductility. These mechanical differences are attributed to different microstructural void arrangements and void sizes, as discussed in (Lee and Su 1999).
Figure 4-21: Shapes of the initial yield surfaces at the $\pi$ plane for AISI 4340 steel with different hardness.
CHAPTER 5: EXTENSION AND APPLICATION OF MICROMECHANICS BASED GURSON TYPE MODEL

5.1 Model Framework and Previous Studies

Throughout the last few decades, many ductile fracture models have been proposed. Some of these models can be classified into micromechanical or damage based models. The ductile fracture can be defined as a process that involves growth, nucleation, and coalescence of microvoids (Wilsdorf 1975). The mechanisms of void behavior were first identified by Tipper (Tipper 1949). Studies on the micromechanics level of isolated voids in a plastically deforming solid are usually attributed to the works of McClintock who analyzed circular cylindrical voids (McClintock 1968). Conversely, Rice and Tracey analyzed spherical voids (Rice JR 1969). Their analysis can overestimate the strain at which coalescence occurs due to internal necking between cavities interrupted by localized shear (Tvergaard and Needleman 1984). Many studies indicate that coalescence by localized shear occurs when the void spacing is a constant of order unity times the void length (Rogers 1960, Cox and Low 1974, Hancock and Mackenzie 1976, Knott and Green 1976). Theories of microvoid growth using notch round bar specimens were verified by Hancock and Mackenzie (Hancock and Mackenzie 1976), Hancock and Brown (Hancock and Brown 1983).

Further developments of previous models were successfully carried out by (Gurson 1977) in which he proposed a yield function that links the effects of preexisting voids along with matrix material. (Chu and Needleman 1980, Tvergaard 1981, Tvergaard 1982, LauNielsen and Tvergaard 2009) modified Gurson’s work and included some constant parameters that control the plastic flow potential. Nevertheless, a new void nucleation mechanism was developed in addition to void
growth. This model is often referred to as GTN model or porous metal plasticity model and is expressed as the following:

\[
\phi = \left( \frac{\sigma}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left( \frac{-3q_2 p}{2\sigma_y} \right) - (1 + q_3 f^{*2}) = 0
\]  \hspace{1cm} (5-1)

Where \( f^* \) is defined as the following:

\[
\begin{cases} 
  f & \text{if } f \leq f_c \\
  f_c + \frac{f_f - f_c}{f_f - f_c} (f - f_c) & \text{if } f_c < f < f_f \\
  f_f & \text{if } f \geq f_f \\
\end{cases}
\]  \hspace{1cm} (5-2)

\[
\bar{f}_F = \frac{q_1 + \sqrt{q_1^2 + q_3}}{q_3}
\]

\[
\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation}
\]  \hspace{1cm} (5-3)

where \( \dot{f}_{growth} \) is based on the conservation of mass and is expressed in terms of void volume fraction:

\[
\dot{f}_{growth} = (1 - f)\dot{\varepsilon}^{pl} \cdot I
\]  \hspace{1cm} (5-4)

The \( \dot{f}_{nucleation} \) is given as:

\[
\dot{f}_{nucleation} = \frac{f_n}{s_n \sqrt{2\pi}} \exp \left[ \frac{-1}{2} \left( \frac{\dot{\varepsilon}_{m}^{pl} - \varepsilon_N}{\varepsilon^m} \right)^2 \right] \varepsilon_m^{pl}
\]  \hspace{1cm} (5-5)

where \( \sigma_y \) is the yield stress of the matrix material and \( p \) is the pressure. The GTN model has a total of 9 constant parameters that need calibration. These constants include \((q_1, q_2, q_3)\), which are
parameters in the plastic flow potential to account for interactions between cavities added by Tvergaard (Tvergaard 1981). \( f_0 \) is the initial void volume fraction. \( f_c \) is a critical volume void fraction that when reached, the material’s strength starts to decay more rapidly (i.e., softening region). \( f_f \) is the value of the void volume fraction at which there is a complete loss of strength. \( \varepsilon_N \) is the nucleation strain mean value. \( s_n \) is the standard deviation, and finally, \( f_n \) is the volume fraction of the nucleated voids. The GTN model defines void volume fraction as the summation of void growth and void nucleation terms as shown in Eq. (5-3). However, when using high x-ray tomography resolutions; voids that appear to nucleate are nothing more than small voids expanding (Landron 2011). Please note that void nucleation effects are often neglected (Gao, Faleskog et al. 1998, Zhang, Thaulow et al. 2000, Xue, Pontin et al. 2010).

It has been argued that the critical volume fraction \( (f_c) \) is not constant, and is a function of stress triaxiality (Zhang and Niemi 1994). The GTN model assumes that this \( (f_c) \) is constant. In general, \( (f_c) \) is inversely related to stress triaxiality. In fact, \( (f_c) \) is also dependent on the initially assumed void volume fraction (Koplik and Needleman 1988, Tvergaard 1990). This constant critical void volume fraction is usually found by trial and error of smooth axisymmetric round bar specimen or by unit cell modeling. This indicates that this \( (f_c) \) is not unique. Zhang using the (Thomason 1990) criterion modified the GTN failure criterion to be independent of this \( f_c \). The coalescence-failure criterion developed by Zhang is related to void geometries reaching an unstable limit. The main advantage of this criterion is its ability to predict lower void volume fractions at low-stress triaxiality compared to the traditional GTN. However, many assumptions and approximations have been made. Furthermore, this criterion is limited to volume void fractions
of 0.2. The applicability of this criterion for negative hydrostatic stress is also questionable and needs to be addressed. It is worth noting that this criterion is also dependent on the initial void volume fraction since it involves the dimensions of the void and the unit cell.

A main drawback to the GTN model is its unsuitability to model localization, and fracture for low triaxiality and shear dominated deformations (Nahshon K 2008). Recent studies have also shown the importance of Lode parameter as well as triaxiality to correctly capture the plasticity and fracture behaviors of materials (Bao and Wierzbicki 2004, Bai Massachusetts Institute of Technology ;2008).

Nahshon and Hutchinson (Nahshon K 2008) modified the GTN model to include the effects of what was defined as “void shear” which is another failure (damage) mechanism to describe the microvoids shear localization with low triaxiality. This modification is purely phenomenological, meaning that the parameter $f$ is a damage parameter rather than representing the void volume fraction. It is important to note that the “void shear” is formulated such that it vanishes under an axisymmetric stress state, i.e., retrieving the original GTN model. This new damage term “void shear” considers the Lode angle parameter effects.

(Nielsen and Tvergaard 2009) studied the effects of the shear modified Gurson model developed by Nahshon and Hutchinson on damage development in friction stir welding tensile specimens and concluded that the extra damage parameter has the expected effect of promoting fracture at moderate levels of stress triaxiality.
The initial void size has some great significance effects on void growth behavior as shown in recent literature (Fond, Lobbrecht et al. 1996, Fleck 1997, Shu 1998, Huo, Zheng et al. 1999, Zhang, Bai et al. 1999, Zhang and Hsia 2001). Huang (Wen, Hwang et al. 2005) extended the Gurson model based on the Taylor dislocation model to account for void size effects by considering the intrinsic material length $l$ as a parameter representing voids. The extended Gurson becomes handy when void sizes are thought to be significant especially at extreme loading conditions. It is also important to note that different void shapes have been considered by various researchers (Gologanu, Leblond et al. 1995, Pardoen and Hutchinson 2000). However, many of these extensions have only considered voids under axisymmetric loading conditions and did not address shear dominated deformations (Nahshon K 2008).

(Liu, Wong et al. 2016) studied the effects of Lode angle, stress triaxiality and initial void volume fraction on voided unit cell from an energy perspective. The critical strain to onset void collapse and coalescence was established by examining the energetics of the unit cell system throughout the loading process. The primary goal of the study was to investigate the existence of a transition zone between void collapse and coalescence which was proven.

To better understand the different mechanisms of void failure at the microscopic level, utilization of x-ray tomography comes at a value. (Landron 2011) used x-ray tomography during several fracture tests under different complex loading processes. Landron studied the effects of void coalescence for the first time for an industrial. Landron concluded that the Thomason criterion (Thomason 1990) is the most suitable criteria for coalescence for dual phase (DP) steels. Landron also investigated the effects of void nucleation and concluded that bi-phase microstructure of DP
steels has a much faster nucleation rate when compared to single phase steels. Therefore, the mechanism of void nucleation is highly dependent on the material’s microstructure. Lastly, Landron proposed the idea of measuring the diameters of the largest twenty pores in the population and taking the mean as a representative of the measurement of single voids for void growth mechanism.

Instead of focusing only on the mathematical aspect developed by Landron, (Fansi 2013) established a modified GTN model using the works of Landron and Ben Bettia. The new model is physically based on in-situ high-resolution X-ray tomography. Fansi’s model has three main contributions.

- Developed a new kinetic law for void nucleation predicting the evolution of void density considering stress triaxiality effects.
- The void growth model was based on the mean diameter of the population studied and instead of modeling one void; many voids were modeled using equal spacing between them.
- The critical volume fraction $f_c$ has been provided by measuring the mean distance between two cavities provided by Landron.

This model was implemented in FORTRAN and linked to ABAQUS-EXPLICIT using VUMAT subroutine. The model illustrates the effect and influence stress triaxiality has on void growth and void density. Fansi observed that with different notch sizes the crack develops at a different location. He noted that deep notches develop cracks near the bottom edge, while specimens with larger notches tend to move the crack towards the center of the specimen.
The main drawback of Fansi’s model is the need of high accurate X-ray tomography as well as the increasing number parameters and constants that needs calibration. The main contributions of Landron and Fansi’s work is a better understanding of the material’s behavior at the microscopic level.

A parametric study is essential to get a clear image of the influence of each GTN parameter on material plasticity and fracture behavior. (Slimane, Bouchouicha et al. 2015) and his team performed a parametric study on GTN model using three notched specimens of different sizes. The study was to evaluate the effects of mesh size, \( f_n \) and q parameters influence. It was observed that elastoplastic part of the material is mesh insensitive. However, the softening region is mesh sensitive. A finer mesh results in earlier fracture due to the enhancement of void initiation. The study also concluded that increasing the \( f_n \) parameter has no influence on the elastoplastic region. Contrary, fracture occurrence is earlier for higher values. Lastly, the growth of q’s increases the effects of void volume fraction, which causes a severe decrease in tensile strength. \( q_1 \) parameter variation to values greater than unity causes a loss of strength of GTN material, but this loss is not significant when compared to the variation of parameter \( q_2 \). Increasing \( q_2 \) to values greater than unity causes a premature drop in strength and fracture initiation (i.e., less ductility and rapid loss of strength indicating fracture). The study concluded that q parameters are of significant influence. Thus, many authors held to Tvergaard suggested values. Please refer to Table 5-1 for better understanding.
Table 5-1: Effects of varying GTN model parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Elastic region</th>
<th>Plastic region</th>
<th>Softening region (Fracture)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh size</td>
<td>No effects</td>
<td>No effects</td>
<td>Finer meshes cause earlier fracture</td>
</tr>
<tr>
<td>$f_n$</td>
<td>No effects</td>
<td>No effects</td>
<td>Increasing values cause earlier fracture</td>
</tr>
<tr>
<td>$q_1$</td>
<td>No effects</td>
<td>Little effects near the softening region</td>
<td>Increasing values cause earlier fracture</td>
</tr>
<tr>
<td>$q_2$</td>
<td>No effects</td>
<td>Great effect. Values higher than unity results in a premature plastic region development (less ductility)</td>
<td>Increasing values cause extreme premature fracture</td>
</tr>
</tbody>
</table>

(Kiran and Khandelwal 2014) reevaluated the capability of the GTN model in calibrating ASTM A992 steels under different complex stress states. Stress triaxiality was investigated by designing specimens of different notch sizes. Kiran illustrated the non-uniqueness of the material parameters ($\epsilon_N, f_N$) indicating that ductile fracture initiation can be simulated as early nucleation or delayed nucleation of large volume voids. Thus, a new physical criterion for determining a unique void nucleation term ($\epsilon_N$) was established and recommended. The calibration process laid out by Kiran is as following:
1. **Calibration of nucleation parameters ($\varepsilon_N, f_N$):**

These parameters govern the plastic strain level at which damage is imitated and evolved. Kiran has shown the non-uniqueness of these parameters (i.e., many combinations in the domain will simulate the behavior of microvoid imitation and growth). Increasing $\varepsilon_N$ will result in delayed nucleation. Besides, a high value of $f_N$ indicates substantial nucleation of voids occurring. Since there is little physical meaning to a unique set, Kiran proposed a new method for calibrating $\varepsilon_N$. **He proposed that $\varepsilon_N$ corresponds to the point of divergence of experimental data from $J_2$ plasticity** see Figure 5-1. This technique adds some physical meaning for $\varepsilon_N$ since the divergence of curves ($J_2$ and experimental) indicate significant void activity because $J_2$ assumes no damage parameters and no voids. After calibrating $\varepsilon_N$, $f_N$ is chosen in a merit way that best fits the experimental load-displacement data.

2. **Calibration of ($f_c, f_f$):**

These parameters govern the accelerated void volume growth and final volumetric failure. Once void volume fraction reaches the critical $f_c$, an accelerated void growth occurs until fracture. A parametric study of this unique set is required to get the best calibration. It is worth noting that the ($f_c, f_f$) parameters are highly mesh sensitive.

3. **Calibration of ($q_1, q_2, q_3, f_0, s_N$):**

($q_1, q_2, q_3$) are parameters that are often chosen as suggested by Tvergaard (i.e., $q_1 = 1.5, q_2 = 1, q_3 = 2.25$). On the other hand, ($f_0, s_N$) values are often chosen to match literature.
Figure 5-1: Experimental load-displacement curve and the corresponding FEA results with J2 plasticity model (Kiran and Khandelwal 2014).
In this paper, the plasticity and fracture behaviors of AISI steel 4340 samples under different tempering conditions will be studied. The Gurson-Tvergaard Needleman (GTN) is used to describe the material plasticity and fracture. Also, a modified Gurson-Tvergaard Needleman plasticity model coupled with modified Mohr-Coulomb fracture model denoted as (GTN-MMC) is used for the plasticity and fracture modeling consecutively. As mentioned previously, the Lode angle parameter is crucial, especially when modeling shear dominated regions. Thus, the effects of this parameter on matrix material should be included. A series of mechanical tests to fracture were conducted using the MTS machine with Digital Image Correlation (DIC) analysis. The (GTN-MMC) model was implemented into Abaqus/explicit by means of a user-defined subroutine (VUMAT). The final analysis shows good correspondence between experimental and simulated results.

5.2 Theoretical Improvements to the GTN Model

The Gurson-Tvergaard-Needleman (GTN) model neglects the effects of the third deviatoric stress invariant (or Lode angle parameter) on material plasticity. It is found that the GTN model does not imitate the experimental data for plane strain specimens for the 16 Rockwell Hardness (HRC) case as will be shown in section 5.3. Therefore, the GTN model needs improvements to account for Lode angle dependency on matrix material. It is important to note that the proposed modified model is a decoupled model, unlike the coupled GTN model. This section attempts to address the former issues by proposing the following:
5.2.1 Modified GTN Plasticity Model with Lode Angle Matrix Material Dependence

The GTN matrix material is considered a homogeneous, incompressible, rigid plastic, von Mises material. We propose the following yield criterion suggested by Bai and Wierzbicki (Bai 2008):

$$
\sigma_{y,mat} = \bar{\sigma}(\bar{\varepsilon}_{pl}) \left[ c^s_0 + \left( c^{ax}_0 - c^s_0 \right) \left( \frac{m + 1}{m} \right) \left( \gamma - \frac{\gamma^{m+1}}{m + 1} \right) \right]
$$

(5-6)

Where, $\bar{\sigma}(\bar{\varepsilon}_{pl})$ can be described by many hardening laws such as Swift, Ludwik, Voce, Isotropic hardening, etc.

The $c^s_0$, $c^{ax}_0$ and $m$ are constant coefficients that controls the effects of Lode angle on material plasticity. The ($\gamma$) parameter is the difference between Von Mises and Tresca in the deviatoric stress plane. The $\gamma$ and $c^{ax}_0$ are expressed as follows:

$$
\gamma = 6.4641 \left[ \sec(\theta - \pi/6) - 1 \right]
$$

(5-7)

$$
c^{ax}_0 = \begin{cases} 
c^t_0 & \text{for } \bar{\theta} \geq 0 \\
c^s_0 & \text{for } \bar{\theta} < 0 
\end{cases}
$$

(5-8)

This yield criterion has gained much attention in the recent literature (Wierzbicki, Bao et al. 2005, Beese, Luo et al. 2010, Algarni 2015, Bai 2015, Jia and Bai 2016, Algarni, Choi et al. 2017). It also enables easy calibration of plane strain specimens using the GTN model. Using the associated flow rule, a new plastic flow potential is adapted as follows:

$$
de^{ij,pl} = d\lambda \left( \frac{\partial f}{\partial \sigma_{ij}} \right)
$$

(5-9)
Where \( f \) and \( q \) are defined as the following:

\[
f = q - \sigma_y \left[ c_0^s + (c_0^{ax} - c_0^s) \left( \frac{m + 1}{m} \right) \right] \\
\left( \gamma - \frac{\gamma^{m+1}}{m + 1} \right) \sqrt{\left( 1 + q_3 q^{*2} \right) - 2q_1 f^* \cosh \left( \frac{-3q_2 \sigma_m}{2\sigma_y} \right)}
\]

Please note that this \( f \) is not the void volume fraction.

\[
q = \bar{\sigma} = \sqrt{3} f_2 = \sqrt{\frac{3}{2}} [S] : [S] = \frac{1}{\sqrt{2}} (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2
\]

5.2.2 Modified Evolution Rule of Microvoid Volume Void Fraction (\( f \))

The works of (Landron 2011, Fansi 2013) prove the linkage between void nucleation and stress triaxiality. Higher stress triaxiality results in more nucleation activity. The current GTN model doesn’t account for such linkage. Since nucleation is usually activated once \( f_c \) reaches a constant value, it is hard to control the softening and fracture regions for different stress triaxialities using the same \( f_c \) for all cases. The proposed new volumetric void fraction is as follows:

\[
f = f_0 + (f_{\text{growth}} + f_{\text{nucleation}}) \ F_1(\bar{\theta}, \eta)
\]

The \( F(\bar{\theta}, \eta) \) can take many forms. However, the proposed form is as following:

\[
F_1(\bar{\theta}, \eta) = \left( 1 - c_p \left( \eta - \eta_o \right) \right) \left( 1 + m_1 (1 - \bar{\theta})^{m_2} \right)
\]
By choosing \( c_p = m_1 = 0 \), the GTN void volume fraction is retrieved. Introducing such terms enables better calibration of specimens under different stress states. \( \eta_o \) is chosen from the reference test (i.e., \( \eta_o = \frac{1}{3} \) for the smooth bar test). If the smooth bar is selected as the reference test \( c_p \) will be chosen in such a way to give the best fit for other specimens under different stress triaxiality state. The advantage of using the Lode angle parameter \( \bar{\theta} \) is its simplicity. \( \bar{\theta} = 1, -1 \) for axial symmetric tension and compression consecutively. \( \bar{\theta} = 0 \) for simple shear cases (Bai 2008). Since \( \bar{\theta} = 0 \) for flat grooved specimens, \( m_1 \) and \( m_2 \) are chosen in such a way to obtain the best fit among different grooves under different stress states. However, for simplicity and to attain the physical meaning of \( f \) the original GTN void volume fraction is used for calibration (i.e., \( c_p = m_1 = m_2 = 0 \)).

5.2.3 Modified Criterion of Critical Microvoid Volume Fraction to Fracture \( (f_f) \) Depending on Different Stress States

To obtain the best fit possible for different specimens under different loading conditions. It was observed that neither \( f_c \) nor \( f_f \) is constant under different loading conditions (i.e., choosing a fixed pair of \( f_c \) and \( f_f \) to calibrate specific loading conditions, for example, smooth bar, results in under/over predicting fracture phenomena for different loading conditions). This behavior has also been reported by (Kiran and Khandelwal 2014). Nahshon and Hutchinson have also noted that many metallic materials have a fracture strain in shear that is well below fracture strain for axisymmetric loadings (Nahshon K 2008). They suggested the addition of a new damage parameter that could be called “void shear”. However, this new damage parameter changes the physical meaning of void volume fraction \( f \) in the sense that “void shear” is a phenomenological
term. Therefore, a new decoupled fracture criterion that keeps the physical meaning of the $f$ parameter is proposed and developed. This decoupled fracture criterion is similar to the extended Mohr-Coulomb fracture criteria developed by Bai and Wierzbicki (Bai and Wierzbicki 2009).

The modified Mohr-Coulomb fracture criteria (MMC) has been gaining much attention in recent literature (Bai and Wierzbicki 2009, Li, Luo et al. 2010, Luo and Wierzbicki 2010, Dunand and Mohr 2011, Areias and Rabczuk 2013, Algarni 2015, Jia and Bai 2016, Jia, Ghazali et al. 2017, Jia, Qiao et al. 2017, Qiao, Liu et al. 2017). The Mohr-Coulomb (MC) is a physical simple fracture criterion that has been widely used throughout literature (Zhao 2000, Chung, Ma et al. 2011, Algarni, Jia et al. 2015, Jia, Qiao et al. 2017). In this section a new decoupled fracture criterion that links ($f, \eta, \tilde{\theta}$) is developed as follows:

The Gurson-Tvergaard-Needleman GTN model is expressed as follows:

$$\phi = \left( \frac{\bar{\sigma}}{\sigma_{y,mat}} \right)^2 + 2q_1 f \cosh \left( \frac{-3q_2 p}{2\sigma_{y,mat}} \right) - (1 + q_3 f^2) = 0 \tag{5-14}$$

Noting that the $f^*$ in the GTN has been replaced by $f$ making it a complete decoupled model.

Solving Eq. (5-14) for $\bar{\sigma}$:

$$\bar{\sigma} = \sigma_{y,mat} \sqrt{(1 + q_3 f^2) - 2q_1 f \cosh \left( \frac{-3q_2 \sigma_m}{2\sigma_{y,mat}} \right)} \tag{5-15}$$

Expanding Eq. (5-15) and considering tiny volume void fractions $f$ thus, $f^2 \cong 0$
\[ \sqrt{(1 + q_3 f^2) - 2 q_1 f \cosh\left(\frac{-3 q_2 \sigma_m}{2 \sigma_{y,mat}}\right)} = 1 + \frac{1}{2} \left[ q_3 f^2 - 2 q_1 f \cosh\left(\frac{-3 q_2 \sigma_m}{2 \sigma_{y,mat}}\right) \right] \] (5-16)

\[ = \left[ 1 - q_1 f \cosh\left(\frac{-3 q_2 \sigma_m}{2 \sigma_{y,mat}}\right) \right] \]

Thus, the GTN is reduced to the following:

\[ \tilde{\sigma} = \sigma_{y,mat} \sqrt{(1 + q_3 f^2) - 2 q_1 f \cosh\left(\frac{-3 q_2 \sigma_m}{2 \sigma_{y,mat}}\right)} \] (5-17)

\[ \equiv \sigma_{y,mat} \left[ 1 - q_1 f \cosh\left(\frac{-3 q_2 \sigma_m}{2 \sigma_{y,mat}}\right) \right] \]

The Mohr-Coulomb in terms of \((\tilde{\sigma}, \eta, \theta)\) is expressed as follows (Bai and Wierzbicki 2009):

\[ \tilde{\sigma} = c_2 \left[ \left(\frac{1 + c_1}{3}\right) \cos\left(\frac{\pi}{6} - \theta\right) + c_1 \left(\eta + \frac{1}{3} \sin\left(\frac{\pi}{6} - \theta\right)\right) \right]^{-1} \] (5-18)

\(\theta\) can be expressed as (Bai 2008) for simplicity:

\[ \theta = \frac{\pi}{6} (1 - \tilde{\theta}) \] (5-19)

Equating (5-17), (5-18) and (5-19) and denoting that the volume void fraction at the point of fracture is \(f_f\)
\[ f_f = \left[ q_1 \cosh \left( \frac{-3q_2 \sigma_m}{2 \sigma_{y,\text{mat}}} \right) \right]^{-1} \left[ 1 \right. \\
- \frac{c_2}{\sigma_{y,\text{mat}}} \left[ \sqrt{\left( \frac{1 + c_1}{3} \right)} \cos \left( \frac{\pi \bar{\theta}}{6} \right) + c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\pi \bar{\theta}}{6} \right) \right) \right]^{-1} \]

Where (Bai and Wierzbicki 2008);

\[ \sigma_{y,\text{mat}} = \sigma_{y,J2} \left[ \tilde{\epsilon}_0^{s} + (\tilde{\epsilon}_0^{ax} - \tilde{\epsilon}_0^{s}) \left( \frac{\sqrt{3}}{2 - \sqrt{3}} \left[ \sec \left( \frac{\pi \bar{\theta}}{6} \right) - 1 \right) \right] \right] \]

We define \( \eta \) and \( \tilde{c}_2 \) following:

\[ \eta = \frac{\sigma_m}{\sigma_{y,J2}} \] (5-22)

\[ \tilde{c}_2 = \frac{c_2}{\sigma_{y,\text{mat}}} \] (5-23)

Thus (5-20) is reduced to the following:

\[ f_f = \left[ q_1 \cosh \left( \frac{-3q_2 \eta}{2 \left[ \tilde{\epsilon}_0^{s} + (\tilde{\epsilon}_0^{ax} - \tilde{\epsilon}_0^{s}) \left( \frac{\sqrt{3}}{2 - \sqrt{3}} \left[ \sec \left( \frac{\pi \bar{\theta}}{6} \right) - 1 \right) \right] \right) \right]^{-1} \left[ 1 \right. \\
- \tilde{c}_2 \left[ \sqrt{\left( \frac{1 + c_1}{3} \right)} \cos \left( \frac{\pi \bar{\theta}}{6} \right) + c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\pi \bar{\theta}}{6} \right) \right) \right]^{-1} \]

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This decoupled fracture criterion has a total of six parameters \( (q_1, q_2, c_1, \tilde{c}_2, \tilde{c}_0^2, \tilde{c}_0^{ax}) \) that need to be found.

5.3 Experimental Results of AISI 4340 Steel and Numerical Validation of the Proposed (GTN-MMC) Model

In this section, an attempt to calibrate AISI 4340 steel of hardness 16 HRC, 32HRC and 39 HRC is carried out using the hardening curves shown in Figure 5-2 and the parameters shown at Table 5-2. Note that the solid curves in Figure 5-2 indicate pre-necking behaviors, while the dashed curves indicate the post necking. The post necking data was extrapolated using Swift hardening law, and then data was adjusted through iteration to get the best FEA correlation with experiments (trial and error method).

![Figure 5-2: Hardening curves for 16 HRC, 32 HRC and 39 HRC consecutively used in GTN model](image-url)
Table 5-2: GTN parameters used for 16 HRC, 32HRC and 39 HRC consecutively

<table>
<thead>
<tr>
<th>Hardness</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( f_0 )</th>
<th>( f_N )</th>
<th>( \varepsilon_N )</th>
<th>( s_N )</th>
<th>( f_c )</th>
<th>( f_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>1.5</td>
<td>1</td>
<td>( q_1^2 )</td>
<td>0.0065</td>
<td>0.01</td>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>32 HRC</td>
<td>1.5</td>
<td>1</td>
<td>( q_1^2 )</td>
<td>0.0065</td>
<td>0.005</td>
<td>0.1</td>
<td>0.1</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>39HRC</td>
<td>1.5</td>
<td>1</td>
<td>( q_1^2 )</td>
<td>0.0065</td>
<td>0.005</td>
<td>0.1</td>
<td>0.1</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The round specimens (SB, LN, and SN) were simulated using a quarter of the model with an axisymmetric elements (CAX4R) type due to symmetry. While, the flat grooved specimens (LFG, MFG, and SFG) were constructed using a \( \frac{1}{8} \) model with 3D solid elements (C3D8R) type due to symmetry. The small compressive cylinder (SCC) was created using 3D solid elements (C3D8R).

The \( q \) parameters are chosen as suggested by (Tvergaard 1981). \( f_0 \) is chosen to resemble the low porosity of this material. Other parameters are chosen based on the best fit for all cases and within literature suggested values (Kiran and Khandelwal 2014). Note that the smooth bar is used as the reference specimen for calibration purposes (i.e., fracture parameters \( (f_c, f_f) \) are chosen to fit the smooth bar fracture location). It is clear from Figure 5-3, Figure 5-5 and Figure 5-6 that the GTN model does not perceive the correct fracture location (denoted by cross mark) compared with experimental tests (denoted by an asterisk). Choosing a unique set of \( (f_c, f_f) \) to fit the smooth bar will lead to early fracture locations for round bar specimens under higher stress triaxiality states. Picking the same fracture parameter set will also lead to extreme late fracture behaviors in-plane.
strain specimens when compared to the experimental tests. Therefore, the new decoupled fracture criterion is implemented in the upcoming sections.

It can also be seen that round bar specimens give a reasonable plasticity fit using the GTN model (dashed curves) compared with experimental (solid curves). However, the GTN model does not provide adequate plasticity behavior for the cases of 16 HRC flat grooved (plane strain specimens) as seen in Figure 5-3 nor the small compressive cylinder case as seen Figure 5-4. The absence of Lode angle parameter is probably the reason for the inadequate plasticity fit achieved for the flat grooved specimens. On the other hand, the existence of void nucleation terms in the compressive region will affect the attainment of realistic results for the small compressive cylinder case (SCC).

The previous observations are evidence that the current GTN model needs improvements to include Lode angle parameter effects on material plasticity, especially for the 16 HRC flat grooved (plane strain) specimens. However, the cases of 32 HRC (Figure 5-5) and 39 HRC (Figure 5-6) seem to have less plasticity dependence on Lode angle, and $c_0^s$ of unity is therefore chosen. It is worth noting that choosing $c_0^s$ to be unity retrieves the original GTN yield criterion.
Figure 5-3: Force-displacement curves for (16 HRC) GTN model
Figure 5-4: Stress-strain curve of the small compressive cylinder (16 HRC)
32 Rockwell Hardness

Figure 5-5: Force-displacement curves for (32 HRC)
Figure 5-6: Force-displacement curves for (39 HRC)
5.3.1 Validation of Proposed Model (GTN-MMC)

In this section, an attempt to calibrate AISI 4340 steel of hardness 16 HRC, 32HRC and 39 HRC is carried out using the updated yield criterion Eq. (5-6) and decoupled fracture criterion Eq. (5-24) (GTN-MMC).

5.3.2 Plasticity Model Calibration

The GTN matrix material is assumed to be a von Mises material. We suggest that the matrix material follows that of Eq. (5-6). The advantages of using such yield criterion are its simplicity and ease of calibration. In essence; the axial symmetric round bars in tension and compression exhibit a Lode angle parameter of unity \( \bar{\theta} = 1 - 1 \) consecutively. On the other hand, flat grooved specimens in tension (plane strain) exhibit a Lode angle of \( \bar{\theta} = 0 \).

The Lode angle dependence on material plasticity is evident for the (16 HRC) flat grooved specimens as shown in Figure 5-6 where the dashed curves (GTN model) indicates much higher strength than the experimental curves (solid curves). Using \( c_0^s = 0.9 \) (i.e., including Lode angle dependence) results in more realistic strengths (dotted curves) comparable to the experimental tests. Furthermore, it was observed that the GTN model is quite sufficient in capturing the plasticity behavior for the cases of 32 HRC and 39 HRC. This indicates that the material at (32 & 39 HRC) hardness has negligible Lode angle dependence when using the GTN model, therefore \( c_0^s \) of unity is chosen (i.e., retrieving the original GTN model). Please refer to Table 5-3 for detail description of plasticity parameters chosen for modified GTN. Also, please note that small compressive
cylinder tests where only performed at 16 HRC due to machine capability limits. Fracture did not occur at this hardness (16 HRC).

Table 5-3: Modified GTN plasticity parameters for 16 HRC, 32 HRC, and 39 HRC consecutively

<table>
<thead>
<tr>
<th>Hardness</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q_3$</th>
<th>$f_0$</th>
<th>$f_N$</th>
<th>$e_N$</th>
<th>$s_N$</th>
<th>$c_{\theta}^s$</th>
<th>$c_{\theta}^c$</th>
<th>$c_{\theta}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 HRC</td>
<td>1.5</td>
<td>1</td>
<td>$q_1^2$</td>
<td>0.0065</td>
<td>0.01</td>
<td>0.1</td>
<td>0.1</td>
<td>0.9</td>
<td>0.95</td>
<td>1</td>
</tr>
<tr>
<td>32 HRC</td>
<td>1.5</td>
<td>1</td>
<td>$q_1^2$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
<td>N/A</td>
<td>1</td>
</tr>
<tr>
<td>39 HRC</td>
<td>1.5</td>
<td>1</td>
<td>$q_1^2$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
<td>N/A</td>
<td>1</td>
</tr>
</tbody>
</table>

5.3.3 Calibration of Failure Criteria (GTN-MMC)

It is clear from previous discussions and illustrations that the void volume fraction at failure is not constant under different stress states. The existence of impurities in the material due to casting and manufacturing errors could be the cause. Other causes vary from different hardness level within the same material as well as void geometry shape changes. Therefore, a new decoupled fracture criteria Eq. (5-24) is developed and incorporated. The calibrated parameters are shown in Table 5-4. The set of $(c_1, \tilde{c}_2)$ are determined from the best fit of the axisymmetric round bar specimens in tension. While $(\tilde{c}_0^s)$ is determined from the flat grooved specimens.
Table 5-4: GTN-MMC fracture criteria

<table>
<thead>
<tr>
<th>Hardness</th>
<th>16 HRC</th>
<th>32 HRC</th>
<th>39 HRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.018</td>
<td>0.021</td>
<td>0.076</td>
</tr>
<tr>
<td>$\bar{c}_2$</td>
<td>0.480</td>
<td>0.485</td>
<td>0.550</td>
</tr>
<tr>
<td>$\bar{c}_\theta$</td>
<td>0.310</td>
<td>0.320</td>
<td>0.300</td>
</tr>
</tbody>
</table>

5.3.4 Comparison between Tests and Simulation Results

The axial symmetric round bars in tension (SB, LN, SN) determine the fracture locus at $\bar{\theta} = 1$ and the plane strain specimens (LFG, MFG, SFG) determines the locus at $\bar{\theta} = 0$. The small compressive cylinder can give fracture data at $\bar{\theta} = -1$ but the fracture was not observed. Figure 5-7 shows the 2D fracture locus of the axial symmetric tension $\bar{\theta} = 1$ and the plane strain in tension at $\bar{\theta} = 0$, which shows the effects of stress triaxiality and Lode angle parameters. The calibrated 3D fracture loci of different tempering conditions are shown in Figure 5-8, Figure 5-9 and Figure 5-10 consecutively.

It is found that the (GTN-MMC) fracture criterion has the capability of capturing the correct fracture locations of the round bar specimens with reasonable accuracy as seen in Figure 5-7 (solid curves). However, the new fracture criterion still lacks the capability of capturing the correct fracture locations for all flat grooved specimens (dotted curves). The new criterion underestimates the fracture location occurrence for some of these specimens. However, the new decoupled (GTN-MMC) fracture criterion is still better in capturing a more realistic fracture location for plane strain specimens than the original GTN criteria. Please refer to Figure 5-3, Figure 5-5 and Figure 5-6.
where diamond, cross, and asterisk denote the GTN, (GTN-MMC) and experimental fracture locations respectively.

It is also found that round bar specimens under tension experience larger void volume fractions at failure when compared to plane strain (flat grooved) specimens as seen in Figure 5-7. This means that a calibrated GTN fracture parameter set $(f_c, f_f)$ for round bar specimens will, in general, overestimate fracture locations for plane strain specimens as previously mentioned.

Figure 5-7: Calibrated 2D fracture locus of axial symmetric tension (round bar specimens) at $\bar{\theta} = 1$ and plane strain tension (flat grooved specimens) at $\bar{\theta} = 0$
Figure 5-8: Calibrated 3D fracture locus of AISI 4340 steel (16 HRC).

Figure 5-9: Calibrated 3D fracture locus of AISI 4340 steel (32 HRC).
Figure 5-10: Calibrated 3D fracture locus of AISI 4340 steel (39 HRC).
CHAPTER 6:
STUDY OF DUCTILE FRACTURE FROM MATERIAL MICROSCOPIC IMAGES

Fractured surfaces of round bar (SB, LN, and SN) and plane strain (LFG, MFG, SFG) specimens of hardness (16 HRC, 32 HRC, and 39 HRC) were selected for microstructural analysis using a scanning electron microscope (SEM). Fractured surfaces of round bars reveal that the state of stress affects the microvoids the material encounters. In essence, Figure 6-1 cases (a-c), (d-f) and (g-h) indicate that the average microvoid size increases under an increasing stress triaxiality state. Larger void sizes could explain the loss of ductility the material experiences under higher stress triaxiality states. Figure 6-1 also demonstrates that the initial hardness of the material affects void sizes. It is observed that the voids experience an increase in size for harder surface (i.e., please refer to Figure 6-1 cases (a, d, g), (b, e) and (c, f, h)). For different magnifications of fractured round bar specimens, please refer to Figure 6-3 through Figure 6-10.

Fractured surfaces of plane strain (flat grooved) specimen don’t reveal much size change in the microvoids (Figure 6-2). It seems that the dominant failure mechanism for these specimens is void shear. However, it is difficult to assess and distinguish the actual failure mechanism without x-ray tomography testing methods. A study of in situ x-ray tomography similar to the work of (Landron 2011) but for plane strain specimens is highly recommended for future studies. For different magnifications of fractured flat grooved specimens, please refer to Figure 6-11 through Figure 6-18.
Figure 6-1: Fracture surfaces of round bar specimens for 16 HRC, 32 HRC, and 39 HRC consecutively
Figure 6-2: Fracture surfaces of flat grooved specimens for 16 HRC, 32 HRC, and 39 HRC consecutively
Figure 6-3: Fractured smooth bar specimen at different magnifications (16 HRC)
Figure 6-4: Fractured smooth bar specimen at different magnifications (32 HRC)
Figure 6-5: Fractured smooth bar specimen at different magnifications (39 HRC)
Figure 6-6: Fractured Large Notch specimen at different magnifications (16 HRC)
Figure 6-7: Fractured Large Notch specimen at different magnifications (32 HRC)
Figure 6-8: Fractured sharp notch specimen at different magnifications (16 HRC)
Figure 6-9: Fractured sharp notch specimen at different magnifications (32 HRC)
Figure 6-10: Fractured sharp notch specimen at different magnifications (39 HRC)
Figure 6-11: Fractured large flat grooved specimen at different magnifications (16 HRC)
Figure 6-12: Fractured large flat grooved specimen at different magnifications (32 HRC)
Figure 6-13: Fractured large flat grooved specimen at different magnifications (39 HRC)
Figure 6-14: Fractured medium flat grooved specimen at different magnifications (16 HRC)
Figure 6-15: Fractured medium flat grooved specimen at different magnifications (32 HRC)
Figure 6-16: Fractured small flat grooved specimen at different magnifications (16 HRC)
Figure 6-17: Fractured small flat grooved specimen at different magnifications (32 HRC)
Figure 6-18: Fractured small flat grooved specimen at different magnifications (39 HRC)
CHAPTER 7: CONCLUSION AND FUTURE WORK

7.1 Summary of Contributions

Many milestones have been achieved since the beginning of this study. These include specimens’ design, heat-treatments, experimental tests, FEA simulations, data acquisition and data analysis leading to results that have vastly enriched the topic. Below is a summary of the research:

- A full round of numerical and experimental studies was performed to investigate the plasticity and fracture of AISI 4340 steel alloy under multiaxial stress loading conditions at different heat-treated conditions. The models that were used to investigate this material were the following:
  a) A modified form of the Bai-Wierzbicki (also named as PPL) model couple with a modified Mohr-Coulomb (MMC) fracture criterion.
  b) The linear Drucker-Prager Plasticity model.
  c) The Gurson, Tvergaard, and Needleman (GTN) model, also known as the porous metal plasticity model.
  d) A new decoupled form of the GTN representing plasticity and fracture (GTN-MMC) is proposed and validated.

- It is found that this material exhibits both stress triaxiality and Lode angle dependent behaviors, especially for the case of 16 Rockwell Hardness (HRC). Conversely, Lode angle dependence for the (32 & 39 HRC) heat-treated cases was found to be less significant. Furthermore, it was observed that the shape of the yield surface of this
material changes with heat-treatment effects. The PPL model for the unheated case (16 HRC) is contained within Tresca and von Mises yield surfaces. The results of the heat-treated case (32 HRC) exhibit a yield surface shape change and exceeds the von Mises envelope. Further heat-treatment (39 HRC) exhibits similar effects to that of the unheated (16 HRC). This confirms previous studies that heat-treatment not only affects mechanical properties but also changes the yield surface shape. Thus, it is found that the heat-treatment increases the strength while ductility is sacrificed. The 3D fracture surface of this material is revealed, and it is found that heat-treatment had no major significance in changing the shapes of 3D fractured surface (16, 32 and 39 HRC).

- It is found that the GTN model captures the plasticity part of this material with reasonable accuracy for round bar specimens (SB, LN, and SN). Conversely, the GTN model doesn’t capture the plasticity part of the 16 HRC plane strain specimens (LFG, MFG, and SFG) with reasonable accuracy. Furthermore, it was observed that the current fracture parameter set \((f_c, f_f)\) in GTN model don’t take into consideration specimens experiencing different stress states, (for example calibrating \((f_c, f_f)\) for the smooth bar will result in underestimating fracture locations for both large notch and sharp notch also overestimating fracture locations of plane strain specimens). Therefore, a new decoupled fracture criterion (GTN-MMC) that is both Lode angle and stress triaxiality dependent is developed and implemented. This is done by incorporating the Bai and Wierzbicki yield criterion, which incorporates Lode angle dependence, as the matrix material instead of the traditional J2 plasticity used in the GTN. The (GTN-MMC) fracture criterion has the
capability of capturing the correct fracture locations of round bar specimens with reasonable accuracy. However, the new fracture criterion still lacks the capability of capturing the correct fracture locations for all flat grooved specimens respectively. The new criterion underestimates the fracture location occurrence. However, the new decoupled (GTN-MMC) fracture criterion is much better in capturing a more realistic fracture location for plane strain specimens than the original GTN criteria.

- Fracture surface analysis of round bars using (SEM) reveals that the state of stress affects the microvoids the material encounters, i.e., larger microvoids appear for higher stress triaxiality encountered. While, fractured surfaces of plane strain (flat grooved) specimen don’t reveal much size change in the microvoids, which is probably due to microvoid shearing effect.
7.2 Recommended Future Study

Ductile fracture after large plastic deformation become a more and more important subject as the requirements of high fidelity modeling in computers. The microvoid based fundamental mechanisms are not fully understood and they are also critical to future material design and development. Below is a list of suggested topics for future study.

1. **Effects of strain rate, temperature and anisotropy**: The PPL-MMC plasticity/fracture model describes the materials behaviors under different loading conditions considering various stress triaxiality and Lode angle effects. This model has also the capability of correctly capturing various fracture propagation phenomena which makes it a unique candidate for such a study. However, the coupling effects of strain rate and temperature have not fully addressed. There is also a need to include material anisotropic effect.

2. **Heat-treatment effects on Lode angle parameter**: Heat-treatment on AISI 4340 steel seems to restrict the Lode angle dependence behaviors. Heat-treatment and Lode angle dependence should be investigated using different materials.

3. **X-ray tomography**: To better understand the microvoid behaviors and attributes under different loading conditions, A study similar to the work of (Landron 2011) for different loading conditions including axisymmetric smooth and notched round bars, plane strain, and shear specimens are highly recommended for future study.

4. **Void behaviors and attributes**: Void behaviors are complex and different under different stress states. It has been recognized that the void nucleation, growth, and coalescence are the primary attributes at high-stress triaxiality. However, it is also known that the void
shape at low or even negative stress triaxiality is entirely different. The issue of void shape within the unit cell modeling has been gaining much attention recently. The current GTN model does not account for this. (Wen, Hwang et al. 2005) extended the GTN model to account for such dependency. However, the resulting equations are complicated and hard to implement for practical cases. An applicable and easy way to implement criterion should be addressed.
Here are the papers that I have published or submitted for publication:


APPENDIX A:
FLOW RULE DERIVATION CONSIDERING $c_\eta(\theta)$
The plastic flow rule of the Bai-Wierzbicki (also called PPL) plasticity model reads as follows:

\[ f = q - \bar{\sigma}(\bar{\varepsilon}^p) \left[ 1 - c_\eta (\bar{\eta} - \eta_0) \right] \left[ c_\theta^s + (c_\theta^{ax} - c_\theta^s) (\gamma - \frac{\gamma^{m+1}}{m+1}) \left(1 + \frac{1}{m}\right) \right] \]  

(A-1)

where \( q , \bar{\sigma}(\bar{\varepsilon}^p) , c_\eta , \bar{\eta}, \eta_0 , c_\theta^s , m , \gamma \) and \( c_\theta^{ax} \) are parameters defined in section 4.2

Deriving the plastic flow rule \( f \) with respect to \( \sigma_{ij} \) the following is obtained:

\[
\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial q}{\partial \sigma_{ij}} + \bar{\sigma}(\bar{\varepsilon}^p) \left[ \frac{\partial c_\eta}{\partial \sigma_{ij}} (\bar{\eta} - \eta_0) + c_\eta \frac{\partial \bar{\eta}}{\partial \sigma_{ij}} \right] c_\theta^s \\
+ (c_\theta^{ax} - c_\theta^s) \left( \gamma - \frac{\gamma^{m+1}}{m+1} \right) \left(1 + \frac{1}{m}\right) \\
- \bar{\sigma}(\bar{\varepsilon}^p) \left[ 1 - c_\eta (\bar{\eta} - \eta_0) \right] (c_\theta^{ax} - c_\theta^s) \left(1 + \frac{1}{m}\right) \left(1 - \gamma^m\right) \frac{\partial \gamma}{\partial \sigma_{ij}}
\]

(A-2)

Assuming that the \( c_\eta \) is a function of Lode angle \( c_\eta \) is expressed as:

\[ c_\eta = M_1 - M_2 f(\bar{\theta}) \]  

(A-3)

Where, \( M_1 , M_2 \) are constant parameters.

\[ f(\bar{\theta}) = \frac{1}{1 + e^{-k(\theta_0 - \bar{\theta})}} \]  

(A-4)

Deriving \( c_\eta \) with respect to \( \sigma_{ij} \) the following is obtained:

\[ \frac{\partial c_\eta}{\partial \sigma_{ij}} = -M_2 \frac{\partial f(\bar{\theta})}{\partial \sigma_{ij}} \]  

(A-5)

\[ \frac{\partial f(\bar{\theta})}{\partial \sigma_{ij}} \] is expressed as follows:
\[
\frac{\partial f(\bar{\theta})}{\partial \sigma_{ij}} = -k \frac{\partial \bar{\theta}}{\partial \sigma_{ij}} \frac{e^{-k(\theta_0 - \bar{\theta})}}{(e^{-k(\theta_0 - \bar{\theta})} + 1)^2}
\]  \hspace{1cm} (A-6)

Substituting Eq. (A-6) into Eq. (A-5):

\[
\frac{\partial c_\eta}{\partial \sigma_{ij}} = -M_2 \frac{k}{(e^{-k(\theta_0 - \bar{\theta})} + 1)^2} \frac{\partial \bar{\theta}}{\partial \sigma_{ij}} \frac{e^{-k(\theta_0 - \bar{\theta})}}{\pi \sqrt{1 - \xi^2}} \frac{\partial \xi}{\partial \sigma_{ij}}
\]  \hspace{1cm} (A-7)

The Lode angle parameter \( \bar{\theta} \) is expressed as the following:

\[
\bar{\theta} = 1 - \frac{2}{\pi} \cos^{-1} \xi
\]  \hspace{1cm} (A-8)

Deriving \( \bar{\theta} \) with respect to \( \sigma_{ij} \) the following is obtained:

\[
\frac{\partial \bar{\theta}}{\partial \sigma_{ij}} = \frac{2}{\pi \sqrt{1 - \xi^2}} \frac{\partial \xi}{\partial \sigma_{ij}}
\]  \hspace{1cm} (A-9)

Substituting Eq. (A-9) into Eq. (A-7):

\[
\frac{\partial c_\eta}{\partial \sigma_{ij}} = \frac{2 M_2 k}{\pi \sqrt{1 - \xi^2} (e^{-k(\theta_0 - \bar{\theta})} + 1)^2} \frac{\partial \xi}{\partial \sigma_{ij}}
\]  \hspace{1cm} (A-10)

\( \xi \) is defined as follows:

\[
\xi = \left(\frac{r}{q}\right)^3
\]  \hspace{1cm} (A-11)

Deriving \( \xi \) with respect to \( \sigma_{ij} \) the following is obtained:

\[
\frac{\partial \xi}{\partial \sigma_{ij}} = \frac{3 r^2}{q^4} \left( q \frac{\partial r}{\partial \sigma_{ij}} - r \frac{\partial q}{\partial \sigma_{ij}} \right)
\]  \hspace{1cm} (A-12)

The goal here is to find \( \frac{\partial r}{\partial \sigma_{ij}} \) and \( \frac{\partial q}{\partial \sigma_{ij}} \) therefore \( r \) and \( J_3 \) are defined as follows:

\[
r = \left( \frac{9}{2} S \cdot [S] : [S] \right)^{1/3}
\]  \hspace{1cm} (A-13)
\[ J_3 = \frac{1}{3} S_{ij} S_{jk} S_{kl} = \text{det} \ [S] \] (A-14)

Expressing \( r \) as a function of \( J_3 \) the following is obtained:
\[ r = (13.5 J_3)^{1/3} = (13.5 \text{ det} \ [S])^{1/3} \] (A-15)

Now we need to express \( q \) as a function of \( J_2 \) and it is expressed as follows:
\[ J_2 = \frac{1}{2} S_{ij} S_{ij} = \frac{q^2}{3} \] (A-16)

Taking derivatives of the previous parameters results in the following:
\[ \frac{\partial J_3}{\partial \sigma_{ij}} = S_{jk} S_{kl} - \frac{1}{3} S_{ij} S_{ij} \delta_{ij} = S_{jk} S_{kl} - \frac{2}{3} J_2 \delta_{ij} \] (A-17)

\[ \frac{\partial r}{\partial \sigma_{ij}} = (13.5)^{1/3} (J_3)^{-2/3} \frac{\partial J_3}{\partial \sigma_{ij}} = (13.5)^{1/3} (J_3)^{-2/3} \left( S_{jk} S_{kl} - \frac{2}{3} J_2 \delta_{ij} \right) \] (A-18)

\[ \frac{\partial r}{\partial \sigma_{ij}} = \left( \frac{1}{2} \right)^{1/3} \left( \text{det} \ [S] \right)^{-2/3} \left( S_{jk} S_{kl} - \frac{2}{3} J_2 \delta_{ij} \right) \] (A-19)

\[ \frac{\partial q}{\partial \sigma_{ij}} = \frac{3}{2} q \ s_{ij} \] (A-20)

From Eqn. (A-2) the \( \frac{\partial \tilde{\eta}}{\partial \sigma_{ij}} \) and \( \frac{\partial \gamma}{\partial \sigma_{ij}} \) parameters still need to be found.

They are expressed as follows:
\[ \frac{\partial \tilde{\eta}}{\partial \sigma_{ij}} = \frac{\delta_{ij}}{3 \sigma_o} \] (A-21)

\[ \frac{\partial \gamma}{\partial \sigma_{ij}} = \left( \frac{3\sqrt{3}}{2 - \sqrt{3}} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \frac{1}{q \sin(3\theta)} \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta)}{2q} \ s_{ij} - \frac{3}{2q^2} S_{lk} S_{kj} \right) \] (A-22)
Please note that to satisfy the assumption of plastic incompressibility, the term \( \frac{\delta_{ij}}{3 \alpha_e} \) in Eqn. (A-21) is removed.

Substituting Eqns. (A-19) and (A-20) into Eq. (A-12):

\[
\frac{\partial \xi}{\partial \sigma_{ij}} = \frac{3 r^2 \left( q \left( \frac{1}{2} \right)^{1/3} \left( \det[S] \right)^{-2} \left( S_{jk} S_{kl} - \frac{2}{9} q^2 \delta_{ij} \right) \right) - r \left( \frac{3 \delta_{ij}}{2 q} \right)}{q^4}
\]  
(A-23)

Substituting Eqns. (A-15) into Eq. (A-23):

\[
\frac{\partial \xi}{\partial \sigma_{ij}} = \frac{(13.5 \det[S])^{2/3} \left( q \left( \frac{1}{2} \right)^{1/3} \left( \det[S] \right)^{-2} \left( S_{jk} S_{kl} - \frac{2 q^2 \delta_{ij}}{9} \right) \right) - (13.5 \det[S])^{1/3} \left( \frac{3 \delta_{ij}}{2 q} \right)}{q^{4/3}}
\]  
(A-24)

Substituting Eq. (A-24) into (A-10):

\[
\frac{\partial c_\eta}{\partial \sigma_{ij}} = \left( \frac{2 M_2 k \left( e^{-(\theta_0 - \bar{\theta})} \right)}{\pi \sqrt{1 - \xi^2} \left( e^{-(\theta_0 - \bar{\theta})} + 1 \right)^2} \right)
\]

\[
\times \left( (13.5 \det[S])^{2/3} \left( q \left( \frac{1}{2} \right)^{1/3} \left( \det[S] \right)^{-2} \left( S_{jk} S_{kl} - \frac{2 q^2 \delta_{ij}}{9} \right) \right) - (13.5 \det[S])^{1/3} \left( \frac{3 \delta_{ij}}{2 q} \right) \right)
\]

Substituting Eqns. (A-20), (A-21) and (A-22) into (A-2) the following is obtained:
\[
\frac{\partial f}{\partial \sigma_{ij}} = \frac{3s_{ij}}{2q} + \bar{\sigma}(\bar{\varepsilon}^p) \left[ \frac{27M_2 k e^{-k(\theta_0-\bar{\theta})} q^2 \left( S_{jk}S_{kl} - \frac{2q^2 \delta_{ij}}{9} \right) - 60.75 s_{ij} \text{det}[S]}{\pi \sqrt{1 - \xi^2 (e^{-k(\theta_0-\bar{\theta})} + 1)^2 q^5}} (\bar{\eta} - \eta_0) \right] c_{\theta}^s \\
+ \left( c_{\theta}^{ax} - c_{\theta}^s \right) \left( y - \frac{y^{m+1}}{m+1} \right) \left( 1 + \frac{1}{m} \right) - \bar{\sigma}(\bar{\varepsilon}^p) \left[ 1 - c_{\eta} (\bar{\eta} - \eta_0) \right] \left( c_{\theta}^{ax} - c_{\theta}^s \right) \left( 1 - \frac{1}{m} \right) \\
+ \frac{1}{m} \left( \frac{3\sqrt{3} (1 - y^m)}{q \sin(3\theta)(2 - \sqrt{3})} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta) s_{ij}}{2q} - \frac{3 s_{ik}s_{kj}}{2q^2} \right) 
\]

To satisfy the assumption of plastic incompressibility, the term \( \frac{\delta_{ij}}{3} \) is also removed. Therefore, a deviatoric associative flow rule is developed as follows:

\[
d\varepsilon^{pl} = d\lambda \left[ \frac{\partial f}{\partial \sigma_{ij}} \right] \tag{A-27}
\]
\[d\varepsilon^{pl} = d\lambda \left[ \frac{3s_{ij}}{2q} \right.\]

\[+ \bar{\sigma}(\varepsilon^p) \left[ \frac{27M_2 k e^{-k(\theta_0 - \bar{\theta})} q^2 \left( S_{jk}S_{ki} - \frac{2q^2 \delta_{ij}}{9} \right) - 60.75 s_{ij} \det[S]}{\pi \sqrt{1 - \xi^2} (e^{-k(\theta_0 - \bar{\theta})} + 1)^2 q^5} (\bar{\eta} - \eta_0) \right] \left[ c_{\theta^s} \right.\]

\[+ (c_{\theta^{ax}} - c_{\theta^s}) \left( \gamma - \frac{\gamma^{m+1}}{m + 1} \right) \left( 1 + \frac{1}{m} \right) - \bar{\sigma}(\varepsilon^p) [1 - c_{\eta}(\bar{\eta} - \eta_0)] \right] \left( c_{\theta^{ax}} - c_{\theta^s} \right) \left( 1 \right.\]

\[+ \frac{1}{m} \left( \frac{3\sqrt{3} (1 - \gamma^m)}{q \sin(3\theta)(2 - \sqrt{3})} \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \left( \frac{\cos(3\theta) s_{ij}}{2q} - \frac{3 s_{ik}s_{kj}}{2q^2} \right) \right.\]

The present formulation can be classified into an intermediate flow rule between the fully associated and non-associated flow rule.
APPENDIX B:  
DETERMINING TRUE STRESS-STRAIN CURVES BASED ON BRIDGMAN’S ROUND NOTCHED BARS USING DIGITAL IMAGING CORRELATION (DIC)
The Goal of this section is to determine the evolution of the minimum cross section “$\alpha$” (p2p5) and the change in radii (O1p2 & O2p5) shown in Figure B-1. Knowing this allows the determination of true stress at every time interval including after necking by using Bridgman solutions for round notched specimens.

Figure B-1: The cross-section of a notched specimen
The following points are defined as follows:

<table>
<thead>
<tr>
<th>Point/vector name</th>
<th>Coordinate system correspondence</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₁</td>
<td>(x₀, y₀)</td>
</tr>
<tr>
<td>O₂</td>
<td>(xᵢ, yᵢ)</td>
</tr>
<tr>
<td>p₁</td>
<td>(x₁, y₁)</td>
</tr>
<tr>
<td>p₂</td>
<td>(x₂, y₂)</td>
</tr>
<tr>
<td>p₃</td>
<td>(x₃, y₃)</td>
</tr>
<tr>
<td>p₄</td>
<td>(x₄, y₄)</td>
</tr>
<tr>
<td>p₅</td>
<td>(x₅, y₅)</td>
</tr>
<tr>
<td>p₆</td>
<td>(x₆, y₆)</td>
</tr>
<tr>
<td>O₁p₁</td>
<td>(x₁ - x₀, y₁ - y₀)</td>
</tr>
<tr>
<td>O₁q₁</td>
<td>(\left(\frac{x₁ + x₂}{2} - x₀, \frac{y₁ + y₂}{2} - y₀\right))</td>
</tr>
<tr>
<td>O₁p₂ (Radius 1)</td>
<td>(x₂ - x₀, y₂ - y₀)</td>
</tr>
<tr>
<td>O₁q₂</td>
<td>(\left(\frac{x₃ + x₂}{2} - x₀, \frac{y₃ + y₂}{2} - y₀\right))</td>
</tr>
<tr>
<td>O₁p₃</td>
<td>(x₃ - x₀, y₃ - y₀)</td>
</tr>
<tr>
<td>O₂p₄</td>
<td>(x₄ - xᵢ, y₄ - yᵢ)</td>
</tr>
<tr>
<td>O₂q₃</td>
<td>(\left(\frac{x₄ + x₅}{2} - xᵢ, \frac{y₄ + y₅}{2} - yᵢ\right))</td>
</tr>
<tr>
<td>O₂p₅ (Radius 2)</td>
<td>(x₅ - xᵢ, y₅ - yᵢ)</td>
</tr>
<tr>
<td>O₂q₄</td>
<td>(\left(\frac{x₅ + x₆}{2} - xᵢ, \frac{y₅ + y₆}{2} - yᵢ\right))</td>
</tr>
<tr>
<td>O₂p₆</td>
<td>(x₆ - xᵢ, y₆ - yᵢ)</td>
</tr>
</tbody>
</table>
Since \( \overrightarrow{O_1q_1} \perp \overrightarrow{q_1p_2} \) and \( \overrightarrow{O_1q_2} \perp \overrightarrow{q_2p_3} \)

\[
\therefore (\overrightarrow{O_1q_1}) \cdot (\overrightarrow{q_1p_2}) = 0 \text{ and } (\overrightarrow{O_1q_2}) \cdot (\overrightarrow{q_2p_3}) = 0
\]

The solution of this equation with respect to \((x_0, y_0)\) defines the change of \((x_0, y_0)\) with respect to the other parameters which are known at every time step. \((x_0, y_0)\) are read as follows:

\[
x_0 = \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2 + 2 y_1 a_1}{a_2} - \frac{2 y_2 a_1}{a_2}
\]

where,

\[
a_1 = \left(\frac{y_2}{2} - \frac{y_3}{2}\right) \left(\frac{y_2}{2} + \frac{y_3}{2}\right) + \left(\frac{x_2}{2} - \frac{x_3}{2}\right) \left(\frac{x_2}{2} + \frac{x_3}{2} - \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2}{2 x_1 - 2 x_2}\right)
\]

\[
a_2 = \left(\frac{y_3}{2} - \frac{y_2}{2}\right) + \left(\frac{x_3}{2} - \frac{x_2}{2}\right) \left(2 y_1 - 2 y_2\right) \cdot \frac{2 x_1 - 2 x_2}{2 x_1 - 2 x_2}
\]

\[
y_0 = \frac{\left(\frac{y_2}{2} - \frac{y_3}{2}\right) \left(\frac{y_2}{2} + \frac{y_3}{2}\right) + a_4 \left(\frac{x_2}{2} + \frac{x_3}{2} - \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2}{a_3}\right)}{\left(\frac{y_3}{2} - \frac{y_2}{2}\right) + \frac{a_4(2 y_1 - 2 y_2)}{a_3}}
\]

where,

\[
a_3 = 2 x_1 - 2 x_2
\]
\[ a_4 = \left(\frac{x_2}{2} - \frac{x_3}{2}\right) \]  \tag{B-6} 

Radius 1 reads as:

\[
R_1 = \left( x_1 - \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2 + 2 y_1 (a_5) - 2 y_2 (a_5)}{2 x_1 - 2 x_2} \right)^2 + \left( y_1 + \frac{a_5}{a_6} \right)^2 \]  \tag{B-7} 

where,

\[
a_5 = \left(\frac{y_2}{2} - \frac{y_3}{2}\right) \left(\frac{y_2}{2} + \frac{y_3}{2}\right) + \left(\frac{x_2}{2} - \frac{x_3}{2}\right) \left(\frac{x_2}{2} + \frac{x_3}{2} - \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2}{2 x_1 - 2 x_2}\right) \]  \tag{B-8} 

\[
a_6 = \left(\frac{y_3}{2} - \frac{y_2}{2}\right) + \left(\frac{x_2}{2} - \frac{x_3}{2}\right) \left(2 y_1 - 2 y_2\right) \]  \tag{B-9} 

The same argument goes for the left side of the notch \((x_f, y_f)\) and radius 2 read as follows:

\[
x_f = \frac{x_4^2 - x_5^2 + y_4^2 - y_5^2 + \frac{2 y_4 (b_1)}{b_2} - \frac{2 y_5 (b_1)}{b_2}}{2 x_4 - 2 x_5} \]  \tag{B-10} 

where,
\[
b_1 = \left(\frac{y_5}{2} - \frac{y_6}{2}\right) \left(\frac{y_5}{2} + \frac{y_6}{2}\right) + \left(\frac{x_5}{2} - \frac{x_6}{2}\right) \left(\frac{x_5}{2} + \frac{x_6}{2} - \frac{x_4^2 - x_5^2 + y_4^2 - y_5^2}{2x_4 - 2x_5}\right) \\
\text{(B-11)}
\]

\[
b_2 = \left(\frac{y_6}{2} - \frac{y_5}{2}\right) + \frac{\left(\frac{x_5}{2} - \frac{x_6}{2}\right)(2y_4 - 2y_5)}{2x_4 - 2x_5} \\
\text{(B-12)}
\]

\[
y_f = \frac{\left(\frac{y_5}{2} - \frac{y_6}{2}\right) \left(\frac{y_5}{2} + \frac{y_6}{2}\right) + b_4 \left(\frac{x_5}{2} + \frac{x_6}{2} - \frac{x_4^2 - x_5^2 + y_4^2 - y_5^2}{b_3}\right)}{\left(\frac{y_6}{2} - \frac{y_5}{2}\right) + \frac{b_4(2y_4 - 2y_5)}{b_3}} \\
\text{(B-13)}
\]

where,

\[
b_3 = 2x_4 - 2x_5 \\
\text{(B-14)}
\]

\[
b_4 = \left(\frac{x_5}{2} - \frac{x_6}{2}\right) \\
\text{(B-15)}
\]

Radius 2 reads as:

\[
R_2 = \sqrt{\left(\frac{x_4}{2} - \frac{x_5}{2} + \frac{y_4}{2} - \frac{y_5}{2} + \frac{2y_4(b_5)}{a_6} - \frac{2y_5(b_5)}{b_6}\right)^2 + \left(\frac{y_4}{b_6}\right)^2} \\
\text{(B-16)}
\]
where,

\[ b_5 = \left( \frac{y_5}{2} - \frac{y_6}{2} \right) \left( \frac{y_5}{2} + \frac{y_6}{2} \right) + \left( \frac{x_5}{2} - \frac{x_6}{2} \right) \left( \frac{x_5}{2} + \frac{x_6}{2} - \frac{x_4^2 - x_5^2 + y_4^2 - y_5^2}{2 x_4 - 2 x_5} \right) \] (B-17)

\[ b_6 = \left( \frac{y_6}{2} - \frac{y_5}{2} \right) + \left( \frac{x_5}{2} - \frac{x_6}{2} \right) \left( \frac{2 y_4 - 2 y_5}{2 x_4 - 2 x_5} \right) \] (B-18)

Since both radii are known as a function of 6 known points the minimum cross section “\( \alpha \)” is known and reads as follows:

\[ \alpha = \frac{1}{2} \sqrt{\left| \frac{a_5}{a_6} - \frac{b_5}{b_6} \right|^2} - \frac{1}{2} \sqrt{\left( |x_4 - c_1|^2 - \left| y_4 + \frac{b_5}{b_6} \right|^2 \right)} \]

\[ - \frac{1}{2} \sqrt{\left( |x_1 - c_2|^2 - \left| y_1 + \frac{a_5}{a_6} \right|^2 \right)} \] (B-19)

where,

\[ c_1 = \left( \frac{x_4^2 - x_5^2 + y_4^2 - y_5^2 + 2 y_4 (b_5) - 2 y_5 (b_5)}{b_6} \right) \] (B-20)
The changes in radii and minimum cross-sectional thicknesses are known at every time step. The user must track six points in the DIC. True stress strain is readily determined by Bridgman stress-strain solutions.
Matlab Code:

%Clearing commands

clc

format compact

clear all

syms x1 x2 x3 x0 y0 y1 y2 y3 xa ya x4 y4 x6 y6 xf yf

% --------------------------------------------------------------

% dealing with the three points on the right plane (p1,p2,p3)
oq1=[(x1+x2)/2-x0,(y2+y1)/2-y0];
q1p2=[x2-(x1+x2)/2,y2-(y1+y2)/2];
dot1x=oq1(1,1)*q1p2(1,1);
dot1y=oq1(1,2)*q1p2(1,2);
dot1=dot1x+dot1y;
new_x0=solve(dot1==0,x0);
oq2=[(x2+x3)/2-x0,(y2+y3)/2-y0];
q2p3=[x3-(x3+x2)/2,y3-(y3+y2)/2];
dot2x=oq2(1,1)*q2p3(1,1);
dot2y=oq2(1,2)*q2p3(1,2);
dot2=dot2x+dot2y;
% now solving for \( x_0, y_0 \) in terms of three points selected

\[
dot2_{\text{new}} = \text{subs}(\dot{2}, x_0, \text{new}\_x0);
\]

\[
\text{new}\_y0 = \text{solve}(\dot{2}_{\text{new}} == 0, y_0);
\]

\[
\text{las}\_x0 = \text{subs}(\text{new}\_x0, y_0, \text{new}\_y0);
\]

% now solving for radius in terms of selected points

\[
\text{radius\_x1} = x_1 - \text{las}\_x0;
\]

\[
\text{radius\_y1} = y_1 - \text{new}\_y0;
\]

\[
\text{radius} = [\text{radius}\_x1, \text{radius}\_y1];
\]

\[
\text{radius} = \text{norm}(\text{radius});
\]

% ---------------------------------

% now dealing with the other 3 points on the left plane (p4, p5, p6)

% same argument performed on the right plane points is done here

\[
\text{oq3} = [(x_4 + x_5)/2 - x_f, (y_5 + y_4)/2 - y_f];
\]

\[
\text{q3p5} = [x_5 - (x_4 + x_5)/2, y_5 - (y_4 + y_5)/2];
\]

\[
\text{dot3x} = \text{oq3}(1,1) \times \text{q3p5}(1,1);
\]

\[
\text{dot3y} = \text{oq3}(1,2) \times \text{q3p5}(1,2);
\]

\[
\text{dot3} = \text{dot3x} + \text{dot3y};
\]

\[
\text{new}\_x_f = \text{solve}(\text{dot3} == 0, x_f);
\]

\[
\text{oq4} = [(x_5 + x_6)/2 - x_f, (y_5 + y_6)/2 - y_f];
\]

\[
\text{q4p6} = [x_6 - (x_6 + x_5)/2, y_6 - (y_6 + y_5)/2];
\]

\[
\text{dot4x} = \text{oq4}(1,1) \times \text{q4p6}(1,1);
\]
dot4y = oq4(1,2)*q4p6(1,2);
dot4 = dot4x + dot4y;
dot4_new = subs(dot4, xf, new_xf);
new_yf = solve(dot4_new == 0, yf);
las_xf = subs(new_xf, yf, new_yf);
radius_2_x = x4 - las_xf;
radius_2_y = y4 - new_yf;
radius_2 = [radius_2_x, radius_2_y];
radius_2 = norm(radius_2);

% now determining the total length from point O1 to O2
length_tot = [x0 - xf, y0 - yf];
length_tot = norm(length_tot);

% now determining the minimum cross-sectional thickness
a = length_tot - radius_2 - radius;
a = subs(a, xf, las_xf);
a = subs(a, yf, new_yf);
a = subs(a, x0, las_x0);
a = subs(a, y0, new_y0);

% a is redefined as half thickness
a = 0.5 * a;

%%%%%%%
fprintf(' x0 =\n\n')
pretty(las_x0)
fprintf(' y0 =\n\n')
pretty(new_y0)
fprintf(' radius_1 =\n\n')
pretty(radius)
fprintf(' xf =\n\n')
pretty(las_xf)
fprintf(' yf =\n\n')
pretty(new_yf)
fprintf(' radius_2 =\n\n')
pretty(radius_2)
fprintf(' a =\n\n')
pretty(a)

% --------------------------------------------------------------
% material parameter input
load('force_sb.mat')
load('data_sb.mat')
% specifying all 6 points (x, y) coordinates
while (i < 14)
i=i+1;
end
x1=data_sb(i,1);
y1=data_sb(i,2);
x2=data_sb(i,3);
y2=data_sb(i,4);
x3=data_sb(i,5);
y3=data_sb(i,6);
x4=data_sb(i,7);
y4=data_sb(i,8);
x5=data_sb(i,9);
y5=data_sb(i,10);
x6=data_sb(i,11);
y6=data_sb(i,12);

gage_length_start=data_sb(1,4)-data_sb(1,2);
gage_length_step(i)=data_sb(i,4)-data_sb(i,2);
delta(i)=gage_length_step(i)-gage_length_start;
a_evolv(i)=abs(eval(a)).*10^{-3}; % in m
radius_evolv(i)=eval(radius).*10^{-3}; % in m
radius_evolv_2(i)=eval(radius_2).*10^{-3}; % in m

% to determine true stress-strain at every point
true_stress=0; % initially
a_initial=0.00254;
if true_stress <= 838.48

    eng_strain(i)=delta(i)./gage_length_start;
    true_strain(i)=log(1+eng_strain(i));
    eng_stress(i)=force_sb(1,i)./(pi.*a_initial.^2);
    true_stress(i)=eng_stress(i).*(1+eng_strain(i));
    true_plastic_strain(i)=true_strain(i)-true_stress(i)./204*10^-3;

else

    num(i)=force_sb(1,i)./(pi*a_evolv(i).^2);
    den(i)=(1+2*radius_evolv(i)/a_evolv(i))*log(1+a_evolv(i)/(2*radius_evolv(i)));

        %if radius_evolv(i)>=0 & radius_evolv_2(i)>=0

        stress_bridgman(i)=num(i)/den(i)
        plastic_strain(i)=2*log(abs(a_evolv(i))./0.00254)*-1;

        % end

scatter(plastic_strain,stress_bridgman)

end

end
APPENDIX C:
PLASTIC FLOW POTENTIAL DERIVATION FOR PROPOSED MODIFIED GTN MODEL
The plastic flow rule of the Gurson-Tvergaard-Needleman (GTN) model reads as follows:

$$F = q - \sigma_y \sqrt{(1 + q_3 f^*^2) - 2q_1 f^* \cosh\left(\frac{-3q_2 \sigma_m}{2\sigma_y}\right)} = 0$$  \hspace{1cm} (C-1)

The original GTN model assumes \(\sigma_y\) to follow the Von Mises criterion. Here we assume that the \(\sigma_y\) is expressed as follows:

$$\sigma_y = \sigma_y \left[ c_0^s + (c_0^{ax} - c_0^s) \left(\frac{m + 1}{m}\right) \left(\gamma - \frac{\gamma^{m+1}}{m + 1}\right) \right]$$  \hspace{1cm} (C-2)

where \(q, c_0^s, m, \gamma, c_0^{ax}\) are parameters defined in section 4.2.

The flow rule is reformulated into the following:

$$F = q - \sigma_y \left[ c_0^s + (c_0^{ax} - c_0^s) \left(\frac{m + 1}{m}\right) \left(\gamma - \frac{\gamma^{m+1}}{m + 1}\right) \right] \sqrt{(1 + q_3 f^*^2) - 2q_1 f^* \cosh\left(\frac{-3q_2 \sigma_m}{2\sigma_y}\right)} = 0$$  \hspace{1cm} (C-3)

Deriving \(F\) with respect to \(\sigma_{ij}\) we obtain the following:

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{\partial q}{\partial \sigma_{ij}} - \frac{3q_1 q_2 f^* \sinh\left(\frac{3q_2 \sigma_m}{2\sigma_0}\right) \delta_{ij}}{2 \sqrt{(1 + (q_1 f^*)^2) - 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_m}{2\sigma_0}\right)}}$$  \hspace{1cm} (C-4)

\[
\left[ c_0^s + (c_0^{ax} - c_0^s) \left(\frac{m + 1}{m}\right) \left(\gamma - \frac{\gamma^{m+1}}{m + 1}\right) \right] + \frac{\partial \gamma}{\partial \sigma_{ij}} \sigma_y \\
\left[ (c_0^{ax} - c_0^s) \left(\frac{m + 1}{m}\right) (1 - \gamma^m) \right] \\
\sqrt{(1 + q_3 f^*^2) - 2q_1 f^* \cosh\left(\frac{-3q_2 \sigma_m}{2\sigma_y}\right)}
\]
The terms \( \frac{\partial q}{\partial \sigma_{ij}} \) and \( \frac{\partial \gamma}{\partial \sigma_{ij}} \) can be expressed as follows:

\[
\frac{\partial q}{\partial \sigma_{ij}} = \frac{3}{2} q s_{ij}
\]  
\( (C-5) \)

\[
\frac{\partial \gamma}{\partial \sigma_{ij}} = \left( \frac{3\sqrt{3}}{2 - \sqrt{3}} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \frac{1}{q \sin(3\theta)} \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta)}{2q} s_{ij} - \frac{3}{2q^2} s_{ik} s_{kj} \right)
\]  
\( (C-6) \)

Substituting Eqn.’s (C-5) and (C-6) into (C-4) and assuming an associative flow rule the following plastic strain is developed:

\[
d\varepsilon_{ij}^{pl} = d\lambda \left( \frac{3s_{ij}}{2q} - \frac{3q_1 q_2 f^* \sinh\left(\frac{3q_2 \sigma_m}{2\sigma_0}\right)}{2\sqrt{1 + (q_1 f^*)^2} - 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_m}{2\sigma_0}\right)} \delta_{ij} \right) \left[ c^o_0 + (c^{ax}_0 - c^o_0) \left( \frac{m + 1}{m} \right) \left( \gamma - \frac{\gamma_m}{m + 1} \right) \right]
\]

\[
+ \left( \frac{3\sqrt{3}}{2 - \sqrt{3}} \right) \left( \frac{\tan(\theta - \frac{\pi}{6})}{\cos(\theta - \frac{\pi}{6})} \right) \frac{1}{q \sin(3\theta)} \left( \frac{\delta_{ij}}{3} + \frac{\cos(3\theta)}{2q} s_{ij} - \frac{3}{2q^2} s_{ik} s_{kj} \right) \sigma_y
\]

\[
\times \left[ (c^{ax}_0 - c^o_0) \left( \frac{m + 1}{m} \right) (1 - \gamma_m) \right] \sqrt{\left( 1 + q_3 f^* \right) - 2q_1 f^* \cosh\left(\frac{-3q_2 \sigma_m}{2\sigma_y}\right)}
\]

\( (C-7) \)
APPENDIX D:
HEAT-TREATMENT CERTIFICATIONS
Braddock Metallurgical Inc. - Daytona Certification

To:
UNIVERSITY OF CENTRAL FLORIDA
12424 RESEARCH PARKWAY
SUITE 300
ORLANDO FL 32826-3249

Purchase Order No.: 129499
Packing List No.: 1
Material: 4340

We certify that the listed Parts / Material were processed as follows:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Part Number / Part Name / Part Description</th>
<th>Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>FLG1 and FLG2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>RB1 and RB2</td>
<td>2.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inspect Type</th>
<th>Scale</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Number</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Requirements:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardness</td>
<td>HRC</td>
<td>32</td>
<td>34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results:

<table>
<thead>
<tr>
<th>Inspect Type</th>
<th>Scale</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Number</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness</td>
<td>HRC</td>
<td>32</td>
<td>33</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parts were preheated in furnace #543 at 1000°F ±10°F for 60 minutes then loaded into hardening furnace hot.
Parts were hardened in furnace #533 at 1525°F ±25°F for 60 minutes then oil quenched.
Parts were tempered in furnace #532 at 1100°F ±15°F for 120 minutes and air cooled to room temperature.
Parts Inspected: 3
PARTS WERE PROCESSED IN A MERCURY FREE ENVIRONMENT.
These parts comply with your purchase order and specification. All Heat Treating was conducted within the pyrometry requirements of AMS 2750E.
We certify that the listed Parts / Materials were processed as follows:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Part Number / Part Name / Part Description</th>
<th>Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>flat block</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>ROUND SPECIMANS</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Insp. Type</th>
<th>Scale</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Number</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Requirements:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardness</td>
<td>HRC</td>
<td>39.</td>
<td>41.</td>
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<td></td>
</tr>
</tbody>
</table>

Results:

<table>
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<tr>
<th>Insp. Type</th>
<th>Scale</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Number</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness</td>
<td>HRC</td>
<td>39.</td>
<td>40.</td>
<td></td>
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</tr>
</tbody>
</table>

Order was heat treated to 39/41 HRC in accordance with the requirements of the purchase order.

- Pre-heat: Equipment ID: 631; 1200°F (±25°F) for 120 minutes.
- Hardening: Equipment ID: 630; 1525°F (±25°F) for 60 minutes. .40% Atmosphere.
- Quench
- Tempering: Equipment ID: 632; 950°F (±15°F) for 120 minutes. Air cool

Parts Inspected: 4

These parts comply with your purchase order and specification. All Heat Treating was conducted within the pyrometry requirements of AMS 2750E.
REFERENCES


Bai, Y. (Massachusetts Institute of Technology ;2008). "Effect of loading history on necking and fracture [PhD thesis]."


Fansi, J. (2013). Ecole nationale supérieure d'arts et métiers-ENSAM; Université de Liège.


