

ERROR RATES IN NARROW-BAND DIGITAL FM SYSTEMS  
OPERATING IN VARIOUS  
INTERFERENCE ENVIRONMENTS

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B.S.E.E., University of Florida, 1972

RESEARCH REPORT

Submitted in partial fulfillment of the requirements  
for the degree of  
Master of Science in Engineering  
in the  
Graduate Studies Program  
of  
Florida Technological University

Orlando, Florida  
1975

172776

## ACKNOWLEDGEMENTS

The author is deeply indebted to Dr. C. Lowery for his invaluable guidance in the conduct of the work reported here. More importantly, the author feels that without the knowledge gained from Dr. Lowery's "Random Processes" and "Telecommunications" courses the present paper would not have been possible.

In addition, the author wishes to express his gratitude to the Southeast Regional Data Center at Florida Atlantic University, and specifically to Dr. Robert Cain of that institution, for the use of their computer facilities during the course of this investigation.

Finally, a word of thanks is due Dr. Brian Petrasko for his encouraging advisement during the planning stages of the author's graduate curriculum which is herein culminated.

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Hialeah, Florida

August, 1975



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## I. INTRODUCTION

Although the literature on error rates in Digital F.M. Systems dates back to 1954 [1], the analyses have dealt almost exclusively with error rates due to on-channel (narrow-band) noise, [1]-[7]; and in most cases, [1], [5]-[7], consideration was given only to the performance of binary (2-level) systems.

Cahn [2] appears to have been the first to consider multilevel systems in the presence of noise, although he was concerned with Phase Modulation (as opposed to Frequency Modulation) Systems. Years later, Salz [3] reported on the performance of Multilevel Digital F.M. Systems in the presence of noise, bringing to light the merits of such systems. A more rigorous theoretical investigation followed, by Mazo and Salz [4], which gave the probability of receiver error in terms of the carrier-to-noise ratio and the number of levels in the system. In a parallel effort, Klapper [5] investigated the effects of center-frequency (carrier) shifts and considered the broad-band case. Although his analysis was general, only results for binary systems were presented.

In later studies, Bobilin and Lindenlaub [6] have optimized transmitter frequency deviation, system filter shape, and system bandwidth with respect to the tradeoffs between additive noise and distortion effects, and between spike (FM clicks) and nonspike noise components of the discriminator output. In addition, Prapinmongkolkarn et. al. [7] have considered error rates under fading conditions in Binary Digital F.M. Systems with various types of predetection diversity combining.





























































































































































































































