DIGITAL IMAGE PROCESSING BY THE TWO-DIMENSIONAL DISCRETE FOURIER TRANSFORM METHOD

BY

LYMAN F. JOELS, JR.
B.S.E.E., University of Colorado, 1961

RESERACH REPORT

Submitted in partial fulfillment of the requirements
for the degree of Master of Science
in Engineering in the Graduate Studies Program of
Florida Technological University, 1973

Orlando, Florida

Table of Contents

Section

List of Figures

1.0 IMAGE PROCESSING

A. Introduction

The objective of image processing is to perform some type of process on an image, such as a photographic transparency or X-ray, so that certain image qualities such as brightness and contrast are enhanced, while undesirable qualities such as blurriness and distortion are attenuated. In the past decade high-speed general-purpose digital computers have reached the stage of development where digital processing of images has become feasible. Formerly, image processing was performed almost entirely by optical means which required elaborate and accurate processing equipment, as well as excessive preparation and set-up time. In contrast, image processing by digital computer requires very little setup time, has increased accuracy, greater operational flexibility, and has the added advantage of performing nonlinear mathematical operations with relative ease.

The theory of image processing by digital computers can best be explained by first discussing the basic theory of optical image processing. Optical image processing is a technique in which two-dimensional image functions are illuminated by a light beam and processed by a system of lenses, photographic transparencies, and other optical elements. A typical optical image processing system is described by Andrews and Pratt [1]. This system which is comprised of lenses, photographic transparencies, and a light source is shown on Figure 1-1. A transparency of the original image in the x_1, y_1 plane with transmittance $f(x_1, y_1)$ is illuminated by a light beam so that the electric field amplitude of the light at the input plane is proportional to $f(x_1, y_1)$. The first spherical lens produces an image of the transparency in the

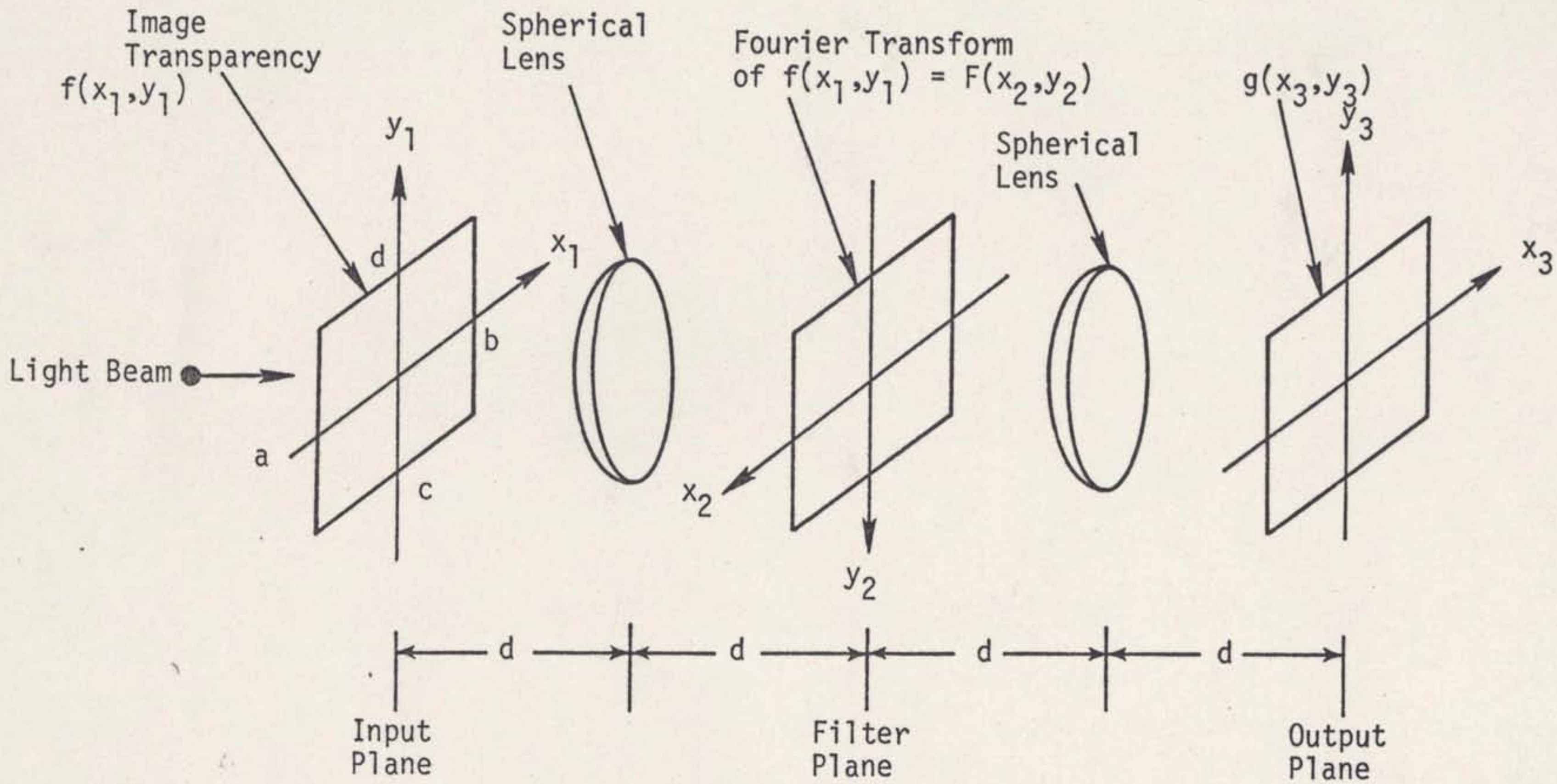

Figure 1-1. Optical Image Processing System

filter plane. The light electric field amplitude, $F(x_2, y_2)$, at the filter plane can be described by the two-dimensional Fourier transform relation

$$F(x_2,y_2) = \int_c^d \int_a^b f(x_1,y_1)e^{j\frac{2\pi}{\lambda d}(x_1x_2+y_1y_2)}\,dx_1dy_1 \qquad (1)$$

where λ is the wavelength of the light illuminating the transparency and d is the focal length of the lens. This relation can also be written as

$$F(u,v) = \int_c^d \int_a^b f(x_1,y_1)e^{j(ux_1+vy_1)}\,dx_1dy_1 \qquad (2)$$

by defining u and v as

$$u = \frac{2\pi}{\lambda d}x_2 \text{ and } v = \frac{2\pi}{\lambda d}y_2 \qquad (3)$$

The variables u and v are called the spatial frequencies in the Fourier transform plane having dimensions of cycles per unit length. The relationship given by equation (2) then permits one to describe an image in terms of its spatial frequencies which result when a Fourier transform is made of the image function. A filter transparency with a transmittance function, $H(u,v)$, can be inserted at the filter plane to modify the amplitude and phase of the light there, and hence modify the spatial frequencies. A second spherical lens, as shown in Figure 1-1, will perform an inverse Fourier transform to return to the spatial domain. The electric field distribution, $g(x_3, y_3)$, in the output plane is then the inverse Fourier transform of the $H(u,v)$ $F(u,v)$ product and represents the processed image.

A major problem experienced with optical processing systems is the construction of filter transparencies. One technique is to let density variations on a transparency represent amplitude functions, and to let thickness variations represent phase functions. Unfortunately, variable thickness filters are rather difficult to construct. Other types of filter transparencies require relatively complex optical systems.

The operation performed by a lens in the optical image processing system can be performed mathematically by a digital computer since the operation can be described by a two-dimensional Fourier transform function. A block diagram of a digital image processing system is shown on Figure 1-2. The image to be processed is scanned by some type of scanner which converts the pictorial information to a two-dimensional continuous analog signal, f(x,y), which represents the gray scale variations of the image. The analog signal is then digitized by an analog to digital converter and stored. The digitized image data is represented by f(m,n) and is nothing more than a two-dimensional number array with elements proportional to the image spatial gray scale values. We will restrict ourselves to a square image so that the f(m,n) number array will always be an NxN array with the m and n integer denoting the array row and column numbers, respectively. A two-dimensional discrete Fourier transform of the f(m,n) array is represented by F(k,l), which is also an NxN array that represents the spatial frequency components of the image in the spatial frequency domain. Some type of mathematical operation is then performed in the frequency domain to alter the original image. This operation, denoted by H(k,l), can be a spatial frequency filter which alters the frequency and magnitude of the spatial frequencies, or it can be some other mathematical operation such as a data correlation operation. The ease with which a linear or nonlinear mathematical operation can be performed in the spacial frequency domain is what makes digital image processing so attractive. An inverse two-